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## CHAPTER I

### DEFINITIONS

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repeated 8 times to measure the length of the room, and the ratio of the length of the room to the unit of measure is represented by the number 8.

If a box has been found to contain 30 doz. of eggs, the unit, 1 doz. eggs, has been counted 30 times to measure the quantity of eggs.

If I buy 3 qt. of milk, the milkman fills and empties the unit of capacity, his one-quart measure, 3 times in order to measure the quantity of milk.

5. In the above instances the wholes to be measured are: the length of the room, the eggs in the box, and the milk sold. The units of measure are 2 ft., 1 doz., and 1 qt. The numbers telling how many of these units are needed are 8, 30, and 3.

The unit of measure may itself be, and in exact computation is, measured or compared with some other unit, for convenience called the *primary* unit; that is, it may be a part or a multiple of such unit.

6. \$1, 1 in., 1 lb., are all primary units. A five-dollar bill, a two-pound weight, a four-inch measure, are examples of *derived* units, because the *primary* units, \$1, 1 lb., 1 in., are repeated 5, 2, and 4 times respectively, to give the derived units.

If a quantity of chestnuts be counted into groups of 4 or 5, the derived units, 4 or 5 chestnuts, and the number of these groups or derived units measures the whole quantity of chestnuts. Similarly, eggs counted by 4's and 6's, and apples by 3's or 5's, are examples of the use of the derived unit. The primary units are 1 egg and 1 apple, while 4 eggs, 6 eggs, 3 apples, and 5 apples are derived units.

7. With reference to 1 ct., \$1 is a derived unit, and 100 is the *number* expressing \$1 in terms of the primary unit, 1 ct.

Similarly, 1 wk. is a derived unit, and 7 is the number expressing 1 wk. in terms of the primary unit, 1 da.

In  $\frac{3}{4}$  ft., the primary unit of reference is 1 ft. The foot is divided into 4 equal parts, one of which is the derived unit of measure. The number 3 shows how many of these derived units make up the given length.

### Exercise 1

1. Name three units of length used to measure short distances, and state the number of times each unit must be repeated to make the next larger.

2. What unit of length is used in stating the distance between two cities?

3. Name instances in which 1 sec. is used as the unit of time. 1 min. 1 hr. 1 da. 1 mo. 1 yr. State how often each of these units must be repeated to make the next larger.

4. What is the *prime* standard unit for money value? For weight? For area? For length? For time? For volume?

5. What unit of area is used to convey a definite idea of the size of a farm? Of a country?

6. What unit is used to measure wood?

7. With what unit of capacity is milk measured? Kerosene?

8. What unit is used to measure a quantity of strawberries? Potatoes? Why are these convenient units for the purpose?

9. State at least three reasons why the bushel would be an inconvenient unit to measure strawberries.

10. Early in the season strawberries are sold in pint boxes; later, in quart boxes. Explain why different units of capacity are chosen.

11. Why is the pint box chosen as the unit to measure red raspberries in preference to the quart?

12. Name different quantities which are weighed and sold by the lb. By the oz. By the T.

13. Give instances in which the following units are used: 1 sheet, 1 quire, 1 doz.

14. In each of the following quantities name the units and give the ratio of each quantity to its primary unit: 5 ft., 4 hr., 6 sq. in., 7 qt., 365 da., 12 oz.

15. What is the quantity which contains the unit 4 times, when the unit is 6 in.? 9 hr.? 8 yr.?

16. If the unit is \$4, and this unit is repeated 6 times, what quantity will be produced?

17. If the ratio of the size of a farm to the unit of area, 8 A., is equal to 6, what is the size of the farm?

18. In the following examples, what are the quantities which contain their respective units the given number of times?

UNITS OF MEASURE	NUMBERS
\$2	6
8 qt.	2
7 da.	3
5 hr.	4
1 doz.	15
4 in.	4

19. Name the primary units of measure, name in two ways the derived units, and state the number of derived units which measure these quantities:  $\$ \frac{7}{10}$ ,  $\frac{5}{6}$  ft.,  $\frac{2}{3}$  yd.,  $\frac{2}{5}$  of a dime,  $\frac{3}{7}$  of a wk.,  $\frac{4}{5}$  of a da.,  $\frac{3}{4}$  of a doz. eggs.

20. Name the coin which gives the derived unit of value in each of the following:  $\$ \frac{9}{10}$ ,  $\$ \frac{3}{4}$ ,  $\$ \frac{17}{20}$ ,  $\$ \frac{23}{50}$ , \$10, \$5, \$20,  $\frac{3}{5}$  of a nickel,  $\frac{1}{2}$  dime,  $\frac{4}{5}$  of a quarter of a dollar,  $\frac{3}{2}$  of half a dollar,  $\frac{2}{3}$  of half a dollar,  $\frac{23}{50}$  of half a dollar.

21. The following quantities contain their respective units how often?

QUANTITY	UNIT
21 da.	7 da.
24 hr.	8 hr.
1 gal.	1 qt.
1 min.	1 sec.
10 dimes	2 dimes
\$18 worth of hats	\$3 for 1 hat
30 ct. worth of milk	6 ct. a qt.

22. What is the unit of measure in reckoning population? How many of these units give the population of the town in which you live?

23. What is the number of times each of the following units must be repeated to make the next higher unit: 1 in., 1 ft., 1 ct., 1 dime, 1 da., 1 hr., 1 qt.?

24. How many times must the following units of measure be repeated to make 3 ft.: 2 in., 3 in., 4 in., 6 in., 9 in., 12 in.?

25. State how each of the following units may be derived from the next higher: 1 ft., 1 in., 50 ct., 25 ct., 1 da., 1 min., 1 qt., and 1 pt.

26. A quantity of cherries is measured by using as the unit as many cherries as will fill a dish holding 3 qt.; 9 of these dishes are filled. How many qt. are there in the whole quantity?

27. At 30 bu. to the A., how many bu. would there be on 10 A.? What is the unit here? What gives this particular unit? If 10 bu. to the A., how many A. to produce an equal quantity?

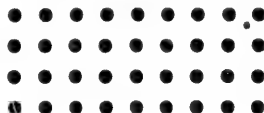
28. What coin is equal in value to 10 of the unit 1 ct.? 100 of the unit 1 ct.? 10 of the unit 1 dime?

29. What coin is equal to 10 of the unit 1 nickel? 5 of the unit 1 nickel? 20 of the same unit?

30. How many ct. are there in 5 of the unit \$1? 6 of the unit \$1? 10 of the unit \$1?

31. What coin is equal in value to 500 of the unit 1 ct.? 100 of the unit 1 nickel?

32. What coin is equal to one-tenth of a five-dollar gold piece?



8. In the preceding diagram we have 36 dots, signifying 36 units of any kind, arranged in 4 rows of 9 dots each, and at the same time 9 rows of 4 dots each. Hence we think of 36 as equal to 4 times 9 or 9 times 4. Arrange the dots to show that 36 is equal to  $3 \times 12$  or  $12 \times 3$ , and also to  $2 \times 18$  or  $18 \times 2$ .

4 and 9 are called *factors* of 36, and 36 is called the *product* of 4 and 9.

This illustrates a law of great importance in Arithmetic.

Thus we think of 24 as equal to  $2 \times 12$  or  $12 \times 2$ ,  $3 \times 8$  or  $8 \times 3$ ,  $4 \times 6$  or  $6 \times 4$ .



9. If in the above arrangement we think of each dot as representing \$1, then the diagram shows that  $\$12 \div \$2 = 6$ .

What other measurement is shown by the *same* arrangement?

### Exercise 2

1. Arrange 30 dots in rows in as many ways as you can, and express the results as in the preceding paragraph.

2. Express the following numbers as products in two ways: 6 (i.e.  $2 \times 3$  or  $3 \times 2$ ), 8, 10, 15, 21, and 35.



3. Express the following numbers as products in as many ways as possible, and arrange the products in corresponding pairs: 12, 16, 20, 28, 42, and 60.

4. Give all the factors of 32, 40, and 48.

5. Place dots to show the measurement of \$ 20 by a \$ 5 unit. What other measurement does it show ?

6. Place dots to show the measurement of \$ 24 by a \$ 2 unit; a \$ 3 unit; a \$ 4 unit. What other measurements are shown ?

7. What is the price of 6 yd. of cheese-cloth at 8 ct. a yd. ?

Explain each of these statements :

The whole price = 6 (8 ct.).

The whole price = 8 (6 ct.).

8. Show that 10 units of \$ 5 each is equal to 5 units of \$ 10 each.

9. If a line 3 ft. long is repeated 6 times to measure the length of a room, how long is the room ? How often would a line 6 ft. long have to be repeated to measure the room ?

10. How many apples will be required to make 7 rows with 9 apples in each row ? What is the unit of measurement ? What other convenient unit might be used ? What would be the ratio of the whole quantity to this unit ?

11. Draw a straight line 36 in. long, cut strips of paper respectively 6 in., 7 in., 8 in., 9 in., 10 in., 11 in., and 12 in. long. Measure along the line 3 times with each of these strips of paper.

Use a yardstick divided into in. to measure your results, and prove the following :

$$3 \times 6 \text{ in.} = 18 \text{ in.}; 3 \times 9 \text{ in.} = 27 \text{ in.};$$

$$3 \times 7 \text{ in.} = 21 \text{ in.}; 3 \times 10 \text{ in.} = 30 \text{ in.};$$

$$3 \times 8 \text{ in.} = 24 \text{ in.}; 3 \times 11 \text{ in.} = 33 \text{ in.};$$

$$3 \times 12 \text{ in.} = 36 \text{ in.}$$

12. Draw a line 36 in. long. Make a 3-in. measure. Measure along the line respectively 6, 7, 8, 9, 10, 11, and 12 times, and prove that:

$$6 \times 3 \text{ in.} = 18 \text{ in.}; \quad 9 \times 3 \text{ in.} = 27 \text{ in.};$$

$$7 \times 3 \text{ in.} = 21 \text{ in.}; \quad 10 \times 3 \text{ in.} = 30 \text{ in.};$$

$$8 \times 3 \text{ in.} = 24 \text{ in.}; \quad 11 \times 3 \text{ in.} = 33 \text{ in.};$$

$$12 \times 3 \text{ in.} = 36 \text{ in.}$$

13. What quantities are measured by the following:  $6 \times 4$  in.,  $4 \times 6$  in.,  $5 \times \$8$ ,  $8 \times \$5$ ,  $10 \times 5$  pears,  $5 \times 10$  pears?

14. What two units of length each longer than 4 in. can be used to measure a line 35 in. long? State in each case the ratio of the length of the whole line to each unit.

15. What are the convenient units of money to pay a debt of \$35? \$80?

16. What are convenient units to pay debts of 75¢? 15¢? 34¢? 87¢?

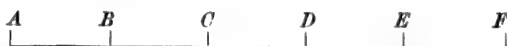
17. A fruit dealer sells apples at the rate of 3 for 5¢. What is the unit to measure his apples? What is the unit of value? How many units are there in 24 apples? What is their selling price?

18. Bananas are sold at the rate of 4 for 5¢. What is the measuring unit for the bananas? What is the unit of value? How many units of value in 20 bananas? 36 bananas? What is their value?

19. Oranges are sold at 20¢ a doz. What are the two measuring units?

20. If A can do a piece of work which is represented by 36 units in 9 da., how much will he do in 1 da.?

21. If a piece of work is represented by 60 units and A can do 5 units in one da., in how many da. can he do the entire work?



22.  $AB$  is a line which represents any primary unit of measure, and  $AC$ ,  $AD$ ,  $AE$ , and  $AF$  are derived units. What part is the

primary unit  $AB$  of each of the derived units? What is the ratio of each derived unit to the primary unit? If  $AF$  is the primary unit, what would  $AB$  be? What is the ratio of the derived unit  $AF$  to the derived unit  $AD$ ? Of  $AD$  to  $AF$ ? Of  $AC$  to  $AE$ ? Of  $AE$  to  $AC$ ? Of  $AF$  to  $AC$ ? Of  $AC$  to  $AF$ ?

23. One field contains 7 units of area, and a second field contains 9. What is the ratio of the area of the first field to the second? Of the second field to the first? Illustrate by a drawing.

24. The distance from  $A$  to  $B$  is divided into 5 parts of 3 mi. each, and that from  $A$  to  $C$  into 6 parts of 3 mi. each. What is the ratio of the distance  $AB$  to  $AC$ ? Of  $AC$  to  $AB$ ? Illustrate your answer by a diagram.

25. The money in my purse is measured by the number 8 and the unit \$5. I owe a debt measured by the number 6 and the unit \$5. What is the ratio of the debt to the money in my purse? What is the ratio of the money in my purse to the debt? How much shall I have left after paying my debt?

26. If the amount of work required to dig a trench 800 yd. long is represented by 40 units, what does 1 unit represent?

## CHAPTER II

### NUMERATION AND NOTATION

**10.** Numeration is counting, or the expression of number in words.

The ordinary system of numeration is the *Decimal System*, so called because it is based on the number ten.

**11.** The names of the first group of numbers in regular succession are: one, two, three, four, five, six, seven, eight, nine.

Other number-names are: ten, hundred, thousand, million, billion, trillion, etc.

**12.** The number one applied to any unit denotes a quantity which consists of a single unit of the kind named.

The number two applied to any unit denotes a quantity which consists of one such unit and one unit more.

The number three applied to any unit denotes a quantity which consists of two such units and one unit more.

And so on with the numbers four, five, six, seven, eight, nine; applied to any unit they denote quantities increasing regularly by one such unit with each successive number.

**13.** The number next following nine is ten, which applied to any unit denotes a quantity consisting of nine such units and one unit more.

Counting now by ten units at a time, as before we counted

by single units, we get the numbers ten, twenty, thirty, forty, . . ., ninety.

The names of the numbers between ten and twenty are, in order: eleven, twelve, thirteen, fourteen, . . ., nineteen.

The names of the numbers between twenty and thirty, thirty and forty, . . ., are formed by placing the names of the numbers one, two, three, . . ., nine, in order after twenty, thirty, . . ., ninety.

**14.** The number hundred applied to any unit denotes a quantity which consists of ten ten-units.

Counting now by a hundred units at a time, as before we counted by single units, we get the numbers one hundred, two hundred, . . ., nine hundred.

The names of the numbers between one hundred and two hundred, two hundred and three hundred, . . ., are formed by placing the names of the numbers from one to ninety-nine in regular succession after one hundred, two hundred, . . ., nine hundred.

**15.** The number thousand applied to any unit denotes a quantity which consists of ten hundred-units.

Counting now by a thousand units at a time, as before we counted by single units, we get the numbers one thousand, two thousand, . . ., nine thousand, ten thousand, eleven thousand, twelve thousand, . . ., twenty thousand, . . ., one hundred thousand, . . ., two hundred thousand, . . ., nine hundred and ninety-nine thousand.

The names of the numbers between one thousand and two thousand, two thousand and three thousand, . . ., are formed by placing in order the names of the numbers from one to nine hundred and ninety-nine, — the numbers preceding a

thousand, — after one thousand, two thousand, . . ., nine hundred and ninety-nine thousand.

**16.** The number million applied to any unit denotes a quantity which consists of a thousand thousand-units.

The number billion applied to any unit denotes a quantity which consists of a thousand million-units.

The number trillion applied to any unit denotes a quantity which consists of a thousand billion-units.

**17.** The number tenth applied to any unit denotes that quantity of which ten make up the unit.

The number hundredth applied to any unit denotes that quantity of which ten make up one tenth of the unit.

Consequently, one hundred of the hundredths of any unit make up that unit.

The number thousandth applied to any unit denotes that quantity of which ten make up the one hundredth of the unit.

Consequently, one thousand of the thousandths of any unit make up that unit; and so on.

**18.** We count by the tenth of a unit at a time, as before we counted from one to nine by a single unit each time; thus: one tenth, two tenths, . . ., nine tenths.

We count by a hundredth of a unit at a time, as before we counted from one to ninety-nine by a single unit each time; thus: one hundredth, two hundredths, . . ., ninety-nine hundredths.

We count the thousandths of a unit, from one to nine hundred and ninety-nine of the same in like manner as we count thousands of the unit, and so on.

**19. Notation** is the art of expressing numbers by means of certain number symbols called numerals or figures.

**20.** The *Arabic Numerals*, styled also *Figures*, are

0, 1, 2, 3, 4, 5, 6, 7, 8, 9,

denoting naught, one, two, three, four, five, six, seven, eight, nine respectively. The first of these is called *naught*, cipher, or zero; the remaining nine are called *digits*. By means of these numerals and a dot called the *decimal point*, we can write down any number expressed decimally. The method of doing so may be described as follows:

A figure immediately to the left of the decimal point denotes so many single units.

A figure immediately to the left of the single-units figure denotes so many tens of the units, while a figure immediately to the right of the single-units figure denotes so many tenths of the unit.

Figures to the left of the tens-figure, taking them in order from right to left, denote so many hundreds of the unit, so many thousands of the unit, so many ten-thousands of the unit, so many hundred-thousands of the unit, so many millions of the unit, etc.

Figures to the right of the tenths-figure, taking them in order from left to right, denote so many hundredths of the unit, so many thousandths of the unit, so many ten-thousandths of the unit, so many hundred-thousandths of the unit, so many millionths of the unit, etc.

The function of the decimal point is to mark the place of the standard unit when the quantity is measured.

**21.** Instead of speaking of "the number denoted by" 5, 75, or 375, we may for brevity speak of the number 5, 75, or 375.

22. Consider the following quantities:

9	yd.
59	"
259	"
3259	"
43259	"
843259	"
59.7	"
59.76	"
59.761	"
59.7613	"
59.76132	"

9 denotes 9 of the unit one yd.

5 " 5 " " " ten yd.

2 " 2 " " " one hundred yd.

3 " 3 " " " one thousand yd.

4 " 4 " " " ten thousand yd.

8 " 8 " " " one hundred thousand yd.

7 " 7 " " " one tenth of a yd.

6 " 6 " " " one one-hundredth of a yd.

1 " 1 " " " one one-thousandth of a yd.

3 " 3 " " " one ten-thousandth of a yd.

2 " 2 " " " one one-hundred thousandth of a yd.

23. The number 8 always denotes 8 of the unit; 86 denotes 8 of the ten-unit or 80 of the unit and 6 of the unit, and is read *eighty-six* of the unit.

865 denotes 8 of the *hundred*-unit and 6 of the *ten*-unit and 5 of the unit; *i.e.* 800, 60, and 5 of the unit, and is read *eight hundred and sixty-five* of the unit.

Thus, the numbers 8, 86, 865 always denote eight, eighty-



six, eight hundred and sixty-five respectively, the position of the figures in each case giving the *unit*. For example:

In the number 865,865,865 each 865 is read eight hundred and sixty-five, the difference being in the unit only; 865 of the million-unit, 865 of the thousand-unit, and 865 of the one-unit, which is the primary unit of reference.

This number is read eight hundred and sixty-five million eight hundred and sixty-five thousand eight hundred and sixty-five.

### Exercise 3

1. Express in words the numbers given in Exercises 11, 13, 19, 20, and 21.

Express in words:

2. 3,409,035; 42,590,709; 6,003,040.

3. 20,394,678; 4,007,890; 8,000,006; 70,001,002.

### Exercise 4

Express in words:

1. .7 ton; .64 ton; .643 ton.    3. 9.403 hr.; 29.04 min.; .09 sec.

2. 4.92 lb.; 8.09 lb.; 2.734 lb.    4. 7.456 A.; 6.7985 A.

5. 8452.69 sq. mi.; 21.4394 A.

6. Express in words the numbers given in Exercises 14 and 24.

### Exercise 5

Write in figures:

1. Three hundred and forty-nine; eight thousand four hundred and sixty-nine; nine thousand five hundred and seventy.

2. Twenty-nine thousand one hundred and thirty-four; fifty thousand eight hundred and seventy-six; seventy-eight thousand three hundred.

3. Nine hundred and fifty-two thousand seven hundred and forty; six hundred and forty-nine thousand nine hundred and five; nine hundred thousand eight hundred and sixty-four.

4. One hundred and sixty-eight thousand six hundred and eighteen; three hundred and twelve thousand seven hundred and forty-two; four hundred and sixty-one thousand eight hundred and twenty-one.

5. Seven hundred and four thousand and thirty; three hundred thousand two hundred and four; one hundred thousand and fifty.

6. Five million two hundred and ninety-five thousand three hundred and three.

7. Sixty-four million seven hundred and eight thousand one hundred and thirty-four.

8. Seventy-eight million four thousand and eighty-five.

9. Six million six thousand and six.

10. What is the value of six thousand units of \$4 each? Of six million units of \$5 each?

### Exercise 6

Express in figures:

1. Five tenths; two, and sixty-seven hundredths; nine hundredths; four thousandths.

2. Nine hundred and twelve thousandths; seven hundred and four thousandths; five thousand four hundred and sixteen ten thousandths.

3. Four hundred and fifty-three, and four hundredths; four hundred and fifty, and one hundred and twenty-six thousandths.

4. Nine hundred and six thousandths; nine hundred and six ten thousandths; twenty, and forty-five thousandths.

5. Seventeen, and seven ten thousandths; three hundred and three, and nine hundred and nine ten thousandths.

## Exercise 7

1. Name each unit in 346 yd. and state the number of units of each kind.

2. In question 1, what is the ratio of each *unit* to the next unit to the right of it?

3. What part is each unit of the next unit to the left?

4. What quantity expressed in single units is indicated by 3 in question 1? By 4? By 6?

5. Express in words 3.46 feet. Name each unit.

6. Fill in the blanks: \$ 679 is equal to 6 units of —, 7 units of —, and 9 units of —.

7. Write as one number, 8 of the *hundred-unit*, 5 of the *ten-unit*, and 6 of the *one-unit*.

8. Name the two chief units in 843,294 and state the number of units of each kind. Express the number in words.

9. Write the name of each of the six units in \$ 294,785 and the number of each unit.

10. In the quantity \$ 666,666, which 6 represents the largest sum of money? Which the smallest?

11. Write as one number 67 of the thousand-unit and 413 of the one-unit.

12. Write as one number 5 of the million-unit, 463 of the thousand-unit, and 768 of the one-unit.

13. Write as one number 349 of the thousand-unit and 258 of the one-unit.

14. Write as one number 365 of the million-unit, 829 of the thousand-unit, and 604 of the one-unit.

15. Which is the largest unit used in giving the population of the largest city in the United States? The length of the largest river? The height of the highest mountain? The area of the largest state? The area of the smallest state?

## THE ROMAN NOTATION

**24.** The Arabic Notation is the one in general use. It was introduced into Europe by the Arabs. The system of notation which was used among the Romans is now used only to denote the chapters and sections of books, etc.

**25.** The following letters are used to denote numbers, and their values are written below:

I.	V.	X.	L.	C.	D.	M.
1.	5.	10.	50.	100.	500.	1000.

**26.** The numbers 6, 8, 15, 20 are represented thus:

VI. VIII. XV. XX.

Hence if a character in the Roman Notation be followed by another of equal or less value, the number denoted by the expression is equal to the *sum* of the simple values.

**27.** The numbers 4, 9, 40, and 90 are represented by IV., IX., XL., XC.

Hence if a character in the Roman Notation is followed by one of greater value than itself, the number denoted by the expression is the *difference* of their simple values.

**28.** Express 1896 in Roman numerals.

$$1896 = 1000, 800, 90, \text{ and } 6.$$

$$1000 = M.$$

$$800 = DCCC.$$

$$90 = XC.$$

$$6 = VI.$$

$$\therefore 1896 = MDCCCXCVI.$$

Hence to write any number in Roman numerals, separate the number into its different parts, and write down the parts in order, beginning at the left.

**Exercise 8**

Write in Roman numerals :

1. 14, 25, 54, 89, 99.
2. 178, 304, 871, 982, 999.
3. 1204, 1590, 1756, 1876, 1895.

Write in figures :

4. XLVI., LXXIX., XCIV., LXXXIII.
5. XCIX., CXXXIX., CLX.
6. DLIV., MDCII., MDCCCXIX., MXC.

## CHAPTER III

### ADDITION

**29.** Let the length of a room be measured by the parts, 2 ft., 3 ft., 4 ft., and 5 ft. Here the common unit of measure, 1 ft., has been repeated 2, 3, 4, and 5 times to measure the parts.

The number of units in all is the sum found primarily by counting 2, 3, 4, and 5, or 14 units of 1 ft. Hence the length of the room which is now definitely measured is 14 ft.

**Addition** may, therefore, be considered as the operation of finding the quantity, which, as a whole, is made up of two or more given quantities as its parts. Each of these quantities must have the same measuring unit. Not only is it impossible to add 5 ft. to 4 min., it is impossible to add 5 ft. to 4 in.; *i.e.* to express without change of unit the whole quantity by a number of either ft. or in.

The parts added are called **Addends**.

The **Sum** is the quantity obtained by adding the quantities expressed in terms of a common unit.

**30.** The Sign of Addition is +, and is read *plus*; thus  $6 + 8$  is read 6 plus 8.

The Sign of Equality is =, and is read *equals* or *equal*; thus  $4 + 5 = 9$  is read 4 plus 5 equals 9.

**31.** I bought 3 farms of 50 A. each, 6 farms of 50 A., and 4 farms of 50 A. How much did I buy altogether?

Here we are required to find the whole quantity measured by the sum of 3, 6, and 4 farms of 50 A.

$\therefore$  the whole quantity = 13 farms of 50 A.

**Exercise 9**

1. What quantity is measured by the parts 2 yd., 6 yd., and 7 yd.?

2. What sum of money is equal to 4 five-cent pieces, 9 five-cent pieces, and 5 five-cent pieces?

3. How much is 8 fifty-dollar bills, 4 fifty-dollar bills, and 9 fifty-dollar bills?

4. I paid out in one day 6 ten-dollar bills, 8 ten-dollar bills, and 5 ten-dollar bills. How much did I spend altogether?

5. If I sell two lots, one for 8 units of value, and the other for 6 units, what do I get for both, the unit of value being \$100?

6. A fruit dealer who arranges his apples in piles of 6 for 5 ct. sells 1 pile to each of a company of 4 persons, and 3 piles to another customer. How much does he sell altogether?

7. A speculator buys 5 farms of 100 A. for \$5000, 6 farms of 100 A. for \$7000, and 3 farms of 100 A. for \$4000. How much land did he buy? If \$1000 is the unit of value, how many units did he pay out for all the farms?

8. What is the quantity denoted by the sum 6, 7, and 5 times the measuring unit?

9.  $2 \text{ in.} + 5 \text{ in.} + 4 \text{ in.} = ?$   $2 \text{ ft.} + 5 \text{ ft.} + 4 \text{ ft.} = ?$

10.  $\$3 + \$4 + \$6 = ?$  3 ten-dollar bills + 4 ten-dollar bills + 6 ten-dollar bills = ?

11. A horse was bought for 10 ten-dollar bills, and a carriage for 12 ten-dollar bills. How much was paid for both?

12. A man paid out at one time 4 five-dollar bills, at another 6 five-dollar bills, and again 3 five-dollar bills. How much did he pay out altogether?

13. A fruit dealer arranges his fruit in piles of 3 each. He has 20 piles of apples, 10 of oranges, and 30 of plums. How much has he altogether? If there were 30 of each kind in a pile, what number would express the aggregate?

14. A fruit dealer sells his apples at the rate of 3 for 5 cents. He sells five cents' worth to each of 8 customers. How much did he sell?

15. What is the sum of two quantities, one denoted by 9 times the measuring unit, and the other by 6 times the measuring unit?

16. If I paid 30 units of \$100 each for a lot, and built a house upon it which cost me 40 units of \$100 each, what was the total cost? If \$1000 is the unit, what is the number expressing the total cost?

17. A horse which travels at the rate of 8 mi. an hr. goes from A to B in 3 hr., from B to C in 2 hr., and from C to D in 4 hr. If 8 mi. is the unit of length, what is the number expressing the distance from A to D?

32. Drill on the following addition combinations to secure accuracy and rapidity:

1 1 1 2 1 2 1 2 3 1 2 3 1 2 3 4  
1, 2, 3, 2; 4, 3; 5, 4, 3; 6, 5, 4; 7, 6, 5, 4;  
 1 2 3 4 1 2 3 4 5 2 3 4 5 3 4 5 6  
8, 7, 6, 5; 9, 8, 7, 6, 5; 9, 8, 7, 6; 9, 8, 7, 6;  
 4 5 6 5 6 7 6 7 7 8 8 9  
9, 8, 7; 9, 8, 7; 9, 8; 9, 8; 9, 9.

Enlarge each combination thus:

8 8 8 8 18 28 18 38 68  
9 19 29, etc.; 9 9 9, etc.; 19 29 39, etc.;  
 80 80 80 180 280  
90 190, etc.; 90 90 90, etc.



Give such problems as the following requiring instantaneous answers:

How many sq. ft. in 1 sq. yd. 8 sq. ft.? 1 sq. yd. 6 sq. ft.? etc.

How many qt. in 1 pk. 4 qt.? etc.

When 7 is the number added, base the problem on days; thus:

How many da. in 1 wk. 4 da.?

When the number is 6, on minutes; thus:

How many sec. in 1 min. 30 sec., and so on.

**Exercise 10**

Using any unit, count to the number next larger than 100.

By:

1. 2's from 0; from 1.
2. 3's from 0; from 2.
3. 4's from 0; from 1, 2, and 3 separately.
4. 5's from 0; from 1, 2, 3, and 4.
5. 6's from 0, 1, 2, 3, 4, and 5.
6. 7's from 0, 1, 2, 3, 4, 5, and 6.
7. 8's from 0, 1, 2, 3, 4, 5, 6, and 7.
8. 9's from 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9.

**33.** A person paid \$38 for a cow, \$146 for a horse, and \$255 for a carriage. Find the cost of all.

\$ 38	In this problem we are required to find the cost which is the
146	whole measured by the parts \$38, \$146, and \$255.
255	This may, for convenience, be broken up into the sum of 5, 6,
\$439	and 8 units of \$1, 5, 4, and 3 units of \$10, and 2 and 1 units
	of \$100. The sum of 5, 6, and 8 units of \$1 = 19 units of \$1 = 1
	unit of \$10 and 9 of the \$1 unit.

Add the 1 unit of \$10 in with the tens' column.

The sum of 1, 5, 4, and 3 units of \$10 = 13 units of \$10, or 1 unit of \$100, and 3 units of \$10.

Add the 1 unit of \$100 in with the hundreds' column.

The sum of 1, 2, and 1 unit of \$100 = 4 units of \$100.

Hence the cost = 4 units of \$100, 3 units of \$10, and 9 units of \$1 = \$439.

**34.** If the primary unit is 1 da., and the derived unit 1 yr., how many primary units of time are there in one derived and 243 primary units?

Here we are required to find the sum of 365 and 243 primary units. The result = 608 primary units.

Find the sum of the following numbers, using the unit employed in stating your age:

234	∴ the sum = 2626 yr., since 1 yr. is the unit of age.
983	Add thus, beginning at the units' column: 9, 12, 16; write down
567	6 and carry 1 to the tens' column; 5, 11, 19, 22; write down 2
842	under the tens' column and carry 2 to the hundreds' column; 10, 15,
	24, 26; write down 26, putting the 6 under the hundreds' column.
2626	To prove the answer correct, add downward. If the same
	answer is obtained, the result is likely to be correct.

### Exercise 11

Find the sum of the following quantities and explain your work clearly. Name all the primary units and also the quantities which measure the parts. Prove each answer correct by beginning at the top and adding down.

- |             |            |            |
|-------------|------------|------------|
| 1. \$441    | 2. 341 ct. | 3. 532 da. |
| 234         | 225 "      | 233 "      |
| 518         | 343 "      | 154 "      |
| 4. 543 min. | 5. 635 hr. | 6. 247 in. |
| 666 "       | 87 "       | 859 "      |
| 752 "       | 256 "      | 23 "       |
| 231 "       | 742 "      | 271 "      |

7. 2576 ft. 3491 " 7743 " <u>8988 "</u>	8. 2598 yd. 6776 " 4259 " <u>7362 "</u>	9. 4397 mi. 8999 " 5637 " <u>8249 "</u>
10. 64251 oz. 3789 " 45278 " 99 " <u>6472 "</u>	11. 89435 lb. 62789 " 576 " 43 " <u>        </u>	12. 850439 T. 973642 " 845867 " 939894 " 768795 " <u>649879 "</u>

13. \$ 453798  
667788  
549763  
438925  
648888  
999999

14. Find the number of days in 1 leap yr. and 257 da.

15. If the unit of length 1 ft. is repeated 5280 times to measure a mi., how many such units are there in 1 mi. and 4754 ft.? In 1 mi. 1 yd. 2 ft.?

16. If 1 mi. is represented by the number 63,360 when the primary unit is 1 in., how many times will the primary unit measure the distance 1 mi. 5769 in.? The distance 1 mi. 1 yd. 1 ft. 8 in.?

Find the sum of the following quantities, the unit of measure being that in selling retail the following: In (17) cherries, (18) kerosene, (19) raspberries, (20) sugar, (21) coal, (22) cloth, (23) potatoes, (24) eggs.

17. 84	18. 99	19. 893	20. 733
93	77	254	842
89	86	767	951
75	25	899	258
91	43	654	365
27	88	473	874
30	76	129	935
<u>45</u>	<u>52</u>	<u>895</u>	<u>273</u>
21. 542	22. 9834	23. 7594	24. 5846
879	729	821	7593
666	8345	2357	3819
257	728	8463	5578
389	3403	1525	2904
983	17	7469	8392
365	295	2856	9576
<u>874</u>	<u>8943</u>	<u>8888</u>	<u>2882</u>

**35.** If the unit 1 mi. contains 5280 ft., how many ft. are there in 4 such units?

Here we are required to find the sum of four equal addends, each of which is 5280 ft.; thus:

$$\begin{array}{r}
 5280 \text{ ft.} \\
 5280 \text{ " } \\
 5280 \text{ " } \\
 5280 \text{ " } \\
 \hline
 21120 \text{ ft.}
 \end{array}$$

### Exercise 12

1. If 1 mi. contains 1760 yd., find, by adding, the number of yd. in 2 mi. In 3 mi. In 4 mi.
2. If the unit 1 mi. contains 320 rd., how many rd. in 9 such units?
3. One sq. ft. contains 144 sq. in. How many sq. in. are there in two oblongs, one containing 3 sq. ft. and the other 4 sq. ft.?

4. One cu. ft. contains 1728 cu. in. Show by adding that 12 volumes, each containing 144 cu. in., will be equal to 1 cu. ft.

5. One gal. contains 231 cu. in. Show that a gallon measure can be filled with water and emptied 7 times into a measure containing 1 cu. ft. without causing it to overflow, but not 8 times.

6. If the unit of area, 1 A., contains 4840 sq. yd., how many sq. yd. are there in 6 such units?

7. If there are 197 school days in 1 yr., find, by adding, the number of school days in 8 yr.

8. One sq. mi. contains 640 A. How many A. are there in 8 sq. mi.?

9. Find, by adding, the number of da. in 6 ordinary yr. of 365 da. each and 2 leap yr. of 366 da. each.

### Exercise 13

In each of the following questions, state in each case which is the whole quantity to be measured and what are the parts measuring it.

1. A spent the following sums of money: \$425, \$342, \$673, and \$897. How much did he spend altogether?

2. An encyclopædia consists of three volumes. In the first there are 693 pages, in the second 745, and the third 892. Find the number of pages in the encyclopædia.

3. Using the table in §171, find the number of days in the first six months of the year. In the last six months. In a year.

4. Find the number of days in the three spring months. In the three summer months. In the three fall months. In the three winter months.

5. If the average number of persons to the sq. mi. is taken as the unit of population, and the unit for California is 8 persons, what is the unit of population for the following states?

A, which has twice, and B, which has three times the average population of California; C, whose unit is the sum of the units of A and B.

6. Find the total area of these lakes:

Lake Erie, area 7,750 sq. mi.;

Lake Ontario, area 6,950 sq. mi.;

Lake Michigan, area 22,000 sq. mi.;

Lake Superior, area 31,500 sq. mi.

7. A merchant bought 150 yd. of cloth for \$ 232, 254 yd. for \$ 175, 1875 yd. for \$ 2395, and 640 yd. for \$ 1966. Find the number of yd. bought and the total cost.

8. How many years are there between the establishment of the Republic of Rome in 509 B.C., and the Declaration of Independence in 1776 A.D.?

9. What is the sum of 4, 6, 9, 7, 5, 8, 3, 6, and 7, when the unit of value is \$ 1? \$ 10? \$ 100?

10. What is the value of the quantity denoted by the sum of 3, 9, 12, 6, and the unit \$ 1000? \$ 10,000? \$ 100,000? \$ 1,000,000?

11. Find the sum of three hundred and seventy-six thousand and fifty-four; one hundred and ninety-seven thousand two hundred and fifty-one; four hundred and fifty-seven thousand six hundred and forty-nine.

12. The population of Maine is 661,086; New Hampshire, 376,530; Vermont, 332,422; Massachusetts, 2,238,943; Rhode Island, 345,506, and Connecticut, 746,258. Find the population of the New England States.

13. According to the census of 1890, the population of the seven largest cities in the United States is: New York, 1,515,301; Chicago, 1,099,850; Philadelphia, 1,046,964; Brooklyn, 806,343;

St. Louis, 451,770; Boston, 448,477, and Baltimore, 434,439. Find the total population.

14. Find the sum of the following measures: one 1-inch unit, one 12-inch unit, one 36-inch unit, one 198-inch unit, and one 63360-inch unit.

15. Find the sum of the following: 2 of the one-unit, 4 of the ten-unit, 6 of the hundred-unit, 3 of the thousand-unit, and 9 of the ten-thousand unit quantities.

16. Find the number of times a clock strikes from a quarter of nine A.M. until a quarter of nine P.M.

17. A, B, and C engaged in trade; A put in \$ 3475, B \$ 4593, and C as much as the other two together. How much money was put into the business?

18. A man, dying, willed to his widow \$ 6875; to his son, \$ 4294, and to his daughter, \$ 3875. What was the value of his estate?

19. The census of 1890 gave the negro population of Georgia 858,996; Mississippi, 744,749; South Carolina, 689,141; Alabama, 679,299; Virginia, 635,858; North Carolina, 562,565; Louisiana, 560,192; Texas, 489,588; Tennessee, 430,881; Arkansas, 309,427. Find the entire negro population of these states.

20. The area of the basin of the Colorado River is 250,000 sq. mi.; Columbia, 250,000; Mackenzie River, 440,000; Missouri-Mississippi, 1,250,000; Nelson, 355,000; Rio Grande, 180,000; St. Lawrence, 350,000. Find the total area of these river basins.

21. What is the area of the New England States; that of Maine being in sq. mi. 33,040, of New Hampshire 9305, of Vermont 9565, of Massachusetts 8315, of Rhode Island 1256, of Connecticut 4990?

22. A father left his eldest son \$ 24,000 more than he left his second son, and the second son \$ 7560 more than the third; to the third he left \$ 60,480. What was the eldest son's portion, and what sum did the father leave to his three sons?

23. If in question 6, 100 sq. mi. = unit of measure, and in question 20, 1000 sq. mi. = the unit, what numbers express the respective aggregates?

24. Add vertically and horizontally the following statement of eight weeks' cash receipts:

	MON.	TUES.	WED.	THUR.	FRI.	SAT.	TOTAL.
1st	\$3862.93	\$1391.76	\$6760.68	\$1098.91	\$1696.65	\$ 43.68	
2d	396.74	6168.37	864.39	964.26	167.69	1864.86	
3d	1768.63	467.89	2035.68	3165.03	691.83	785.97	
4th	3976.98	76.05	364.76	93.68	1948.39	1759.46	
5th	263.76	1035.84	36.10	386.41	3.45	1396.71	
6th	1559.83	1932.57	1268.15	8.37	279.72	67.85	
7th	62.24	318.62	134.36	1763.29	1468.29	543.66	
8th	194.87	3.85	7643.82	685.38	765.42	39.67	
Total							

25. Add vertically and horizontally the following statement:

							TOTAL.
	\$1169.84	\$3650.12	\$ 189.10	\$ 97.22	\$ 26.55	\$ 851.02	
	909.58	866.78	914.19	239.49	297.02	312.60	
	575.72	742.49	1654.70	196.17	859.69	1477.42	
	2678.28	1180.66	119.25	8418.60	2223.42	568.35	
	312.83	1638.24	2016.72	1542.24	5300.20	116.02	
	1052.47	342.65	108.00	349.95	136.97	1214.03	
	339.11	687.23	215.17	1020.00	1124.50	1732.25	
	1732.50	514.02	557.60	600.00	475.00	138.50	
	1237.50	3839.25	777.60	136.70	4656.65	1097.47	
	113.56	1291.98	112.50	1850.14	738.75	1204.74	
	3661.00	973.03	311.20	636.99	243.44	142.91	
	1139.67	670.22	1201.64	7357.51	252.47	694.62	
Total							



**36.** Find the sum of: 2.46, 23.973, 15.025, 643.319, and .468.

$$\begin{array}{r} 2.46 \\ 23.973 \\ 15.025 \\ 643.319 \\ \underline{.468} \\ 685.245 \end{array}$$

Since we can add numbers of the same unit, we write the addends so that units will be under units, tenths under tenths, and so on. This is easily done by placing the decimal points directly below each other. Then, beginning at the right, we add the figures as if they were integers, and place the decimal point in the sum between the units' and tenths' column.

**Exercise 14**

Add:

$$\begin{array}{r} 1. \quad 3.456 \\ \quad 4.593 \\ \quad 7.245 \\ \underline{\quad 9.864} \end{array}$$

$$\begin{array}{r} 2. \quad 27.43 \\ \quad 18.314 \\ \quad 5.687 \\ \underline{\quad 34.986} \end{array}$$

$$\begin{array}{r} 3. \quad 76.425 \\ \quad 39.639 \\ \quad 28.764 \\ \underline{\quad 21.385} \end{array}$$

Write in columns and add:

4.  $4.396 + 7.295 + 6.478 + 5.765$ .

5.  $.432 + .987 + .593 + .666$ .

6.  $84.63 + 46.892 + 24.7 + 95.657$ .

7.  $\$24.375 + \$95.875 + \$16.125 + \$19.50$ .

8. Find the capacity of four bins, the first of which will contain 66.384 bu., the second 89.645 bu., the third 27.437 bu., and the fourth 75.938 bu.

9. What is the area of a farm which is divided into three fields containing, respectively, 25.936 A., 14.56 A., and 24.504 A.?

10. Four towns, A, B, C, D, lie on a road running directly east and west. The distance from A to B is 5.693 mi., from B to C 8.421 mi., from C to D 12.768 mi. Find the distance from A to D.

## CHAPTER IV

### SUBTRACTION

**37.** A man who earned \$14 a week, spends \$5 a week for his board. How much has he left?

We are here given the whole quantity, or \$14, and one part, or \$5, and we are required to find the other part.

The question may be viewed in two ways: How much must be added to \$5 to make \$14? Or how much must be taken from \$14 to leave \$5? The answer in both cases is known from addition. \$5 and \$9 are two quantities making \$14. Therefore, if one of them, \$5, is given, the other must be \$9. Or, in other words, \$9 is the *difference* between \$14 and \$5. It is this view of difference that gives the name *Subtraction*.

**38. Subtraction** may therefore be defined as the operation of finding the part of a given quantity that remains when a given part has been taken from the quantity.

The given quantity is called the **Minuend**, and the given part the **Subtrahend**, while the part that remains is called the **Difference** or **Remainder**.

**39.** The Sign of Subtraction,  $-$ , is called *minus*. Thus  $8 - 6$  is read 8 minus 6, and signifies that 6 is to be subtracted from 8.

**Exercise 15**

Read the following questions, filling in the blanks :

1. 6 and 7 are —, 8 and 9 are —, 4 and 8 are —.
2. 4 and 6 are —, 4 and — are 10, 9 and — are 15.
3. 2 and — are 11, 3 and — are 8, 6 and — are 14.
4. 22 and — are 25, 4 and — are 36, 9 and — are 27.
5. 8 and — are 29, 5 and — are 16, 6 and — are 48.

Subtract (Note: Let the process be *not* 6 from 9 leaves 3, but 6 and 3 are 9):

$$6. \frac{9}{6}; \frac{19}{6}; \frac{29}{6}; \frac{39}{6}; \frac{49}{6}; \frac{69}{6}; \frac{99}{6}.$$

$$7. \frac{90}{60}; \frac{190}{60}; \frac{8}{5}; \frac{28}{5}; \frac{80}{50}; \frac{380}{50}.$$

$$8. \frac{12}{7}; \frac{120}{70}; \frac{32}{7}; \frac{320}{70}; \frac{14}{8}; \frac{54}{8}.$$

What numbers added respectively to 9, 7, 6, 8, 5, and 4, make

9. 12? 10. 15? 11. 17? 12. 14? 13. 18? 14. 16?

**40.** Drill, as in § 32, in addition, on the fundamental subtractions, connecting with corresponding additions, until accuracy and rapidity are secured; thus :

$$\frac{9}{8}; \frac{19}{8}; \frac{29}{8}; \frac{39}{8}; \frac{49}{8}; \frac{59}{8}; \text{ and so on.}$$

$$\frac{90}{80}; \frac{190}{80}; \frac{290}{80}; \frac{390}{80}; \frac{490}{80}; \text{ and so on.}$$

$$\frac{17}{8}; \frac{27}{8}; \frac{37}{8}; \frac{47}{8}; \frac{57}{8}; \text{ and so on.}$$

**41.** A man who owned 18 farms of 50 A., sold 7 of them. How much had he left?

Because  $7 + 11 = 18$ , it is evident that he had left 11 farms of 50 A. each.

## Exercise 16

1.  $9 \text{ ft.} + ? = 16 \text{ ft.}$   $9 \text{ yd.} + ? = 16 \text{ yd.}$
2. How many dimes must be added to 6 dimes to get 14 dimes? What is the difference between 14 dimes and 6 dimes? How much less is 6 five-dollar bills than 14 five-dollar bills?
3. A fruit dealer sold 10 piles of 3 apples each. How many had he left if he had at first 15 piles of 3 apples?
4. A fruit dealer arranges his oranges into 12 piles of 4 oranges each. He sells 8 piles. How many has he left?
5. A fruit dealer who sells apples at the rate of 3 for 5 ct., arranges his apples into 19 groups of 3 each. He sells 5 ct. worth to each of 12 customers. How much has he still remaining?
6. I owe a debt of 12 ten-dollar bills and have 5 ten-dollar bills in my pocket. If I pay this toward the debt, how much do I still owe?
7. What must be added to 15 units to get 20 units? Taken from 20 units to get 15 units? To get 5 units? If I sell my house, which cost 20 units of value of \$100 each, for 25 units, how much did I gain on the transaction?
8. What is the difference between a quantity denoted by 14 times the measuring unit and one denoted by 8 times the measuring unit?
9. A person who has \$50 pays a debt of \$30. How much money has he left? What number expresses this remainder if \$10 is the unit of measure? If \$5 is the unit of measure? If \$4 is the unit? If \$2 is the unit?
10. A speculator bought 12 farms of 100 A. each for \$6000. He sold 4 of these farms for \$3000. How much land had he left? If \$1000 is the unit of money, express in terms of this unit the difference between the buying and the selling price of the 4 farms.

**42.** The following method of subtraction, which is nearly always adopted in making change, is almost universally employed by professional computers and is considered by many teachers the best way to perform subtraction.

It is superior in accuracy and rapidity to the method of the next paragraph.

It is based on the principle that the sum of the subtrahend and remainder is equal to the minuend.

From 875 take 451.

$$\begin{array}{r} 875 \\ 451 \\ \hline 424 \end{array}$$

Thus: 1 and 4 are 5; 5 and 2 are 7; 4 and 4 are 8. In this operation let the pupil fancy that he is doing addition with the sum at the top, and as he works set down the figures, 4, 2, and 4.

**43.** A merchant bought 965 yd. of silk and sold 723 yd. How much had he left?

Here we are required to find the undefined part. This is the difference between the measured whole, or 965 yd., and the given part, 723 yd.

965 yd. = the measured whole.

723 yd. = the measured part.

242 yd. = the difference, which is now definitely known.

**EXPLANATION.** — As in addition, we write *units* under *units*, *tens* under *tens*, and *hundreds* under *hundreds*. Beginning with the units, we say 3 units from 5 units leaves 2 units, which we write below the line in the units' column. Then 2 tens from 6 tens leaves 4 tens. Place *this* in the tens' column. Lastly, 7 hundreds from 9 hundreds leaves 2 hundreds, which we write in the hundreds' column.

This difference, 242 yd., is the other part, which is now definitely measured.

### Exercise 17

Subtract, and prove your answer correct in each case:

1.  $\frac{\$946}{324}$

2.  $\frac{785 \text{ lb.}}{323 \text{ "}}$

3.  $\frac{659 \text{ T}}{236 \text{ "}}$

4.  $\frac{897 \text{ da.}}{683 \text{ "}}$

$$5. \frac{8498 \text{ hr.}}{2361} "$$

$$6. \frac{9999 \text{ min.}}{7265} "$$

$$7. \frac{8395 \text{ sec.}}{4073} "$$

$$8. \frac{\$7684}{6450}$$

$$9. \frac{\$8697}{1082}$$

$$10. \frac{\$2578}{1506}$$

**Exercise 18**

In the following questions, name (1) the undefined part, (2) the whole quantity, (3) the given part.

1. A merchant sold 246 yd. from a piece of cloth 258 yd. in length. How many yd. had he remaining?

2. A person deposited in a bank \$8495, but shortly after drew out \$1035. How much had he left in the bank? If \$10 is the unit of value instead of \$1, what number expresses the amount left in the bank?

3. On Tuesday a merchant deposited in a bank \$3475, on Wednesday \$4690. If he withdrew \$1010 on Thursday, how much did he still have on deposit?

4. What is the difference between 1 yr. and 213 da.?

5. A bankrupt has debts amounting to \$8496; his assets are \$3015. How much more does he owe than he can pay?

6. A man left property to the value of \$36,875 to his two children. The son received \$14,250; what was the daughter's share?

7. At an election the successful candidate received 953 votes, and the unsuccessful candidate 613 votes. Find the majority of the former.

**44. Computers' Method.**

From 94,275 take 67,492.

$$\begin{array}{r} 94275 \\ 67492 \\ \hline 26783 \end{array}$$

Thus: 2 and 3 are 5; 9 and 8, 17; carry 1 to 4 as in addition, making it 5; 5 and 7 are 12; carry 1 to 7, making it 8; 8 and 6 are 14; carry 1 to 6, making it 7; 7 and 2 are 9.

The numbers 3, 8, 7, 6, and 2 are written down in order to give the remainder.

**45. Find the difference between 642 and 375.**

642      As we cannot take 5 units from 2 units, we take 1 *ten* from the 4  
 375      *tens*, and adding this 1 *ten*, which equals 10 units, to the 2 units, we  
 267      have 12 units. Then 5 units from 12 units leaves 7 units, which we  
          write under the units' column. Now as we took 1 *ten* from 4 *tens*,  
          we have left only 3 *tens*; we borrow 1 hundred from the 6 hundreds,  
          and considering the 1 hundred as 10 *tens*, we add it to the 3 *tens*, making 13  
          *tens*; then 7 *tens* from 13 *tens* leaves 6 *tens*, which we write under the *tens*'  
          column.

Now as we took 1 hundred from 6 hundreds, we have left only 5 hundreds;  
 hence we subtract 3 hundreds from 5 hundreds, leaving only 2 hundreds,  
 which we write in the hundreds' column.

The remainder, or difference, is thus 2 hundreds, 6 *tens*, and 7 units,  
 or 267.

**Exercise 19**

In the following questions prove the correctness of your results  
 by adding the two parts and finding the sum equal to the whole  
 quantity.

- |                                                  |                                                   |                                                   |
|--------------------------------------------------|---------------------------------------------------|---------------------------------------------------|
| 1. $\frac{\$653}{269}$                           | 2. $\frac{307 \text{ dimes}}{268 \text{ "}}$      | 3. $\frac{642 \text{ sec.}}{375 \text{ "}}$       |
| 4. $\frac{921 \text{ min.}}{87 \text{ "}}$       | 5. $\frac{255 \text{ hr.}}{99 \text{ "}}$         | 6. $\frac{907 \text{ da.}}{859 \text{ "}}$        |
| 7. $\frac{3849 \text{ yr.}}{2567 \text{ "}}$     | 8. $\frac{9345 \text{ in.}}{8367 \text{ "}}$      | 9. $\frac{7007 \text{ ft.}}{6609 \text{ "}}$      |
| 10. $\frac{8000 \text{ yd.}}{5348 \text{ "}}$    | 11. $\frac{9041 \text{ rd.}}{7385 \text{ "}}$     | 12. $\frac{7968 \text{ mi.}}{2693 \text{ "}}$     |
| 13. $\frac{43970 \text{ pt.}}{26784 \text{ "}}$  | 14. $\frac{50062 \text{ qt.}}{37891 \text{ "}}$   | 15. $\frac{12009 \text{ gal.}}{11376 \text{ "}}$  |
| 16. $\frac{34060 \text{ bu.}}{29143 \text{ "}}$  | 17. $\frac{986403 \text{ oz.}}{728547 \text{ "}}$ | 18. $\frac{620703 \text{ lb.}}{444444 \text{ "}}$ |
| 19. $\frac{850439 \text{ T.}}{473642 \text{ "}}$ | 20. $\frac{759826 \text{ A.}}{378934 \text{ "}}$  |                                                   |

**Exercise 20**

\* Subtract:

- |                                                               |                                                               |                                                                |
|---------------------------------------------------------------|---------------------------------------------------------------|----------------------------------------------------------------|
| 1. $\begin{array}{r} 57261 \\ 38877 \\ \hline \end{array}$    | 2. $\begin{array}{r} 40359 \\ 9998 \\ \hline \end{array}$     | 3. $\begin{array}{r} 10000 \\ 1021 \\ \hline \end{array}$      |
| 4. $\begin{array}{r} 89437 \\ 15790 \\ \hline \end{array}$    | 5. $\begin{array}{r} 67182 \\ 30293 \\ \hline \end{array}$    | 6. $\begin{array}{r} 81349 \\ 47538 \\ \hline \end{array}$     |
| 7. $\begin{array}{r} 654375 \\ 412884 \\ \hline \end{array}$  | 8. $\begin{array}{r} 986392 \\ 826957 \\ \hline \end{array}$  | 9. $\begin{array}{r} 303233 \\ 192001 \\ \hline \end{array}$   |
| 10. $\begin{array}{r} 233826 \\ 204739 \\ \hline \end{array}$ | 11. $\begin{array}{r} 310865 \\ 270326 \\ \hline \end{array}$ | 12. $\begin{array}{r} 605487 \\ 584598 \\ \hline \end{array}$  |
| 13. $\begin{array}{r} 164326 \\ 48476 \\ \hline \end{array}$  | 14. $\begin{array}{r} 982623 \\ 897674 \\ \hline \end{array}$ | 15. $\begin{array}{r} 1000101 \\ 707707 \\ \hline \end{array}$ |

**Exercise 21**

\* Subtract:

- |                                                                 |                                                                 |                                                                 |
|-----------------------------------------------------------------|-----------------------------------------------------------------|-----------------------------------------------------------------|
| 1. $\begin{array}{r} 755903 \\ 699004 \\ \hline \end{array}$    | 2. $\begin{array}{r} 640021 \\ 400569 \\ \hline \end{array}$    | 3. $\begin{array}{r} 716287 \\ 662763 \\ \hline \end{array}$    |
| 4. $\begin{array}{r} 100794 \\ 81685 \\ \hline \end{array}$     | 5. $\begin{array}{r} 143812 \\ 109758 \\ \hline \end{array}$    | 6. $\begin{array}{r} 948735 \\ 473596 \\ \hline \end{array}$    |
| 7. $\begin{array}{r} 4731246 \\ 4342760 \\ \hline \end{array}$  | 8. $\begin{array}{r} 9487352 \\ 5999999 \\ \hline \end{array}$  | 9. $\begin{array}{r} 1737682 \\ 739908 \\ \hline \end{array}$   |
| 10. $\begin{array}{r} 3801572 \\ 2003789 \\ \hline \end{array}$ | 11. $\begin{array}{r} 5745861 \\ 2837154 \\ \hline \end{array}$ | 12. $\begin{array}{r} 5048650 \\ 4243091 \\ \hline \end{array}$ |
| 13. $\begin{array}{r} 1217191 \\ 1038182 \\ \hline \end{array}$ | 14. $\begin{array}{r} 4100293 \\ 1925867 \\ \hline \end{array}$ | 15. $\begin{array}{r} 2047000 \\ 1054888 \\ \hline \end{array}$ |
| 16. $\begin{array}{r} 5468305 \\ 1490673 \\ \hline \end{array}$ | 17. $\begin{array}{r} 7086543 \\ 2889454 \\ \hline \end{array}$ | 18. $\begin{array}{r} 1671498 \\ 536819 \\ \hline \end{array}$  |
| 19. $\begin{array}{r} 8235460 \\ 3530089 \\ \hline \end{array}$ | 20. $\begin{array}{r} 2679953 \\ 1346397 \\ \hline \end{array}$ | 21. $\begin{array}{r} 1521315 \\ 1432568 \\ \hline \end{array}$ |

\* Use computers' method of subtraction.



**Exercise 22**

1. Subtract 231 cu. in. from 2772 cu. in., and from the remainder, and so on, until no remainder is left. If 1 gal. contains 231 cu. in., how many gal. are there in 2772 cu. in.?

2. Subtract 320 rd. from 2880 rd., and from each remainder until none is left. If the unit of length, 1 mi., is equal to 320 rd., how many such units are there in 2880 rd.?

3. Subtract 1760 yd. from 15,840 yd., and from each remainder until none is left. If 1 mi. contains 1760 yd., how many mi. are there in 15,840 yd.?

4. Subtract, as in question 3, 5280 ft. from 68,640 ft. If 1 mi. is equal to 5280 ft., how many mi. are equal to 68,640 ft.?

5. Subtract, as in question 3, 144 sq. in. from 864 sq. in. If 1 sq. ft. contains 144 sq. in., how many sq. ft. are there in 864 sq. in.?

6. Subtract, as in question 3, 4840 sq. yd. from 53,240 sq. yd. If 1 A. contains 4840 sq. yd., what is the number of A. in 53,240 sq. yd.?

7. Subtract, as in question 3, 1728 cu. in. from 15,552 cu. in. If 1 cu. ft. contains 1728 cu. in., how many cu. ft. are there in 15,552 cu. in.?

8. Subtract, as in question 3, 365 da. 3 times, and 366 da. once, from 2922 da. How many yr. are there in 2922 da.?

**Exercise 23**

Solve the following questions and verify your answers.

1. Subtract \$819 from \$918, explaining the process.

2. A speculator sold cattle at a loss of \$3145 and some horses at a gain of \$2578. How much did he lose on both transactions?

3. A merchant exchanges a stock of goods worth \$6725, and a house worth \$3120, with a farmer for a farm valued at \$5900,

the farmer paying the balance in money. What sum must the merchant receive?

4. If the measured quantity be 6743 bbl. of sugar, and one of the parts is 1987 bbl., find the other part.

5. Make and solve a question in which the whole quantity and one part are given.

6. A lends B \$9780; B repays A by giving him bank stock to the amount of \$1946, a farm worth \$6385, and the balance in cash. How much cash did B pay A?

7. A is worth \$6215, B is worth \$876 less than A, and C is worth as much as A and B together, lacking \$2343. How much are B and C worth, respectively? How much are all three worth?

8. How much larger is Lake Erie than Lake Ontario? Lake Superior than Lake Michigan? The total areas of the three smaller lakes than Lake Superior? (For the areas of these lakes see Ex. 13, question 6, in Addition.)

If 100 sq. mi. is the unit, what number expresses these differences?

9. What is the difference between 643 and 579 if the unit of value is \$1? \$10? \$100? \$1000? \$10,000? \$100,000? \$1,000,000?

10. How much greater are 253 units of \$1000 than 1864 units of \$100? What number expresses the difference when \$100 is the unit of measure? When \$10 is the unit of measure?

11. A man bought a house and lot for \$8450. He spent \$1379 in improvements and \$212 for insurance. He then sold the house and lot for \$12,000; did he gain or lose, and how much?

12. A collector received \$1300 from five men; from the first he received \$367, from the second \$194 less than from the first, from the third \$36 more than from the second, from the fourth as much as from the second and third together. How much did he collect from the fifth?

13. From the difference between 784 and 8395, take the difference between 17,012 and 21,410.

14. Two men start from the same point and travel in the same direction. The first travels 84 mi. in one day and the second 69 mi. How far were they apart at the end of the first day? If they had travelled in opposite directions, how far would they have been apart?

15. The population of Texas in 1890 was 2,235,523, and of Illinois, 3,826,351. How much greater was the population of Illinois in 1890 than that of Texas? What is the unit in this question?

16. The length of the St. Lawrence River is 2000 mi., and the area drained by it is 350,000 sq. mi. The length of the Amazon is 4000 mi., and the area drained by it is 2,500,000 sq. mi. What is the ratio of the length of the Amazon to that of the St. Lawrence? Show by subtracting 350,000 sq. mi. successively from 2,500,000 sq. mi., how many times greater is the area drained by the Amazon than that drained by the St. Lawrence, and find the remainder.

17. The area of Texas is 265,780 sq. mi., of England, 50,800 sq. mi., and of Germany, 208,700 sq. mi. How much larger is Texas than the united area of England and Germany?

18. The population of New York State in 1870 was 4,387,464, in 1880 it was 5,082,871, and in 1890 it was 5,997,853. What was the increase in population from 1870 to 1880? From 1880 to 1890? How much greater was the latter increase than the former?

19. If the colored population of South Carolina in 1890 was 689,141, and the total population 1,151,149, how much larger was the colored population in 1890 than the white population?

20. The area of Texas is 265,780 sq. mi., of Illinois 56,650 sq. mi., of Oklahoma 39,030 sq. mi., and of the District of Columbia 70 sq. mi. If 10 sq. mi. be cut off of Texas for an Indian

Reservation, into how many states can the remainder be divided, making as many as possible of the size of Illinois, and then of Oklahoma, and of the District of Columbia?

46. From 25.3846 take 18.6397.

25.3846

18.6397

---

 6.7449

We write units under units, tenths under tenths, and so on. Beginning at the right, we subtract as if the figures were integers, and place the decimal point in the difference between the units' and the tenths' column.

Do this problem by the *computers'* method.

### Exercise 24

Find the difference:

1. 26.437

---

 15.254

2. 94.568

---

 29.783

3. 102.4951

---

 58.2876

From:

4. 75.093 take 34.267.

5. 6.4297 take 3.5824.

6. 41.7453 take 27.937.

7. 3.1111 take 1.4682.

8. 3.1416 take .9885.

9. A car containing 24.875 T. of coal was divided between A and B. A received 11.375 T.; what did B get?

10. Show by successive subtractions that a field containing 52.584 A. can be divided into 6 fields, each containing 8.764 A.

47  
Wha  
2.  
man  
man  
3.  
you  
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The  
5.  
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sum  
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say  
(wit  
8.  
find  
time  
9  
\$ 2

## CHAPTER V

### MULTIPLICATION

47. 1. Beginning with \$2, add by \$2, till you reach \$26. What are the \$2 called? Addends. What the result? Sum.

2. In getting this *sum* have you definitely thought of *how many* \$2 there are? *No*. Do you know from the sum *how many* there are? *No*.

3. If you add \$2 to \$2, etc., till you reach the *sum*, \$182, do you know *how many* twos there are? *No*.

4. How do you look upon the sum \$26 (say) and the \$2? The \$26 is simply the sum of an unknown number of \$2.

5. Now count the number of \$2. There are 13. Did you think of this 13 in the addition process? *No*.

6. Now consider this 13 in relation to the *addend* \$2, and the *sum* \$26, what *new* idea is introduced? The *idea* of *how many times* \$2 is repeated to make \$26?

7. Then what is the *number* which measures \$26? 13. What is the unit of measure? \$2. From what you know of number say what *ratio* 13 is? The ratio of \$26 to \$2. We say at once (without adding) that 13 times \$2 is \$26.

8. In this do we depend at all on addition? *Yes*. We first find the *sum*, and connect this in memory with the *number of times* the addend is repeated.

9. But is it then correct to say that the processes  $\$2 + \$2 + \$2 \dots = \$26$ , is *identical* with the process  $13 \times \$2 = \$26$ ? *No*;

for 13 represents the "new idea" referred to, and \$2 has become a *definite unit of measure*, which with 13 denotes the quantity \$26. The *addend* has become a *factor*, and the *sum* a *product*.

**48.** Find the cost of 9 yd. of cloth at \$5 a yd.

(1) Here we think of \$5 as a *derived unit* measuring the value of 1 yd. Hence the cost of 9 yd. is equal to  $9 \times \$5$ , or to \$45.

(2) Thus 45, the number of primary units in the total cost, is called the *product* of the number of primary units in the derived unit \$5, which is 5, by the number of units, viz. 9, in the given quantity of cloth.

(3) In the above example the total cost was given by 9 units of \$5 each, and after *multiplication* by 45 units of \$1 each. Thus multiplication does not change the total cost (*i.e.* the measured quantity); it changes only the *number* which measures it (in this case from 9 to 45) by changing the unit of measure, \$5, to the primary unit, \$1.

The numbers to be multiplied together, viz. 9 and 5, are called *factors* of the product, *i.e.* of the number that measures the quantity.

**49. Multiplication** is the operation of finding the number of primary units in a quantity expressed by a given number of derived units, or, more briefly,

**Multiplication** is the operation of finding the product of two numbers.

The **Multiplicand** is the derived unit of measure.

The **Multiplier** denotes how many times this unit of measure is to be repeated, *i.e.* it denotes the *ratio* of the measured quantity to the unit of measure.

**50.**  $8 \times \$6$  is read 8 times \$6.

$\$6 \times 8$  is read \$6 multiplied by 8.

$\times$  is the Sign of Multiplication.

MULTIPLICATION TABLE

Twice	Three times	Four times	Five times	Six times	Seven times
1 is 2	1 is 3	1 is 4	1 is 5	1 is 6	1 is 7
2 " 4	2 " 6	2 " 8	2 " 10	2 " 12	2 " 14
3 " 6	3 " 9	3 " 12	3 " 15	3 " 18	3 " 21
4 " 8	4 " 12	4 " 16	4 " 20	4 " 24	4 " 28
5 " 10	5 " 15	5 " 20	5 " 25	5 " 30	5 " 35
6 " 12	6 " 18	6 " 24	6 " 30	6 " 36	6 " 42
7 " 14	7 " 21	7 " 28	7 " 35	7 " 42	7 " 49
8 " 16	8 " 24	8 " 32	8 " 40	8 " 48	8 " 56
9 " 18	9 " 27	9 " 36	9 " 45	9 " 54	9 " 63
10 " 20	10 " 30	10 " 40	10 " 50	10 " 60	10 " 70
11 " 22	11 " 33	11 " 44	11 " 55	11 " 66	11 " 77
12 " 24	12 " 36	12 " 48	12 " 60	12 " 72	12 " 84

Eight times	Nine times	Ten times	Eleven times	Twelve times
1 is 8	1 is 9	1 is 10	1 is 11	1 is 12
2 " 16	2 " 18	2 " 20	2 " 22	2 " 24
3 " 24	3 " 27	3 " 30	3 " 33	3 " 36
4 " 32	4 " 36	4 " 40	4 " 44	4 " 48
5 " 40	5 " 45	5 " 50	5 " 55	5 " 60
6 " 48	6 " 54	6 " 60	6 " 66	6 " 72
7 " 56	7 " 63	7 " 70	7 " 77	7 " 84
8 " 64	8 " 72	8 " 80	8 " 88	8 " 96
9 " 72	9 " 81	9 " 90	9 " 99	9 " 108
10 " 80	10 " 90	10 " 100	10 " 110	10 " 120
11 " 88	11 " 99	11 " 110	11 " 121	11 " 132
12 " 96	12 " 108	12 " 120	12 " 132	12 " 144

**51.** Develop at least a portion of the multiplication table by measuring. Let a line be drawn on the board at least 36 in. long, and let 12 strips of paper be cut, respectively, 1 in., 2 in., 3 in., ..., 12 in. long, representing different units of measure.

To develop, for instance, the table of 3's, measure along the line 3 times with each of these measures. Then with a yard ruler divided into inches measure each result in turn.

Hence

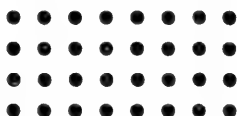
$$3 \times 1 \text{ in.} = 3 \text{ in.}$$

$$3 \times 2 \text{ in.} = 6 \text{ in.}$$

$$3 \times 3 \text{ in.} = 9 \text{ in.}$$

$$\text{etc.} = \text{etc.}$$

$$3 \times 12 \text{ in.} = 36 \text{ in.}$$



**52.** From the above diagram it is evident that 32 is equal to  $4 \times 8$ , or  $8 \times 4$ .

Arrange the dots to show that 32 is equal to  $2 \times 16$ , or  $16 \times 2$ .

How often is 32 measured by 4? 8? 2? 16?

### Exercise 25

1. Arrange dots, representing any units, to show that  $28 = 4 \times 7$ , or  $7 \times 4$ .

2. Give the factors of 45 ( $9 \times 5$  or  $5 \times 9$ ), 66, 56, 72, 96, 63, 90, 54, 99, 84, 132, 108.

3. Give the factors of 9, 16, 25, 36, 49, 64, 81, 100, 121, 144.

4. Arrange 30 dots to show how often 30 is measured by 10. By 3.

5. How often is 72 measured by 9? 8? 6? 12? 4? 18? 3? 24? 2? 36?

6. If one factor of 96 is 12, what is the other? If one factor is 8, what is the other?



7. What will 5 yd. of cloth cost at \$4 a yd.? What will 4 yd. cost at \$5 a yd.?

8. If 9 men can do a piece of work in 6 da., how long will it take 1 man to do it? If 6 men can do a piece of work in 9 da., how long will it take 1 man to do it?

**53. Suggestions regarding the Multiplication Table.**

1. The tables of 10 and 11 present no difficulty, the product being directly associated with the digit.

2. The table of 9 can be similarly remembered. The product is made up of tens and units. In the table (up to  $10 \times 9$ ) the tens' digit is always one less than the number multiplied. The sum of the digits is 9.

3. In the table of 5, the unit digit is 5 for odd multiplicands, and 0 for even ones.

4. By association; thus if  $6 \times 9 = 54$ , then  $9 \times 6 = 54$ .

5. Require the table to be memorized in regular order; also, so that it can be given by the pupil in irregular order, thus:  $9 \times 4 = 36$ ,  $9 \times 6 = 54$ ,  $9 \times 10 = 90$ , etc.

6. Drill, requiring instantaneous oral and written answers to such questions as: What is  $6 \times 7$ ?  $9 \times 8$ ?  $8 \times 9$ ?

7. Drill, requiring instantaneous answers: What is  $6 \times 7 + 4$ ?  $9 \times 8 + 3$ ?  $8 \times 9 + 7$ ?

8. Extend the table thus:  $5 \times 70 = ?$   $7 \times 800 = ?$   $4 \times 120 = ?$

9. Drill thus:  $42 \div 7 = ?$   $84 \div 12 = ?$   $54 \div 8 = ?$

10. Give two factors, each less than 12, of 36, 54, etc.

11. What part of 28 is 12, 16? Of 36 is 20, 28? (using the table of 4 as a basis).

12. What is the quantity whose ratio to the unit \$9 is equal to 6?

13. How many in. in 5 ft. 4 in.? How many da. in 4 wk. 6 da.?


Scale:  $\frac{1}{4}$  in. = 1 in.

**54.** (1) Find the area of an oblong 5 in. long and 3 in. wide.

Let the oblong be divided into 3 strips by lines 1 in. apart, as in the figure.

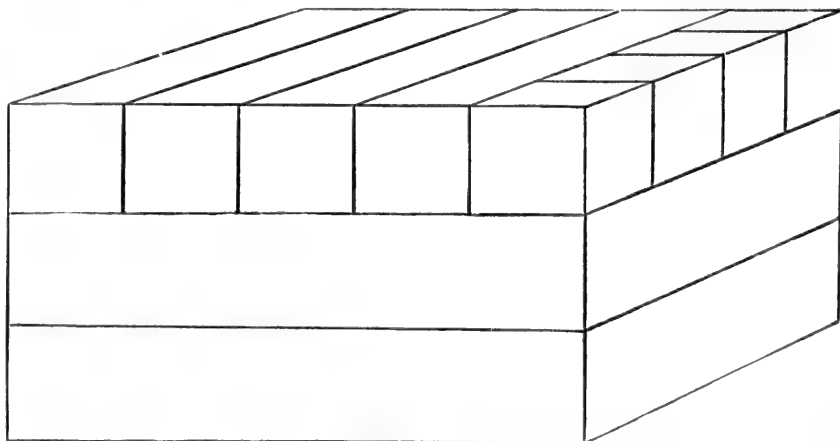
$$\begin{aligned}\text{The area of 1 strip} &= 5 \text{ sq. in.} \\ \therefore \text{the area of the oblong} &= 3 \times 5 \text{ sq. in.} \\ &= 15 \text{ sq. in.}\end{aligned}$$

In this figure there are two units of measurement. The smaller or primary unit of area is 1 sq. in., and is repeated 5 times to make the larger unit, or the strip; the larger or derived unit of one strip is 5 sq. in. and is repeated 3 times to make the oblong which is now measured. Make a rectangle 5 in. long and 3 in. wide. Divide it as in the figure and make a mental picture of the resulting figure.

(2) Reduce 9 pk. 6 qt. to qt.

$$\begin{aligned}9 \text{ pk.} &= 9 \times 8 \text{ qt.} = 72 \text{ qt.} \\ \therefore 9 \text{ pk. 6 qt.} &= 78 \text{ qt.}\end{aligned}$$

Here the problem is to add 6 qt. to 9 units of 8 qt. each.



Scale:  $\frac{1}{2}$  in. = 1 in.

(3) Find the volume of a rectangular solid 5 in. long, 4 in. wide, and 3 in. thick.

Let the solid be divided into 3 slices by horizontal planes 1 in. apart.

Let the upper slice be divided into 5 rows by vertical planes 1 in. apart.

Let the right-hand row be divided into 4 cu. in. by vertical planes 1 in. apart.

The volume of 1 row = 4 cu. in.

The volume of 5 rows or 1 slice =  $5 \times 4$  cu. in.

The volume of 3 slices or the solid =  $3 \times 5 \times 4$  cu. in.  
= 60 cu. in.

In this solid, the three units of volume, in order of size, are the primary unit or 1 cu. in., and the derived units, *i.e.* the rows or 4 cu. in., and the slice or  $5 \times 4$ , *i.e.* 20 cu. in.

The solid is made up of how many units of each kind? Each unit is made up of how many of the next smaller?

### Exercise 26

In the following exercise, where possible, make drawings and mental pictures:

1. Find the area of the following oblongs, draw the figure, and name the primary and derived units of area: 4 in. by 6 in.; 7 in. by 9 in.; 8 in. by 11 in.; 9 in. by 12 in. What is the ratio of the area of the oblong to the primary unit? To the derived unit? What is the ratio of the derived to the primary unit?

2. Find the number of sq. ft. in a sq. yd.; of sq. in. in a sq. ft.

3. Find the area of the floors of the following rooms whose dimensions are: 6 yd., 7 yd.; 6 yd., 8 yd.; 8 yd., 9 yd.; 11 yd., 12 yd.

4. Find the area of squares whose sides are: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, and 12 in., respectively.

5. Find the area of a room made up of 8 units of 9 sq. yd.

6. If the unit of length is 4 ft., what is the unit of area?

7. What is the volume whose smallest unit, 1 cu. in., is repeated 3 times to make the next larger, this 6 times to make the next larger, and this 8 times to make the volume?

8. Find the volumes of rectangular solids whose dimensions are: 2, 3, and 4 in.; 2, 4, and 9 in.; 3, 4, and 7 in.; 2, 5, and 9

in. What are the volumes of the primary and of the derived units of volume?

9. Find the number of cu. ft. in a cu. yd.
10. If 4 in. is the unit of length, what is the unit of volume?
11. Find the volumes of the cubes whose sides are respectively 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12 in.
12. Reduce to lower denominations :
  - (1) 5 yd. 2 ft.; 7 yd. 1 ft.; 8 yd. 2 ft.; 9 yd. 1 ft.; 11 yd. 1 ft.; 12 yd. 2 ft.
  - (2) 5 ft. 4 in.; 8 ft. 10 in.; 7 ft. 8 in.; 11 ft. 3 in.; 9 ft. 7 in.; 12 ft. 6 in.
13. 3 sq. yd. 5 sq. ft.; 5 sq. yd. 7 sq. ft.; 12 sq. yd. 6 sq. ft.; 8 sq. yd. 8 sq. ft.; 9 sq. yd. 2 sq. ft.; 11 sq. yd. 10 sq. ft.
14. 6 qt. 1 pt.; 8 qt. 1 pt.; 11 qt. 1 pt.; 7 pk. 6 qt.; 9 pk. 4 qt.; 9 bu. 3 pk.; 8 bu. 2 pk.; 11 bu. 3 pk.; 7 gal. 2 qt.; 9 gal. 3 qt.
15. 7 wk. 4 da.; 9 wk. 2 da.; 11 wk. 6 da.; 8 wk. 1 da.; 12 wk. 5 da.; 5 hr. 40 min.; 8 hr. 9 min.; 9 hr. 22 min.; 12 hr. 45 min.; 6 da. 4 hr.; 8 da. 2 hr.
16. How many hr. are there in 1 wk.?
17. The perimeter of a room is found by adding twice the width to twice the length. Find the perimeter of rooms whose dimensions are: 6 yd., 8 yd.; 7 yd., 9 yd.; 8 yd., 11 yd.; 11 ft., 12 ft.; 13 ft., 14 ft.

**55.** What is the cost of 6 town lots at \$894 a lot?

Here we think of the whole cost as 6 units of \$894 each.

**EXPLANATION.** — The unit \$894 may be considered as made up of 4 units of one dollar, 9 units of ten dollars, and 8 units of one hundred dollars.

$6 \times 4$ units of one dollar = 24 units of one dollar = 2 units of
$\$894$ ten dollars + 4 units of one dollar.
$6 \times 9$ units of ten dollars = 54 units of ten dollars.
$\$5364$ 54 units of ten dollars + 2 units of ten dollars = 56 units of ten
dollars = 5 units of one hundred dollars + 6 units of ten dollars.
$6 \times 8$ units of one hundred dollars = 48 units of one hundred dollars.

48 units of one hundred dollars + 5 units of one hundred dollars = 53 units of one hundred dollars = 5 units of one thousand dollars + 3 units of one hundred dollars.

5 units of one thousand dollars + 3 units of one hundred dollars + 6 units of ten dollars + 4 units of one dollar = \$ 5364.

**56.** The method generally followed by the pupil is :

6 times 4 = 24 ; 6 times 9 = 54 ; 54 and 2 = 56 ; 6 times 8 = 48 ; 48 and 5 = 53.

This process is too slow. If the pupils have been well drilled in § 53, No. 7, they will be able to add in the digit to be carried instantaneously, and should be trained thus :

$6 \times 4 = 24$  ;  $6 \times 9 = 56$  (adding in the 2 in one process).

$6 \times 8 = 53$  (adding in the 5).

### Exercise 27

Multiply separately :

1. 231 by 2, 4, 6, 8, 10, and 12.
2. 690 by 3, 5, 7, 9, and 11.
3. 897 by 4, 6, 8, 10, and 12.
4. 2463 by 3, 5, 7, 9, and 11.
5. 5781 by 2, 4, 8, 10, and 12.
6. 9654 by 3, 5, 7, 9, and 11.
7. 8267 by 2, 4, 6, 8, 10, and 12.
8. 5280 by 3, 5, 7, 9, and 12.
9. 1728 by 2, 4, 6, 8, 10, and 12.
10. 4840 by 3, 5, 7, 9, and 11.
11. 63,360 by 2, 4, 6, 8, 10, and 12.
12. 24,793 by 3, 5, 7, 9, and 11.
13. 98,654 by 2, 4, 6, 8, 10, and 12.
14. 89,743 by 3, 5, 7, 9, and 11.
15. 64,789 by 2, 4, 6, 8, 10, and 12.
16. Solve the questions in Exercise 12 by multiplication.

## Exercise 28

1. Find the perimeters, and also the areas of the four walls of rooms whose dimensions are:

LENGTH	WIDTH	HEIGHT
16 ft.	14 ft.	8 ft.
17 "	15 "	8 "
20 "	18 "	9 "
22 "	20 "	12 "

2. A cd. of wood is 8 ft. long, 4 ft. wide, 4 ft. high. How many cu. ft. does it contain?

3. Find the number of cu. in. in a cu. ft.

4. A gal. of water will exactly fill a rectangular box 11 in. long, 7 in. wide, and 3 in. high. Find the number of cu. in. in a gal.

5. If there are 1760 yd. in 1 mi., find the number of yd. in 2 mi.; 5 mi.; 8 mi.; 9 mi.; 12 mi.

6. If there are 5280 ft. in 1 mi., find the number of ft. in 3 mi.; 6 mi.; 7 mi.; 9 mi.; 11 mi.

7. Reduce to sq. in.: 5 sq. ft.; 8 sq. ft.; 10 sq. ft.; 12 sq. ft.

8. Reduce to sq. ft.: 3547 sq. yd.; 8426 sq. yd.; 9819 sq. yd.

9. If 1 sq. mi. contains 640 A., find how many A. there are in 6 sq. mi.; 8 sq. mi.; 10 sq. mi.; 12 sq. mi.

10. Reduce to cu. in.: 4 cu. ft.; 5 cu. ft.; 7 cu. ft.; 9 cu. ft.; 11 cu. ft.

11. Reduce to da.: 453 wk.; 769 wk.; 827 wk.; 852 wk.

12. Reduce to qt.: 765 gal.; 917 gal.; 763 gal.; 789 gal.

13. Reduce to qt.: 735 pk.; 892 pk.; 679 pk.; 728 pk.

14. Reduce to da.: 3 yr.; 5 yr.; 6 yr.; 8 yr.; 11 yr.; 12 yr. (1 yr. = 365 da.)

15. If there are 640 A. in 1 sq. mi., find the number of A. in 6 sq. mi.; in 9 sq. mi.; in 12 sq. mi.

16. Show that 4 units of one dollar multiplied by 7 tens is equal to the product found by multiplying 4 units of ten dollars by 7.

17. Show that 9 units of ten dollars multiplied by 7 tens is equal to 63 units of one hundred dollars.

57. If the number expressing the ratio of the measured quantity to the unit of measure \$894 is 76, what is the quantity?

\$ 894      The explanation is similar to that given in § 55. Since, when  
       76      7 is used as a multiplier, the 4 units of one dollar are multiplied  
     5364      by 7 *tens*, the product is the same as that found by multiplying  
     6258      4 units of *ten* dollars by 7. This is 28 units of ten dollars, and is  
     \$ 67944      equal to 2 units of one hundred dollars and 8 units of ten dollars.  
                     Hence the 8 is written under the 6 in the tens' column, and the  
                     2 is carried to be added in the hundreds' column, and so on.

To prove the answer correct, multiply 76 by 894; thus:

$$\begin{array}{r}
 76 \\
 894 \\
 \hline
 304 \\
 684 \\
 608 \\
 \hline
 67944
 \end{array}$$

∴ the answer is correct.

### Exercise 29

Multiply and prove your answers correct:

- |                |                   |
|----------------|-------------------|
| 1. 423 by 36.  | 11. 8647 by 365.  |
| 2. 479 by 32.  | 12. 7245 by 168.  |
| 3. 295 by 16.  | 13. 8939 by 224.  |
| 4. 798 by 24.  | 14. 6558 by 144.  |
| 5. 581 by 52.  | 15. 9275 by 231.  |
| 6. 649 by 27.  | 16. 9475 by 1760. |
| 7. 959 by 24.  | 17. 8213 by 5280. |
| 8. 764 by 31.  | 18. 4781 by 1728. |
| 9. 953 by 56.  | 19. 5893 by 2240. |
| 10. 825 by 48. | 20. 6439 by 1728. |

**Exercise 30**

Multiply :

- |                 |                  |
|-----------------|------------------|
| 1. 8245 by 684. | 6. 8746 by 675.  |
| 2. 7639 by 797. | 7. 9687 by 897.  |
| 3. 5927 by 395. | 8. 4786 by 478.  |
| 4. 4399 by 927. | 9. 9467 by 769.  |
| 5. 8999 by 868. | 10. 8769 by 567. |

**Exercise 31**

In the following questions state which is the unit of measure and which is the number :

1. If 1 mi. contains 320 rd., find the number of rd. in 2897 mi.
2. If 1 mi. contains 1760 yd., find the number of yd. in 1679 mi.
3. If 1 mi. contains 5280 ft., find the number of ft. in 834 mi.
4. If 1 sq. ft. contains 144 sq. in., find the number of sq. in. in a rectangle 27 ft. long and 18 ft. wide.
5. Find the number of sq. ft. in a garden, the shape of an oblong, which is 16 yd. long and 14 yd. wide.
6. If 1 sq. mi. contains 640 A., how many A. are there in a township containing 36 sq. mi.?
7. Find the value of 6 units of land at \$ 85 a unit.
8. If 1 in. be taken as the unit of length, how many units of area are there in the surface of a box whose dimensions are respectively 2, 3, and 4 of the next higher unit of length?
9. If 1 ft. be taken as the primary unit of length, what is the number of the primary units of volume in a rectangular solid whose dimensions contain 4, 5, and 6 of the next higher unit of length?



10. If the unit of length is 2 ft., what is the unit of volume? How many cu. ft. are there in the volume of a rectangular solid whose dimensions contain respectively 3, 4, and 5 such units of length?

11. What is the area of the surface of this solid in square feet?

12. If 1 cu. ft. contains 1728 cu. in., find the number of cu. in. in a rectangular solid 8 ft. long, 4 ft. wide, and 4 ft. high.

13. If 1 cu. yd. contains 27 cu. ft., find the number of cu. ft. in a rectangular solid whose dimensions are 6, 4, and 3 yd.

14. If 1 cd. of wood contains 128 cu. ft., how many cu. ft. are there in 936 cd.?

15. If 1 gal. contains 231 cu. in., how many cu. in. are there in a vessel which contains 888 gal.?

16. A quantity contains the unit 365 da. 896 times. Find the quantity.

58. (1) A drover bought 36 horses at \$145 a head, and 96 cows at \$28 a head. Which cost the more, and how much?

Here the problem is to find the difference between 36 units of \$145 each and 96 units of \$28 each.

(2) A's barn cost \$175, his house 16 times as much, and his farm cost as much as both. What was the cost of all?

Here the problem is to find the cost of  $1 + 16 + 17$ , or 34 units of \$175 each.

### Exercise 32

1. Exemplify the truth that two or more factors will give the same product in whatever order they are multiplied.

2. A speculator bought 150 head of cattle and 47 mules. He made a profit of \$13 a head on the former and \$17 each on the latter. What was gained by the speculation?

3. A ship sailed 56 hr. at the rate of 11 mi. per hr., when she encountered a storm of 16 hours' duration, which drove her back at the rate of 14 mi. per hr. How far from port was she at the expiration of the 72 hr. ?

4. A is worth \$1265, B is worth 4 times as much as A and \$183, and C is worth 3 times as much as A and B lacking \$2343. How much are B and C worth, respectively? How much are they all worth?

5. Make out a bill for the following goods:

23 yd. cotton @ 11¢; 13 yd. gingham @ 23¢;  
25 yd. flannel @ 37¢; 18 yd. tweed @ \$1.50;  
12 yd. serge @ \$1.75; 6 yd. broadcloth @ \$4.50.

6. A produce merchant exchanged 48 bu. of oats at 39¢ per bu. and 13 bbl. of apples at \$3.85 a bbl. for 200 lb. of butter at 37¢ a lb. How much should he pay to settle the account?

7. A grain dealer buys 4795 bu. of wheat in Chicago at 63¢ a bu., and ships it to New York at a cost of 3¢ a bu. Find his gain if he sells it in New York for 71¢ a bu.

8. A man bought 51 horses at \$97 each, and sold them at \$136 each. How much did he gain?

9. Find the amount of the following bill:

63 brooms, at 16¢ each;  
13 yd. print, at 11¢ per yd.;  
17 lb. tea, at 35¢ per lb.;  
4 doz. oranges, at 4¢ each;  
287 lb. sugar, at 5¢ per lb.;  
84 eggs, at 13¢ per doz.

10. Bought oranges at the rate of 18¢ a doz., and sold them at the rate of 6 oranges for 15¢. How much did I gain on 11 boxes, each containing 20 doz.?

11. If in question 10 two boxes were spoiled, what would have been the gain?

12. A grocer buys 150 lb. of coffee at 23¢ a lb., and 39 lb. of chicory at 6¢ a lb. He pays a duty of 2¢ a lb. on each, and mixes and sells the mixture at 33¢ a lb. Find his profit.

13. A book contains 457 pages, each page containing 39 lines, averaging 13 words to a line. Find the number of words in the book.

14. How far will a bicyclist travel in 27 da., if he travels 9 hr. a da. at 12 mi. an hr.?

15. Two vessels start from the same point and travel, the one down a river at the rate of 15 mi. an hr., the other up the river at the rate of 9 mi. an hr. How far will they be apart in 8 hr.?

16. If the first vessel travelled up the river at the rate of 12 mi. an hr., how far apart would they be in 8 hr.?

17. A speculator bought 45 A. of land at \$65 an A., and 63 A. at \$78 an A. If he sold the whole at \$75 an A., how much did he gain or lose?

59. Multiply .948 by 6.

$$\begin{array}{r} .948 \\ 6 \\ \hline 5.688 \end{array}$$

948 thousandths multiplied by 6 equals 5688 thousandths or 5.688.

### Exercise 33

Multiply:

1. .5 by 9.

2. .8 by 3.

3. .26 by 3.

4. .39 by 8.

5. .624 by 3.

6. .842 by 9.

7. .1416 by 25.

8. .988 by 76.

9. 3.54 by 12.
10. .543 by 36.
11. 4.79 by 32.
12. 3.295 by 16.
13. .7568 by 144.
14. 8.754 by 172.
15. The circumference of a circle is 3.1416 times the diameter. What is the circumference of a circle whose diameter is 27 yd. ?
16. Find the number of sq. yd. in 1 sq. ch., there being 30.25 sq. yd. in 1 sq. rd., and 16 sq. rd. in 1 sq. ch.
17. A cu. ft. contains 7.48 gal. of water. Find how many gal. can be poured into a tin-lined box, whose interior dimensions are 3 by 4 by 6 ft.
18. Find the weight of a rectangular solid of oak, 3 ft. by 2 ft. by 1 ft., weighing 47.375 lb. per cu. ft.
19. A drover bought 12 sheep at \$5.375 per head, 36 at \$4.625, and 212 at \$4.125. Find the total cost.
20. Multiply each of the following numbers by 10: .43; .576; 4.23; .017; 89.4263.
21. Write down the product of each of the following numbers multiplied by 10: 7.4; 8.946; 5.32; .008; 62.9347.
22. State how to write the product obtained by multiplying a decimal by 10.
23. Multiply by 100 each of the following: .435; 8.027; 9.12; 46.5928.
24. State how to write the product obtained by multiplying a decimal by 100. By 1000. By 10,000.

## CHAPTER VI

### DIVISION

**60.** What will 4 oranges cost at 5¢ apiece? If 4 oranges cost 20¢, what is the cost of each? What must 5¢ be multiplied by to get 20¢?

At \$5 a yd., how much will 9 yd. of cloth cost? What must \$3 be multiplied by to get \$27? At \$3 a yd., how many yd. can be bought for \$27?

**61.** Of what product are 5 and 6 the factors? (30.) If 5 is one factor of 30, what is the other? Of what product are 12 and 4 the factors? If 4 is one factor of 48, what is the other? If 6 is one factor of 42, what is the other? If 9 is one factor of 63, what is the other? 4 is one factor of each of the following numbers; what are the other factors? 24, 36, 20, 28, and 16.

**62.** In **Multiplication** we are given two factors and we are required to find their product.

In **Division**, on the other hand, we are given the product, and also one of the factors, and we are required to find the other factor.

Thus: Find how many yd. of cloth at \$5 a yd. can be bought for \$45?

In this problem, 45, the number measuring the cost of the cloth, is the product of two factors. One of these is 5, the *given* number, which measures the value of the unit, and the other is 9, which is the number of yards.

If 9 yd. of cloth cost \$45, what will 1 yd. cost?

As before, 45 is the product of two factors. The given factor is 9, and the required factor, 5. Therefore 1 yd. costs \$5.

**63. Division** is the operation of finding either of two factors, when their product and the other factor are given.

The factor found is called the **Quotient**. It shows how often the Divisor is contained in the Dividend.

The given factor is called the **Divisor**.

The given product of the Quotient and Divisor is called the **Dividend**.

When the Divisor is not contained an exact number of times, the excess is called the **Remainder**. See § 71.

**64.** The Sign of Division is  $\div$ ; thus  $\$8 \div \$2 = 4$  is read \$8 divided by \$2 is equal to 4.

$\$8 \div 2 = \$4$ , is read \$8 divided by 2 is equal to \$4.

$9 \div 3$  may also be written  $\frac{9}{3}$ , where 9 is the dividend and 3 the divisor.

**65.** When the divisor does not exceed 12, the operation can be performed mentally, and the process is called Short Division.

When all the different steps of the division are written, the process is called Long Division.

**66. SUGGESTIONS TO THE TEACHER.**— Give questions similar to the following, in order to secure facility in interpreting results and accuracy and rapidity in using the multiplication table.

Thus: (1) A product is 72; one factor is 8. Find the other.

(2) What is  $\$36 \div 4$ ?  $\$72 \div \$9$ ?  $\$72 \div 9$ ?  $132 \div 11$ ?

Associate simple practical questions with these numbers.

(3) Extend the table thus: Divide 210 by 7; 3500 by 5; 450 by 90; 840 by 12.

(4) Give the quotient and remainder when 86 is divided by 7; 93 by 12; 43 by 6.

(5) Reduce to the next higher denomination: 32 qt.; 40¢; 96 in.; 45 da.; 450 min.

(6) The unit of area is 9 sq. rd. What number expresses the ratio of the area of a field containing 270 sq. rd. to the unit of area?

**67.** If 1 T. of coal costs \$6, how many T. will \$4764 buy?

Here the product is \$4764; one factor is \$6, and we are required to find the other factor, which is the number of T.

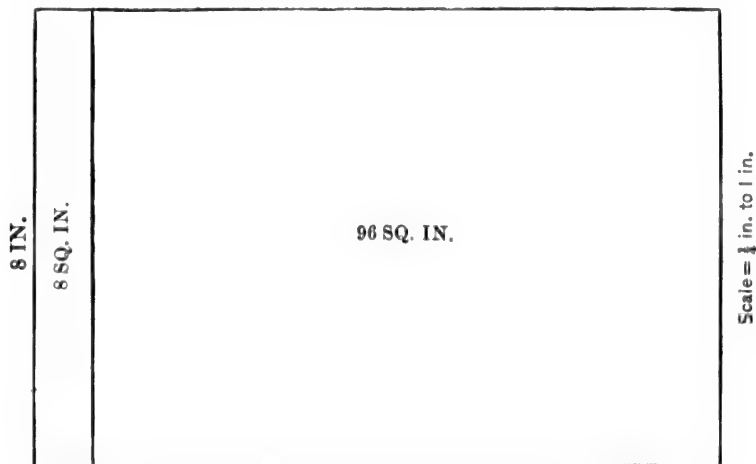
$$\begin{array}{r} 6 \overline{)4764} \\ 794 \end{array}$$

6 divides 47 of the hundreds' unit 7 times in the hundreds' place, with a remainder 5 of the hundreds' unit; 5 of the hundreds' unit and 6 of the tens' unit equal 56 of the tens' unit.

6 divides 56 of the tens' unit 9 times in the tens' place, with a remainder 2 of the tens' unit. 2 of the tens' unit and 4 of the one-unit, equal 24, which divided by 6 equals 4.

∴ the number of T. = 794.

**68.** (1) Find the length of an oblong which contains 96 sq. in. and is 8 in. wide.



Cut off from the oblong a strip 1 ft. wide. This strip contains 8 sq. in.

Here we are given the whole quantity, or 96 sq. in., and the measuring unit, 8 sq. in. The number 12 gives the number of primary units of 1 in. contained in the length. Therefore the length is 12 in.

The area of 1 strip 1 in. wide = 8 sq. in.

The number of strips 1 in. wide =  $96 \text{ sq. in.} \div 8 \text{ sq. in.} = 12$ .

$\therefore$  the length = 12 in.

(2) Find the number of yd. of carpet required to carpet a room 32 ft. long and 26 ft. wide, the carpet running lengthwise, if each strip is 2 ft. wide.

The number of strips of carpet =  $26 \text{ ft.} \div 2 \text{ ft.} = 13$ .

The length of the carpet =  $13 \times 32 \text{ ft.} = 416 \text{ ft.}$

= 138 yd. 2 ft.

$\therefore$  138 yd. 2 ft. of carpet are needed.

Make a diagram for this question on the scale of  $\frac{1}{4}$  in. to 1 ft.

(3) Reduce 6498 da. to wk.

In this question the time is expressed in terms of the primary unit, 1 da., and we are required to express it in terms of the derived unit, 1 wk. or 7 da. Dividing 6498 da. by 7 da., the result is 928 wk. 2 da.

### Exercise 34

In the following exercise prove the correctness of your answers:

1. Reduce to qt.: 36 pt.; 78 pt.; 96 pt.; 65 pt.; and 257 pts.

2. Find the number of strips of carpet 2 ft. wide required to carpet rooms respectively 24, 28, and 32 ft. wide.

3. Reduce to yd.: 384 ft.; 456 ft.; 723 ft.; 897 ft.; 5280 ft.



4. Find the number of strips of carpet 3 ft. wide required to carpet rooms respectively 27, 33, and 39 ft. wide.

5. Find the number of yards of carpet required to carpet a room 24 ft. long and 18 ft. wide, the carpet being 2 ft. wide and running lengthwise. Draw a diagram.

6. In question 5, find the number of yards required in case the carpet runs across the room, and draw the diagram.

7. Reduce to gal.: 576 qt.; 893 qt.; 798 qt.; 962 qt.

8. Divide by 5: 475; 827; 593; 890; 646.

9. Divide by 6: 252; 435; 728; 846; 777.

10. Reduce to wk.: 245 da.; 365 da.; 678 da.; 899 da.; 987 da.

11. Reduce to pk: 32 qt.; 892 qt.; 958 qt.; 2456 qt.; 9472 qt.

12. Reduce to sq. yd.: 756 sq. ft.; 894 sq. ft.; 3478 sq. ft.; 9864 sq. ft.

13. Reduce to dimes: 620¢; 840¢; 729¢; 843¢; 5246¢; 8795¢.

14. Divide by 11: 451; 628; 847; 956; 8297; 7887.

15. How many dozen are there in 842 units? 957 units? 1459 units? 4596 units?

16. Reduce to ft.: 459 in.; 897 in.; 2641 in.; 63,360 in.

17. Find the length of the sides of squares whose areas are respectively 1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, and 144 sq. in.

18. Find the length of an oblong 6 ft. wide which contains 258 sq. ft.

19. What is the width of an oblong which contains 15 units of area, the length containing 5 units of length, and the unit of length being 4 ft.?

## Exercise 35

Find the quotient and remainder and prove your results correct :

1.  $36 \div 2$ ;  $48 \div 3$ ;  $72 \div 4$ ;  $65 \div 5$ ;  $84 \div 6$ .
2.  $56 \div 7$ ;  $96 \div 8$ ;  $81 \div 9$ ;  $80 \div 10$ ;  $77 \div 11$ ;  $48 \div 12$ .
3.  $249 \div 3$ ;  $842 \div 6$ ;  $941 \div 8$ ;  $654 \div 9$ .
4.  $137 \div 2$ ;  $439 \div 5$ ;  $849 \div 7$ ;  $999 \div 12$ .
5.  $6 \overline{)743}$        $11 \overline{)682}$        $9 \overline{)847}$        $10 \overline{)895}$
6.  $5 \overline{)679}$        $4 \overline{)673}$        $8 \overline{)976}$        $12 \overline{)899}$
7.  $7 \overline{)2457}$        $10 \overline{)6430}$        $9 \overline{)1978}$        $12 \overline{)4994}$
8.  $6 \overline{)3975}$        $2 \overline{)2557}$        $11 \overline{)7381}$        $3 \overline{)4565}$
9.  $6 \overline{)4544}$        $12 \overline{)8256}$        $5 \overline{)1935}$        $4 \overline{)3191}$
10.  $11 \overline{)8149}$        $8 \overline{)7919}$        $9 \overline{)4676}$        $7 \overline{)6769}$
11.  $4 \overline{)2319}$        $5 \overline{)9847}$        $6 \overline{)1764}$        $12 \overline{)9543}$
12.  $11 \overline{)8682}$        $8 \overline{)7798}$        $9 \overline{)7992}$        $12 \overline{)63360}$

Divide by short division :

13.  $200 \overline{)14800}$        $200 \overline{)14876}$        $300 \overline{)14976}$
14.  $300 \overline{)29654}$        $400 \overline{)98743}$        $500 \overline{)25931}$

**69.** (1) A father dying left an estate valued at \$48,832 to be divided equally among his wife, his two sons, and his four daughters. What was the share of each ?

In this problem we are required to find the share of each, which is the unit of measure. In order to find this, we are given the value of the estate, which is the whole quantity, and one factor, which is the number of shares ; viz.  $1 + 2 + 4$ , or 7. Therefore, dividing the whole quantity by 7, we find the share of each to be \$6976.

(2) The area of the four walls of a room whose dimensions are 8 yd. and 6 yd. is 112 sq. yd. Find the height of the room.

We are here required to find the number of yd. in the height of the room. In order to find it, we are given the area, which is the measured whole.

We are also given the dimensions with which we can find the perimeter of the rooms, and thus find the measuring unit as in § 54.

The perimeter = twice the sum of 6 + 8, or 14 yd. = 28 yd.

The area of a strip 1 yd. wide running around the room = 28 sq. yd.

The number of yd. in the height =  $112 \text{ sq. yd.} \div 28 \text{ sq. yd.} = 4$ .

$\therefore$  the height = 4 yd.

**70.** If we multiply 59 by 724, we do so by the following process :

$\begin{array}{r} 59 \\ 724 \\ \hline 236 \\ 118 \\ 413 \\ \hline 42716 \end{array}$	or, rearranging the work, we have this :	$\begin{array}{r} 59 \\ 724 \\ \hline 413 \\ 118 \\ 236 \\ \hline 42716 \end{array}$
--------------------------------------------------------------------------------------	---------------------------------------------	--------------------------------------------------------------------------------------

We now wish to arrive at a method of recovering 724 from the dividend 42,716 and the divisor 59. It is evident that if from 42,716 we subtract 413 of the hundreds' digit, which is the product of 59 and 7 times the hundreds' digit, and from the remainder, 118 of the tens' digit, which is the product of 59 and twice the tens' digit, and from this remainder, 236, which is the product of 59 and 4, we shall have no remainder.

Hence to divide 42,716 by 59, we have the following method :

First divide 59 into 427 to get the quotient 7 of the hundreds' digit, multiply 57 by 7, and subtract the product 413 from 427, leaving 14 of the hundreds' digit, which with the 1 of the tens' digit makes 141 of the tens' digit.

Divide this 141 by 59, and we have the quotient 2 of the tens' digit. Multiply 59 by 2, and subtract the product from 141, leaving 23 of the tens' digit, which with the 6 makes 236. Divide 236 by 59, and we have the quotient 4. Multiply 59 by 4, and subtract the product from 236, and there is no remainder.

$$\begin{array}{r}
 59 \overline{) 42716} \text{ (724} \\
 \underline{413} \phantom{00} \\
 141 \phantom{00} \\
 \underline{118} \phantom{00} \\
 236 \phantom{00} \\
 \underline{236} \phantom{00} \\
 0
 \end{array}$$

**71. (1)**

Divisor Dividend Quotient

$$31 \overline{) 7598} \text{ ( 245}$$

$$\underline{62}$$

$$139$$

$$\underline{124}$$

$$158$$

$$\underline{155}$$

3 Remainder

31 divides 75 of the hundreds' unit two hundred times. Put 2 as the first term in the quotient.

Multiply 31 by 2, and subtract the product 62 from 75. The remainder is 13 of the hundreds' unit. Annex the 9 tens of the dividend, making 139 tens. 31 divides 139 tens 4 tens times. Put 4 as the second term in the quotient. Multiply 31 by 4, and subtract the product 124 from 139. The remainder is 15 of the tens' unit. Annex the 8 units, making 158 units. 31 divides 158 units 5 times. Put 5 as the third term in the quotient. Multiply 31 by 5, and subtract the product 155 from 158. The remainder is 3.

What is the quotient on dividing 7598 by 31? What does it show? What is the remainder? What does it show? See § 63.

(2) To prove that the answer in the last example is correct:

$$\begin{array}{r}
 245 \text{ Quotient} \\
 31 \text{ Divisor} \\
 \hline
 245 \\
 735 \\
 \hline
 7595 \text{ Product} \\
 3 \text{ Remainder} \\
 \hline
 7598 \text{ Dividend}
 \end{array}$$

∴ the answer is correct. Or thus, by division,

$$\begin{array}{r}
 245 \overline{)7598} 31 \\
 \underline{735} \\
 248 \\
 \underline{245} \\
 3
 \end{array}$$

∴ 245 quotient and 3 remainder is the correct answer.

**72.** Divide 39,726 by 87.

$$87 \overline{)39726} (456$$

$$\begin{array}{r}
 348 \\
 \hline
 492 \\
 435 \\
 \hline
 576 \\
 522 \\
 \hline
 54
 \end{array}$$

In this division name the unit to which each remainder and each partial dividend belongs.

**73.** *Trial divisor and trial dividend.*

The work of finding the quotients can be much simplified by using the trial divisor and trial dividend.

Thus in § 71, as 31 is nearer 30 than 40, the trial divisor is 3. Dividing 3 into the trial dividends 7, 13, and 15, the quotients are 2, 4, and 5.

In § 72, as 87 is nearer 90 than 80, the trial divisor is 9. Dividing 9 into the trial dividends 39, 49, and 57, the quotients are 4, 5, and 6.

In general, if the divisor is 61, 62, 63, 615, 627, or 634, the trial divisor is 6.

If the divisor is 67, 68, 69, 675, 689, or 697, the trial divisor is 7.

If the divisor is 64, 65, or 66, the use of the trial divisor is less certain; but the rule is to use 6 as the trial divisor for 64, 7 for 66, and either 6 or 7 for 65.

Name the trial divisors in the next exercise.

### Exercise 36

Find the quotients and remainders of the following, and prove the answers to the odd numbers correct by multiplying, and the even numbers correct by dividing.

- |                      |                        |                        |
|----------------------|------------------------|------------------------|
| 1. $712 \div 31$ .   | 13. $9989 \div 93$ .   | 25. $36,989 \div 67$ . |
| 2. $2341 \div 51$ .  | 14. $7948 \div 29$ .   | 26. $52,298 \div 87$ . |
| 3. $6287 \div 71$ .  | 15. $8543 \div 49$ .   | 27. $75,643 \div 97$ . |
| 4. $2195 \div 80$ .  | 16. $9765 \div 69$ .   | 28. $23,877 \div 24$ . |
| 5. $5894 \div 91$ .  | 17. $8720 \div 89$ .   | 29. $38,753 \div 34$ . |
| 6. $2068 \div 22$ .  | 18. $8888 \div 38$ .   | 30. $63,056 \div 64$ . |
| 7. $3572 \div 42$ .  | 19. $9894 \div 18$ .   | 31. $74,111 \div 25$ . |
| 8. $1576 \div 62$ .  | 20. $9320 \div 58$ .   | 32. $96,433 \div 75$ . |
| 9. $8189 \div 82$ .  | 21. $16,324 \div 78$ . | 33. $56,159 \div 95$ . |
| 10. $6285 \div 23$ . | 22. $30,086 \div 98$ . | 34. $27,766 \div 36$ . |
| 11. $7549 \div 53$ . | 23. $18,874 \div 27$ . | 35. $56,139 \div 56$ . |
| 12. $8476 \div 63$ . | 24. $21,803 \div 37$ . | 36. $78,045 \div 76$ . |
37. Reduce 9.872 qt. to bu. (1 bu. = 32 qt.).
38. Make simple practical problems based on questions 26-36.

State how to find the quotient and remainder in any question in long division. In the following exercise, before dividing, make a careful guess as to what the quotient will be.

**Exercise 37**

- |                         |                         |                         |
|-------------------------|-------------------------|-------------------------|
| 1. $10,377 \div 13$ .   | 9. $37,847 \div 86$ .   | 17. $854,300 \div 49$ . |
| 2. $29,452 \div 14$ .   | 10. $84,374 \div 45$ .  | 18. $537,047 \div 36$ . |
| 3. $99,624 \div 15$ .   | 11. $22,158 \div 23$ .  | 19. $624,839 \div 75$ . |
| 4. $87,643 \div 16$ .   | 12. $84,999 \div 69$ .  | 20. $802,666 \div 33$ . |
| 5. $63,277 \div 17$ .   | 13. $15,273 \div 34$ .  | 21. $263,204 \div 54$ . |
| 6. $64,935 \div 18$ .   | 14. $42,965 \div 88$ .  | 22. $467,989 \div 68$ . |
| 7. $99,658 \div 19$ .   | 15. $335,296 \div 47$ . | 23. $467,989 \div 67$ . |
| 8. $29,943 \div 99$ .   | 16. $582,934 \div 56$ . | 24. $633,600 \div 76$ . |
| 25. $604,826 \div 29$ . | 27. $494,358 \div 65$ . |                         |
| 26. $253,789 \div 96$ . | 28. $832,016 \div 79$ . |                         |

In the following exercise, before dividing, make a careful guess as to what the quotient will be.

**Exercise 38**

- |                                                              |                                                        |
|--------------------------------------------------------------|--------------------------------------------------------|
| 1. $395,267 \div 105$ .                                      | 12. $\$ 367,989 \div 476$ .                            |
| 2. $300,498 \div 207$ .                                      | 13. $578,243 \text{ cu. in.} \div 231 \text{ cu. in.}$ |
| 3. $227,876 \div 121$ .                                      | 14. $578,243 \text{ rd.} \div 320 \text{ rd.}$         |
| 4. $407,253 \div 309$ .                                      | 15. $987,655 \text{ cu. ft.} \div 128 \text{ cu. ft.}$ |
| 5. $839,428 \div 224$ .                                      | 16. $\$ 128,821 \div 360$ .                            |
| 6. $719,888 \div 421$ .                                      | 17. $599,647 \div 176$ .                               |
| 7. $584,287 \div 593$ .                                      | 18. $313,947 \text{ da.} \div 365 \text{ da.}$         |
| 8. $495,638 \div 784$ .                                      | 19. $444,555 \div 366$ .                               |
| 9. $597,445 \div 656$ .                                      | 20. $574,381 \text{ A.} \div 640 \text{ A.}$           |
| 10. $386,777 \div 921$ .                                     | 21. $987,432 \text{ sq. in.} \div 144 \text{ sq. in.}$ |
| 11. $811,394 \div 675$ .                                     | 22. $358,049 \div 528$ .                               |
| 23. Solve Exercise 22 by division.                           |                                                        |
| 24. Make simple practical problems based on questions 12-22. |                                                        |

**Exercise 39**

1. Divide \$324 among A, B, and C, giving B twice as much as A, and C three times as much as B.
2. A rod 540 in. long has a piece of 8 in. cut off from it, then another piece of the same length, then another, and so on. How often may this be done, and what is the length of the piece remaining at last?
3. The sum of \$15,108 was paid for a number of sheep at \$6 apiece. Find the number of sheep.
4. What is the object of division when the measuring unit and the measured quantity are given? When the number and the measured quantity are given?
5. A merchant sold a quantity of silk at \$3 a yd., and an equal quantity at \$5 a yd. How much did he sell of each kind if he received \$3816 for the goods?
6. A merchant sold a quantity of cloth at \$3 a yd., and twice as much at \$2 a yd., the whole amounting to \$2065. How much did he sell altogether?
7. What number must be added to 91 to make it exactly divisible by 8?
8. A farmer mixed 15 bu. of oats, worth 40¢ per bu., with 5 bu. of corn, worth 80¢ per bu. What is the mixture worth per bu.?
9. The expense of carpeting a room was \$45; but if the breadth had been 3 ft. less than it was, the expense would have been \$36. Find the breadth of the room.
10. How much water must be mixed with 600 gal. of wine, at \$2.50 per gal., in order to make the mixture worth \$2 per gal.?
11. Divide \$448 among 2 men, 3 women, and 4 children, giving each man three times and each woman twice as much as each child.



**Exercise 40**

- |                                        |                          |
|----------------------------------------|--------------------------|
| 1. 493,287 yd. $\div$ 1760 yd.         | 8. 819,634 $\div$ 4972.  |
| 2. 298,456 ft. $\div$ 5280 ft.         | 9. 819,634 $\div$ 3264.  |
| 3. 140,008 cu. in. $\div$ 1728 cu. in. | 10. 205,639 $\div$ 7459. |
| 4. 680,442 cu. in. $\div$ 2150 cu. in. | 11. 726,998 $\div$ 9543. |
| 5. 998,209 lb. $\div$ 2240 lb.         | 12. 337,877 $\div$ 9961. |
| 6. 857,864 gr. $\div$ 5760 gr.         | 13. 698,206 $\div$ 8456. |
| 7. 398,125 gr. $\div$ 7000 gr.         | 14. 729,453 $\div$ 5879. |

**Exercise 41**

- Find the number of mi. in 56,978 rd.
- Find the number of mi. in 86,300 yd.
- Find the number of mi. in 34,720 ft.
- Find the number of mi. in 746,360 in.
- Find the number of sq. ft. in a room 263 in. long and 248 in. wide.
- Find the number of townships, each containing 36 sq. mi., which can be made out of a section of land in the form of an oblong 27 mi. long and 12 mi. wide.
- If 1 cu. ft. contains 1728 cu. in., find the number of cu. ft. in 632,194 cu. in.
- If 1 gal. contains 231 cu. in., find the number of cu. ft. in 576 gal.
- Find the largest number of gal. of water which can be emptied into a vessel containing 1 cu. ft., without overflowing.
- How many gal. of water will fill a vessel containing 77 cu. ft.?
- Find the number of yr., of 365 da. each, in 84,678 da.
- Find the wages due a workman who has worked 423 hr. at \$1.50 a da., of 9 hr. each.

13. Bought 640 bu. of barley at the rate of 32 bu. for \$ 20.08, and sold it at the rate of 10 bu. for \$ 8.75. Find my profit on the transaction.

14. The earth's polar diameter contains 41,707,796 ft., and the difference between the equatorial and the polar is one-292nd part of the latter. Find the difference between the two in miles.

15. A farmer bought land from B at \$ 60 per A., and the same quantity from C at \$ 85 per A. The whole amounted to \$ 53,215. How many A. did he buy from each?

16. What number must be added to 7,869,456 to render it exactly divisible by 8975?

17. \$ 90.90 are shared among 4 men, 5 women, and 6 children, so as to give to each man twice as much as to each woman, and to each woman three times as much as to a child. What do the women get?

18. A farmer laid out \$ 71,778 in purchasing an equal number of sheep, hogs, and cows. Each sheep cost \$ 6, each hog twice as much as a sheep, and each cow twice as much as a hog. How many of each did he buy?

19. A speculator gave \$ 18,810 for horses, and sold a certain number of them for \$ 7990, at \$ 85 each, losing thereby \$ 10 each. For how much each must he sell the remainder so as to gain \$ 2180 on the whole?

20. In 161,384 in. he made

21. If 68 bales of linen contain 67,048 yd., and each bale contains 34 pieces, and each piece the same number of yd., how many yd. are there in each piece?

22. If the quotient is 5000, when the divisor is 2001 and the remainder 100, what is the dividend?

23. Divide 10,149 by 7, and the quotient by 5; thence deduce the true remainder, and show that it is the same as after the division of 10,149 by 35.

24. What number is that to which if 38 and 5 times 38 be added, and the sum so found be increased by 7 times itself, the total sum is 2400?

25. A dealer in horses gave \$9900 for a certain number, and sold a part of them for \$3825 at \$85 each, and by so doing lost \$5 a head. For how much per head must he sell the remainder so as to gain \$1140 on the whole?

26. A receives on 225 shares of mining stock an annual dividend of \$96 a share; and B receives the same total annual dividend on 270 shares of oil stock. Find the annual dividend on one share of B's stock.

27. A drover bought a number of cattle for \$17,100 and sold a certain number of them for \$12,474 at \$126 a head, gaining on those he sold \$2574. How many did he buy at first, and how much did he gain on each sold?

28. The fore wheel of a carriage is 8 ft. in circumference, and in a distance of 13 mi. makes 2340 revolutions more than the hind wheel. Find the circumference of the hind wheel.

29. Divide \$2640.75 among 4 men, 6 women, and 8 children, giving to each child double a woman's share, and to each woman triple a man's share.

30. A grain merchant bought 40,640 lb. of wheat at \$1.20 per bu., and shipped it to New York at an expense of 3¢ per bu. Before he sold it there was a loss in handling, etc., of  $\frac{1}{80}$  of the original weight; his profit on the transaction was \$69.85. At what price did he sell the wheat? (A bu. of wheat weighs 60 lb.)

31. How many sq. ft. of glass are required to glaze 5 windows, each containing 14 panes of glass, the panes measuring 17 in. by 15 in.?

32. If the average number of people to the sq. mi. is called the unit of population, find to the nearest integer this unit for your own state and also for the following states:

STATE	AREA IN SQUARE MILES	POPULATION (1890)
Massachusetts,	8,315	2,238,943
New York,	49,170	5,997,853
Illinois,	56,650	3,826,351
Texas,	265,780	2,235,523
California,	158,360	1,208,130
South Dakota,	77,650	328,808

Account for the difference in the units of population.

33. If the unit of population for one state is 16 times as great as that of another, but the area of the second is 4 times as great as the first, compare the population of the two states.

34. The length of the Missouri-Mississippi River is 4200 mi., and the area drained by it is 1,250,000 sq. mi. If this area is conceived of as a rectangle whose length is 4200 mi., find its width to the nearest integer.

35. Find, as in question 34, the width of the rectangle for these rivers:

RIVER	LENGTH IN MILES	AREA DRAINED IN SQUARE MILES
St. Lawrence,	2000	350,000
Amazon,	4000	2,500,000
La Plata,	2300	1,250,000

74. Divide 77.968 by 8.

8)77.968      8 is contained in 77 units 9 times with remainder 5 units. 8 is  
      9.746      contained in 59 tenths 7 tenths times with remainder 3 tenths.  
                   8 is contained in 36 hundredths 4 hundredths times with remain-  
 der 4 hundredths. 8 is contained in 48 thousandths 6 thousandths times with  
 no remainder.

Hence the operation of dividing a dividend containing a decimal is similar to that of dividing when the dividend does not contain a decimal. Care must be taken to insert the decimal point in the quotient as in the example, immediately after the units' figure is used in the dividend.

**Exercise 42**

Divide :

1. 12.6 by 6.
2. 7.56 by 7.
3. 16.38 by 13.
4. 239.76 by 37.
5. 596.36 by 17.
6. 889.92 by 72.
7. 195.2544 by 473.
8. 192.4947 by 171.
9. Divide 389.904 A. of land equally among 16 persons.
10. If 43 bu. of wheat cost \$37.625, find the cost of 1 bu.
11. Divide the following numbers by 10 :  
47.39 ; 543.21 ; 62 ; 7.64.
12. Write down the quotients obtained on dividing the following numbers by 10 :  
64.52 ; 3.9 ; 742.63 ; 95.614.
13. State how to find the quotient without actual division, when a decimal is divided by 10.
14. Divide by 100 :  
792.6 ; 8943.62 ; 54.15 ; 89467.1.
15. State how to find the quotient without actual division, when a decimal is divided by 100. By 1000.

## CHAPTER VII

### COMPARISON OF NUMBERS

**75.** If \$4 be multiplied in turn by the numbers 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, the products will be \$4, \$8, \$12, \$16, \$20, \$24, \$28, \$32, \$36, \$40, \$44, \$48.

Hence, taking \$4 as the unit of measure, and noting how often the unit is repeated to produce \$4, \$12, \$8, \$20, \$36, we find that these quantities are represented by 1, 3, 2, 5, and 9 units, respectively.

#### Exercise 43

What is the largest unit which will measure each of the following quantities, and what is the number of units in each of them?

1. \$27, \$24, \$12, \$30, \$33, \$36 (unit \$3).
2. \$45, \$10, \$20, \$55, \$35, \$60 (unit \$5).
3. 42, 18, 36, 6, 54, and 60 sheep.
4. 63, 84, 14, 35, 49, and 77 bbl. of flour.
5. 32, 16, 48, 56, 80, and 96 min.
6. 54, 27, 9, 81, 36, and 99 men.
7. 80, 20, 90, 110, 70, and 120 bu. of oats.
8. 99, 66, 33, 55, 121, and 132 T. of coal.
9. 108, 24, 72, 132, 48, and 60 lb. of tea.

**76.** If 25 sheep cost \$120, what will 10 sheep cost?

Taking 5 sheep as the unit of value, 25 sheep contain 5 units, and 10 sheep 2 units.

5 units cost \$ 120,  
1 unit costs \$ 24.  
∴ 2 units cost \$ 48.

**Exercise 44**

1. If 36 yd. of cloth cost \$42, what will 24 yd. cost at the same rate?

2. A miller sold 35 bbl. of flour for \$126. How much will he receive for 15 bbl. at the same rate?

3. A train runs 32 mi. in 48 min. At the same rate, what distance will it run in 54 min.?

4. If 84 men can dig a trench in 36 da., how long will it take 168 men to dig a trench of the same size?

5. If 88 horses eat 33 bu. of oats in 1 da., how many bu. will 48 horses eat in the same time?

6. If 45 men can reap a field of 36 A. in a certain time, how many A. would 25 men reap in the same time?

7. A bankrupt pays \$35 out of every \$63 owed. How much shall I receive if I am his debtor to the extent of \$81?

8. If 32 T. of coal cost \$184, what will 88 T. cost at the same rate?

9. If 56 men can do a piece of work in 21 da., how long will it take 24 men to do it?

10. How many lb. of tea can be bought for \$56, at the rate of \$16 for 34 lb.?

11. Tea is bought at 72¢ a lb., and sold for 84¢ a lb. The buying price is what part of the selling price?

12. The cost of fencing 132 rd. of railway is \$117. What is the cost of fencing 83 rd.?

13. If a 12-qt. pail is just filled by 6 units of milk, how many qt. are there in a pail which will hold 5 units? What is the unit?

14. Fifty-four min. are represented by 9 units of time. How many min. are there in 7 units?

15. If 4 ft. is the unit of length, what is the unit of area?

16. If 4 ft. is the unit of length, how many units of area are there in an oblong whose dimensions are 24 and 36 ft.?

17. If 5 ft. is the unit of length, what is the unit of volume?

18. If 5 ft. is the unit of length, how many units of volume are there in a rectangular solid 30 ft. long, 25 ft. wide, and 20 ft. high?

19. Divide \$84 among two persons, giving the first \$3 for every \$4 the second will get.

20. Divide \$88 between A, B, and C, so that A will get \$2 and B \$4 for every \$5 C gets.

21. A sum of money is divided between A and B. If A, who gets \$2 for every \$5 B gets, receives \$24, how much does B receive? What sum was divided?

22. A sum of money is divided between A and B. A receives \$2 for every \$5 B receives. If B gets \$24 more than A, how much did each receive? What sum was divided?

23. Make and solve problems similar to 19, 20, 21, and 22.

24. A man whose weekly income is \$15 spends \$2 out of every \$5 he earns for board. What does his board cost him per wk.?

77. (1) What is the ratio of \$20 to \$45?

Let \$5 be taken as the unit of measure.

Then \$20 is measured by 4 times the unit.

And \$45 is measured by 9 times the unit.

$\therefore$  \$20 is  $\frac{4}{9}$  of \$45.



(2) If 22 yd. of cloth cost \$16, what will 33 yd. cost at the same rate?

Take 11 yd. as the unit of length.

Then 33 yd. =  $\frac{3}{2}$  of 22 yd.

$\therefore$  33 yd. cost  $\frac{3}{2}$  of \$16 or \$24.

#### Exercise 45

1. What is the ratio of \$18 to \$24? \$35 to \$55? \$28 to \$63?
2. What is the ratio of 16 hr. to 56 hr.? 72 hr. to 45 hr.?
3. What is the ratio of 60 mi. to 25 mi.? 99 A. to 55 A.?
4. If 45 cd. of wood cost \$162, what will 20 cd. cost?
5. If 21 T. of hay cost \$174, what will 70 T. cost?
6. If 63 men can dig a trench in 16 da., how long will it take 18 men to dig it?
7. If a piece of cloth 15 ft. long and 3 ft. wide costs \$18, what will a similar piece 20 ft. long and 4 ft. wide cost?
8. Divide \$96 between A and B so that A will get \$5 for every \$7 B will get.
9. Divide \$240 between A and B so that the two parts will be in the ratio of their ages, which are 8 and 12 yr.
10. Divide \$460 among three persons, A, B, and C, so that the three portions will be to each other as the numbers 5, 8, and 7, respectively.
11. A bankrupt has three creditors, to whom the sums due are as the numbers 2, 3, and 4. If his assets are valued at \$540, find the sum each will receive.
12. A tract of land is divided into two farms in the ratio of 2 to 3. If the whole tract contains 480 A., what is the size of each farm?

## CHAPTER VIII

### SQUARE ROOT

**78.** The product of 3 and 3 is 9; of 5 and 5 is 25. The squares whose sides measure 3 and 5 units of length contain 9 and 25 units of square measure. We say that 9 is the square of 3 and that 25 is the square of 5; that 3 is the square root of 9 and 5 the square root of 25.

The square of 3 is written  $3^2$ , and the square root of 9 is indicated thus:  $\sqrt{9}$ .

2 is called the **Exponent**, and  $\sqrt{\phantom{x}}$  the **Radical Sign**.  $3^2$  is also called the second **Power** of 3.

**79.** The **Square** of a number is the product found by multiplying the number by itself.

Thus the squares of 1, 2, 3, 4, 5, 6, 7, 8, 9, 10,  
are 1, 4, 9, 16, 25, 36, 49, 64, 81, 100.

**80.** The square root of a number is that number which multiplied by itself is equal to the given number.

Thus the square roots of 1, 4, 9, 16, 25, 36, 49, 64, 81, 100,  
are 1, 2, 3, 4, 5, 6, 7, 8, 9, 10.

**81.** Pupils should memorize the tables in the two preceding paragraphs and be able to answer instantly such questions as the following:

What is the first figure in the square root of 27? 58? 76? 43? 80?

#### Exercise 46

Write the following products as powers:

1.  $5 \times 5$ .

2.  $7 \times 7$ .

3.  $24 \times 24$ .

Write the following powers as products and find their values :

4.  $8^2$ .

6.  $46^2$ .

8.  $(\frac{7}{8})^2$ .

5.  $13^2$ .

7.  $(\frac{3}{4})^2$ .

9.  $(\frac{25}{34})^2$ .

Prove the following statements :

10.  $\sqrt{49} = 7$ .

13.  $\sqrt{324} = 18$ .

16.  $\sqrt{\frac{289}{361}} = \frac{17}{19}$ .

11.  $\sqrt{169} = 13$ .

14.  $\sqrt{961} = 31$ .

17.  $\sqrt{\frac{1369}{1849}} = \frac{37}{43}$ .

12.  $\sqrt{729} = 27$ .

15.  $\sqrt{2025} = 45$ .

**Exercise 47**

1. Find the squares of the numbers from 10 to 20 and commit the results to memory.

2. Find the squares of 25, 28, 54, 75, and 99.

3. From the results in § 79 state how many digits are found in the square of a number of 1 digit.

4. From the results obtained in questions 1 and 2, state how many digits are found in the square of a number of 2 digits.

5. Find the squares of 175, 199, 246, 402, 814, 999.

6. From the results in question 5 state how many digits are found in the square of a number of 3 digits.

7. Judging from the results obtained in questions 1, 2, and 5, state the number of digits in the square root of a square number that contains 3 digits; 4 digits; 6 digits; 7 digits; 8 digits.

8. How many digits in the square correspond to 1 digit in the square root?

9. What is the square root of 400? Of 900?

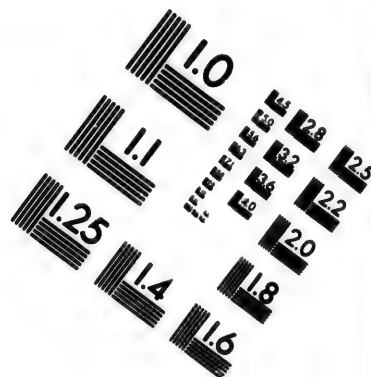
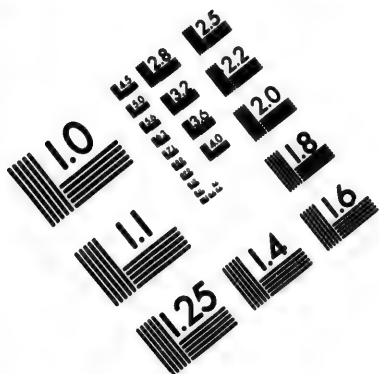
10. The square root of 625 lies between what two numbers?

11. Find the square of 10, 20, 30, 40, 50, 60, 70, 80, and 90.

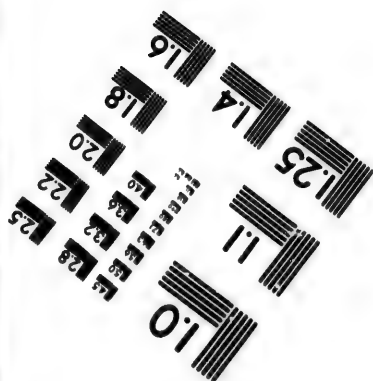
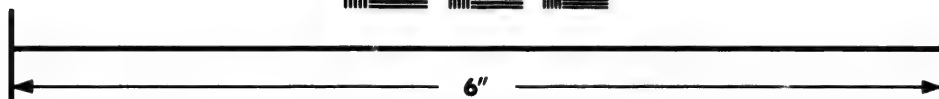
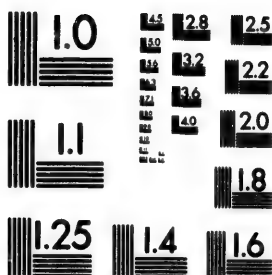
12. Between what numbers does the square root of 1225 lie? Of 4225? Of 2304? Of 8281? Of 8704?

13. What is the square of 200? Of 300?





# **IMAGE EVALUATION TEST TARGET (MT-3)**



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1.8  
2.0  
2.2  
2.5  
2.8  
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3.6  
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40.0  
45.0  
50.0  
56.0  
63.0  
71.0  
80.0  
90.0  
100.0

01

14. The square root of 71,289 lies between what two numbers?  
 15. Find the squares of 100, 200, 300, etc., up to 900.  
 16. Between what two numbers does the square root of 271,441 lie? Of 795,664?

**82.** The following explanation will make clear the method of finding the square root of a number of 3 or 4 digits.

24 Thus 24, which is made up of two parts, 20 and 4, has for its  
 24 square 576, which is seen to be made up of 400, the square of 20;  
 16 16, the square of 4; and twice the product of 20 and 4.  
 80 Now to recover 24 from 576, we know that its hundreds' digit 5,  
 80 showing that the number is between 400 and 900, gives the tens' digit of  
 400 the root, so that we know one of the parts of the root, viz. 20. The  
 576 square of 20 is 400, and the rest of the given number, 176, must be  
 2 times 20, multiplied by the other part, together with the square of  
 the other part. Multiplying 20 by 2, and using the product 40 as a

$$\begin{array}{r} 20 \overline{) 576} \quad (20 + 4 \\ \underline{400} \\ 40 \overline{) 176} \\ \underline{160} \\ 4 \overline{) 16} \\ \underline{16} \end{array}$$

divisor with 176 as dividend, we get the quotient 4. Multiplying 40 by 4 and subtracting the product 160 from 176, we get the remainder 16, which is the square of 4. Therefore 20 + 4 or 24 is the square root of 576.

The work of extracting the square root may be simplified by leaving out the unnecessary zeros, thus:

$$\begin{array}{r} 2 \overline{) 5'76} \quad (24 \\ \underline{4} \\ 44 \overline{) 176} \\ \underline{176} \end{array}$$

**83.** (1) The method of discovering the square root of a number of 5 or 6 digits is similar to that for finding the square root of numbers of 3 or 4 digits.

$$\begin{array}{r} 246 \\ 246 \\ \underline{36} \\ 1440 \\ 1440 \\ \underline{57600} \\ 60516 \end{array}$$

Thus 246, which is made up of two parts, 240 and 6, has for its square 60,516, which is seen to be made up of 57,600, the square of 240; 36, the square of 6; and twice the product of 240 and 6.

$$\begin{array}{r}
 2 \overline{) 6'05'16(240 + 6} \\
 \underline{4} \\
 44 \overline{) 205} \\
 \underline{176} \\
 480 \overline{) 2916} \\
 \underline{2880} \\
 6 \overline{) 36} \\
 \underline{36}
 \end{array}$$

(2) Hence proceeding with 605 as in § 82 with 576, we get in the square root 24 tens or 240. Multiplying 240 by 2 and using the product 480 as a divisor with 2916 as a dividend, the quotient is found to be 6.

Multiplying the 480 by 6, and subtracting the product 2880 from 2916 we have the remainder 36, which is the square of 6. There-

$$\begin{array}{r}
 2 \overline{) 6'05'16(246} \\
 \underline{4} \\
 44 \overline{) 205} \\
 \underline{176} \\
 486 \overline{) 2916} \\
 \underline{2916}
 \end{array}$$

fore  $240 + 6$  or 246 is the square root of 60516.

Leaving out the unnecessary zeros, the work may be simplified as in the contracted form.

The number whose square root is to be subtracted should be pointed off into groups of two figures, as in the preceding examples, beginning with the units' figure.

**84.** To find the square root of  $\frac{289}{625}$ .

The square roots of 289 and 625 are found to be 17 and 25.

Hence  $\sqrt{\frac{289}{625}} = \frac{17}{25}$ .

### Exercise 48

Find the square root and prove your answer correct:

- |          |          |              |                          |
|----------|----------|--------------|--------------------------|
| 1. 324.  | 5. 3025. | 9. 71,824.   | 13. $\frac{64}{289}$ .   |
| 2. 529.  | 6. 6889. | 10. 101,761. | 14. $\frac{729}{2704}$ . |
| 3. 841.  | 7. 4096. | 11. 465,124. | 15. $30\frac{1}{4}$ .    |
| 4. 1156. | 8. 9409. | 12. 998,001. | 16. $22\frac{9}{16}$ .   |

17. Compare the process of extracting the square root of a number with long division, stating in what respect the process is similar, and where it is different.

**85.** (1) Find the length of the side of a square containing 4225 sq. in.

The square is measured by 4225 units of 1 sq. in.; therefore the side is measured by  $\sqrt{4225}$ , or 65 units of 1 in., i.e. the length of the square is 65 in.



(2) The sides of a rectangular field containing 735 sq. rd. are as 3 to 5. Find their length.

The field contains  $3 \times 5$ , or 15 units of area.

The area of one unit =  $735 \text{ sq. rd.} \div 15$ , or 49 sq. rd.

The side of a square containing 49 sq. rd. = 7 rd.

$\therefore$  the sides of the field are  $3 \times 7 \text{ rd.}$ , or 21 rd., and  $5 \times 7 \text{ rd.}$ , or 35 rd.

#### STATEMENT OF SOLUTION

First find the number of units of area in the field. Divide this number into the area, and find the unit of area. Then find the unit of length, which is the length of the side of the unit of area.

Multiply the unit of length by 3 and 5 respectively, to find the sides of the field.

(3) To find the area of a triangle, the lengths of whose sides are given, find one-half the sum of the number of units of length in the sides. Subtract from this the number of units of length in each side separately. Find the product of these four results. The square root of this product is the number of units of area in the given triangle.

Find the area of a triangle whose sides are 5 in., 12 in., and 13 in. respectively.

The sum =  $5 + 12 + 13 = 30$ .

One-half this sum = 15.

$$15 - 5 = 10.$$

$$15 - 12 = 3.$$

$$15 - 13 = 2.$$

$$15 \times 10 \times 3 \times 2 = 900.$$

$$\sqrt{900} = 30.$$

$$\therefore \text{the area of the triangle} = 30 \text{ sq. in.}$$

#### Exercise 49

1. Find the length of the side of an enclosure in the form of a square containing 386 sq. yd. 7 sq. ft.

2. A park contains 9408 sq. yd., and it is 3 times as long as it is wide. Find its length and width.

3. A merchant bought a number of yards of cloth, paying as many cents for each yard as there were yards. The whole cost \$56.25. How many yards did he buy, and at what price per yard?

4. What is one of the two equal factors of 24,336 ?
5. A rectangular field, the sides of which are in the ratio of 4 to 7, contains 4032 sq. rd. Find the length of each side, and the cost of fencing it at \$ 4 per rd.
6. A body of soldiers in column form 567 ranks, 7 abreast. If they were drawn up in solid square, how many would there be on each side ?
7. Find the side of a square which is equal in area to the sum of the area of two squares, the sides of which are 6 and 8 in. long.
8. Draw two lines respectively 6 and 8 in. long, at right angles. Join their extremities by a straight line. Measure this line and show that it is equal to the side of the square found in question 7.
9. Work problems similar to 7 and 8, using the following as the lengths of the sides of the smaller squares: 3 in., 4 in.; 5 in., 12 in.; 8 in., 15 in.
10. From the preceding three questions make a rule showing how to find the length of the hypotenuse of a right triangle when the lengths of the other two sides are known.
11. What is the hypotenuse of a right triangle whose sides are 21 ft. and 28 ft. ? 15 ft., 36 ft. ? 56 ft., 105 ft. ?
12. Find the side of a square equal in area to the difference of the area of the two squares whose sides are 41 ft. and 9 ft.
13. What is the altitude of a right triangle whose hypotenuse and base are 34 ft., 16 ft. ? 205 ft., 45 ft. ? 136 ft., 64 ft. ?
14. The top of a ladder rests against the side of a building 84 ft. from the ground, and its foot is 35 ft. from the wall. Find the length of the ladder.
15. A ladder 51 ft. long stands close against a building. How far must the foot be drawn out that the top may be lowered 6 ft. ?
16. Find the diagonal of a rectangular field whose sides are 144 yd. and 60 yd.

17. Find the side of a square equal in area to a rectangle whose sides are 148 yd. and 333 yd. Find the difference between the perimeters of the rectangle and square.

18. Find the area of the largest rectangle which can be enclosed by a line 36 in. long.

19. A field in the form of a rectangle whose sides are as 3 to 4 contains 432 sq. rd. How much do I save by crossing along its diagonal instead of going along its two sides?

20. State in as few words as possible how you have solved each of the preceding questions.

21. The sides of a triangle are 8 in., 15 in., and 17 in. Find its area.

22. Find the area of an oblong whose sides are 8 in. and 15 in. What is the ratio of its area to that of the triangle given in question 21? Why is this so?

23. Find the areas of the triangles whose sides are :

21 in., 28 in., 35 in.

24 in., 45 in., 51 in.

9 in., 40 in., 41 in.

24. The sides of a rectangle containing 34,992 sq. ft. are as 4 to 3. Find them.

25. One side of an oblong is  $\frac{3}{5}$  as long as the other, and its area is 67,335 sq. yd. Find the length of each side.

## CHAPTER IX

### GREATEST COMMON MEASURE AND LEAST COMMON MULTIPLE

**86.** Name all the units of length which will exactly measure 15 in.

They are 1 in., 3 in., 5 in., and 15 in.

**87.** Find all the different units of length that will exactly measure 12 ft. and 18 ft.

The measures of 12 ft. are 1, 2, 3, 4, 6, and 12 ft.

The measures of 18 ft. are 1, 2, 3, 6, 9, and 18 ft.

It is evident that all the *common* measures of 12 ft. and 18 ft. are 1, 2, 3, and 6 ft., and that the *greatest* common measure is 6 ft.

A **Common Measure** of two or more quantities is a unit that will exactly divide each of them.

The **Greatest Common Measure** (G. C. M.) of two or more quantities is the largest unit which will exactly divide each of them.

For convenience we speak of the common measure or the greatest common measure of two or more numbers.

#### Exercise 50

Find all the common measures and the greatest common measure of:

- |                   |                    |                   |
|-------------------|--------------------|-------------------|
| 1. 16 ft., 28 ft. | 3. 54 yd., 72 yd.  | 5. 32 qt., 56 qt. |
| 2. \$ 60, \$ 90.  | 4. 42 mi., 105 mi. | 6. 27 oz., 47 oz. |
| 7. 21 pt., 91 pt. | 8. 84 bu., 91 bu.  |                   |

9. Find all the measures that can be used to measure the capacity of each of two baskets containing 20 qt. and 32 qt.

10. Find the lengths of the two longest boards that can be used to build a fence around a garden 30 ft. long and 24 ft. wide.

11. Make questions similar to 9 and 10, using the quantities in problems 1 to 8.

### PRIME NUMBERS

**88.** A **Prime Number** is one that can be divided only by unity and itself, as 5, 11, and 13.

Select the prime numbers: 2, 3, 4, 5, 6, 7, 8, 9, 10, 11.

The *prime factors* of a number are the prime numbers which when multiplied together give it; thus, 3, 3, and 5 are the prime factors of 45.

**89.** Find the prime factors of 168.

$$\begin{array}{r|l} 2 & 168 \\ 2 & 84 \\ 2 & 42 \\ 3 & 21 \\ & 7 \end{array}$$

$$\begin{aligned} \text{That is, } 168 &= 2 \times 84; = 2 \times 2 \times 42; = 2 \times 2 \times 2 \times 21; \\ &= 2 \times 2 \times 2 \times 3 \times 7; \\ &= 2^3 \times 3 \times 7. \end{aligned}$$

$\therefore$  The prime factors of 168 are 2, 3, and 7.

$2^3$  is a short way of writing  $2 \times 2 \times 2$ .

3 is called the *exponent* of 2, and denotes that 2 has been taken as a factor three times.

### Exercise 51

1. Name the even numbers from 1 to 100.
2. Name the odd numbers from 1 to 100.
3. Name the prime numbers from 12 to 100.

Find the prime factors of:

4. 30; 36; 56; 48; 84; 66; 196; 195; 231.

5. 86; 147; 104; 132; 78; 135; 342; 255.  
 6. 336; 408; 372; 565; 342; 484; 375; 861.  
 7. What prime factors are common to 30 and 36? 66 and 132? 147 and 336? 135 and 255?

**90.** (1) A man owns a rectangular lot 210 ft. long and 144 ft. wide. Find the length of the longest board that can be used to fence it.

We are required to find the length of the longest board, *i.e.* the G. C. M. of 144 ft. and 210 ft.

$$144 = 2 \times 2 \times 2 \times 2 \times 3 \times 3.$$

$$210 = 2 \times 3 \times 5 \times 7.$$

Thus, the G. C. M. of 144 and 210 =  $2 \times 3 = 6$ .

$\therefore$  the length of the longest board is 6 ft.

To prove the answer correct:

The number of boards required for the length =  $210 \div 6 = 35$ .

The number of boards required for the width =  $144 \div 6 = 24$ .

35 and 24 have no common measure except unity,  $\therefore$  6 ft. is the correct answer.

(2) A certain school consists of 132 pupils in the high school, 154 in the grammar, and 198 in the primary grades. If each group is divided into sections of the same number containing as many pupils as possible, how many pupils will there be in each section?

We are required to find the number of pupils in each section, *i.e.* the G. C. M. of 132, 154, and 198 pupils.

2	132	154	198
11	66	77	99
	6	7	9

Since 2 and 11 are the only common factors, the G. C. M. of 132, 154, and 198 is  $2 \times 11$ , or 22.

$\therefore$  each section will contain 22 pupils.

### Exercise 52

1. Draw two lines, one 15 in. and the other 21 in. long. What is the longest line that can be used to measure both lines?

2. What is the longest line that will exactly measure two lines 28 and 32 in. long?
3. What is the longest line that will exactly measure three lines respectively 20, 30, and 45 in. long?
4. What is the largest unit of capacity that can be used to measure the quantity of oil in each of two vessels, one containing 16 qt. and the other 36 qt.?
5. What is the largest unit of money that can be used to pay each of two debts, one of \$45 and the other of \$80?

### Exercise 53

1. A certain school consists of 132 junior and 99 senior students. How might each of the two classes be divided so that the whole school should be distributed into equal sections?
2. A gentleman has a piece of ground, the sides of which measure 225 ft., 297 ft., and 369 ft. He wishes to enclose it with a fence having panels of uniform length. What is the longest panel that can be used for that purpose?
3. A teacher having a school of 144 boys and 128 girls divided it into the largest possible equal classes, so that each class of girls should number the same as each class of boys. What was the number of classes?
4. There is a street 354 rd. long, and the land on one side of this street is owned by three persons, A, B, and C. A has 102 rd. fronting the street, B 114 rd., and C 138 rd. They agree to divide their land into village lots in such a manner that the lots shall be of the greatest width that will allow each person to form an exact number of lots out of his land. What is this width?
5. A farmer has 240 bu. of wheat and 920 bu. of oats, which he desires to put into the least number of boxes of the same capacity, without mixing the two kinds of grain. Find how many bu. each box must hold.

6. If 1 lb. Avoirdupois contains 7000 gr., and 1 lb. Troy 5760 gr., find the greatest weight that will measure both 1 lb. Troy and 1 lb. Avoirdupois.

7. A farmer has 66 bu. of corn and 90 bu. of wheat, which he wishes to put into sacks of equal size, and without mixing the two kinds of grain. How many bu. must each sack contain in order to be as large as possible?

#### Exercise 54

Find the G. C. M. of:

- |              |                    |
|--------------|--------------------|
| 1. 40, 56.   | 9. 210, 455.       |
| 2. 42, 54.   | 10. 287, 369.      |
| 3. 81, 105.  | 11. 230, 506.      |
| 4. 108, 162. | 12. 42, 72, 180.   |
| 5. 63, 91.   | 13. 60, 135, 165.  |
| 6. 90, 105.  | 14. 210, 462, 546. |
| 7. 102, 114. | 15. 395, 474, 632. |
| 8. 75, 175.  | 16. 666, 738, 954. |

17. Prove that your answer is a common factor by dividing it into each of the numbers. Prove that it is the greatest common measure by examining your quotients and finding that they have no common measure except unity.

18. State how to find the G. C. M. of two numbers. Of three numbers.

19. Make questions similar to those in the previous exercise.

91. Seven divides 126 and 35. It also divides their sum, 161, 23 times, and their difference, 91, 13 times. Seven also divides the sum of 35 and  $4 \times 126$ , or 539, 77 times, and the difference between 126 and  $3 \times 35$ , or 21, 3 times.

Thus any number, as 7, that divides two other numbers, as 126 and 35, will divide their sum or difference. It will also divide the sum or difference of any multiples of these numbers.



**Exercise 55**

Prove the principle stated above :

1. Divisor 6, numbers 84, 30.
2. Divisor 8, numbers 88, 24.
3. Divisor 13, numbers 65, 26.
4. Divisor 19, numbers 133, 38.

**92.** When the factors of the number cannot be easily found, the following method is used :

$$\begin{array}{r}
 741)893(1 \\
 \underline{741} \\
 152)741(4 \\
 \underline{608} \\
 133)152(1 \\
 \underline{133} \\
 19)133(7 \\
 \underline{133}
 \end{array}$$

$\therefore$  the G. C. M. of 741 and 893 is 19.

To prove that 19 is the G. C. M. of 741 and 893 :

*First.* 19 divides 133, and therefore it divides  $19 + 133$ , or 152. 19 divides 133 and 152, and therefore it divides 133 and 4 times 152, or 741. 19 divides 152 and 741, and therefore it divides  $152 + 741$ , or 893. Therefore 19 is a common measure of 741 and 893.

*Again.* It is also the G. C. M. Any number which divides 893 and 741 will divide their difference, or 152. Any number which divides 741 and 152 will divide the difference between 741 and 4 times 152, or 133. Any number which divides 152 and 133 will divide their difference, or 19. But 19 is the largest number that divides 19; therefore 19 is the G. C. M. of 741 and 893.

**93.** The following more compact form may be used after the pupils understand the method given above. It will be observed that the quotients are omitted and that the divisors 741, 152, 133, and 19 are alternately on the left and right side of the vertical line.

$$\begin{array}{r|l}
 741 & 893 \\
 608 & 741 \\
 133 & 152 \\
 133 & 133 \\
 \hline
 & 19 \text{ G. C. M.}
 \end{array}$$

**94.** To find the G. C. M. of three numbers, find first the G. C. M. of two of them, and then of the result and the third number.

**Exercise 56**

Find the G. C. M. of:

- |                                                    |                             |
|----------------------------------------------------|-----------------------------|
| 1. 145, 203.                                       | 7. 11,682, 19,626.          |
| 2. 344, 559.                                       | 8. 31,416, 54,593.          |
| 3. 465, 682.                                       | 9. 2487, 8413.              |
| 4. 1781, 4384.                                     | 10. 495, 891, 1155.         |
| 5. 3423, 3248.                                     | 11. 3066, 4818, 8541.       |
| 6. 4807, 9545.                                     | 12. 15,561, 11,115, 13,585. |
| 13. State how to find the G. C. M. of two numbers. |                             |

**Exercise 57**

1. What is meant by saying that one number is a *common measure* of two or more numbers? Also, the *greatest common measure*?

2. Show by means of the examples in the preceding exercise, that the greatest common measure of two numbers can never exceed the difference of the numbers.

3. In the following pairs of numbers, select those that are prime to each other, and those that are not prime to each other: 12, 18; 8, 15; 12, 17; 20, 21; 28, 35; 13, 29; 36, 48; 31, 47.

4. Explain what is meant by one number being prime to another. When two numbers are prime to each other are they necessarily prime? Give examples.

5. What is the product of the three consecutive numbers, 11, 12, 13?

6. The product of three consecutive numbers is 120. Find the numbers.

7. The product of four consecutive numbers is 1680. Find them.

8. Find the G. C. M. of 90, 150, and 168 by resolving the numbers into their prime factors. When several numbers have been resolved into their prime factors, which of these factors must be taken to form by their product the greatest common measure of the numbers?

9. Find all the common measures of 210 and 462. Prove that every common measure of these two numbers is a measure of their difference.

10. Find the G. C. M. of 13,515 and 13,787.

11. How many rails will enclose a field 3143 ft. long by 2471 ft. wide, provided the fence is straight and 8 rails high, and the longest that can be used?

12. Find the G. C. M. of 169,037 and 66,429.

#### LEAST COMMON MULTIPLE

95. The quantity 15 in. is measured by the unit 5 in., 3 times, and is therefore called a *multiple* of 5 in.

The quantity 18 lb. is exactly divisible by the units 1 lb., 2 lb., 3 lb., 6 lb., and 9 lb., and is a multiple of each one of them. Thus 18 lb. is equal to 18(1 lb.), 9(2 lb.), 6(3 lb.), or 3(6 lb.).

Select from the following quantities the multiples of the unit \$3: \$12, \$16, \$18, \$25, and \$27. Name all the units that will exactly measure the quantity 24 hr.

Any quantity is a *multiple* of a unit of measure when it is exactly divisible by the unit.

96. Thirty days is exactly divisible by the units 3 da. and 5 da., and is, therefore, a *common multiple* of 3 da. and 5 da.

One quantity is a *common multiple* of two or more units when the former is exactly divisible by each of the latter.

Thirty days is the least quantity that is exactly divisible by the units 6 da. and 10 da., and is, therefore, *the least common multiple* of 6 da. and 10 da.

The **Least Common Multiple** (L. C. M.) of two or more units is the least quantity that is exactly divisible by each of them.

For convenience we speak of one number being a multiple of another, or a common multiple, or the least common multiple of two or more numbers.

**97.** In problems in Greatest Common Measure, we are given two or more quantities and are required to find the largest unit that will measure each of them. In problems in Least Common Multiple, we are given two or more units of measure and are required to find the least quantity that can be measured by each of the units.

**98.** 3, 6, 9, 12, 15, 18, 21, 24, are multiples of 3.

2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, are multiples of 2.

$\therefore$  6, 12, 18, 24, are common multiples of 2 and 3, and it is evident that 6 is the L. C. M. of 2 and 3.

The second common multiple, 12, is  $2 \times 6$ .

The third common multiple, 18, is  $3 \times 6$ .

The fourth common multiple, 24, is  $4 \times 6$ .

What are the fifth and sixth common multiples of 2 and 3? In the same way find common multiples and the L. C. M. of 2 and 5, 3 and 4, 4 and 6, 8 and 12.

**99.** The preceding paragraph shows that the L. C. M. of two prime numbers, as 2 and 3, 2 and 5, 3 and 4, are their products 6, 10, 12, while the L. C. M. of two numbers not prime, as 4 and 6, and 8 and 12, which are 12 and 24, are less than their products, and are in both cases equal to the product of the numbers divided by their G. C. M.

**100.** (1) Find the shortest distance which can be exactly measured by two lines respectively 36 ft. and 48 ft. long.

We are here required to find the shortest distance, *i.e.* the L. C. M., of the units 36 ft. and 48 ft.

$$36 = 2 \times 2 \times 3 \times 3.$$

$$48 = 2 \times 2 \times 2 \times 2 \times 3.$$

Thus the L. C. M. of 36 and 48 =  $2 \times 2 \times 2 \times 2 \times 3 \times 3 = 144$ .

$\therefore$  the shortest distance is 144 ft.

(2) Find the L. C. M. of 24, 30, 36.

6	24	30	36
2	4	5	6
	2	5	3

Here 6 and 2 are the factors common to two or more of the numbers, and 2, 5, and 3 are the factors not common.

$$\therefore \text{the L. C. M.} = 6 \times 2 \times 2 \times 5 \times 3 = 360.$$

State how to find the L. C. M. of two or more numbers.

Show by division that 24, 30, and 36 are all factors of their L. C. M. 360.

(3) What is the least number of bu. of wheat that will make an exact number of full loads for three drays, hauling respectively 24, 30, and 36 bu. a load?

We are required to find the least number of bu., *i.e.* the L. C. M. of the units 24, 30, and 36 bu., which is 360 bu.

To prove the answer correct:

Dividing 360 bu. by 24, 30, and 36 bu. respectively, we find the number of loads to be 15, 12, and 10. 15, 12, and 10 have no common factor.

$\therefore$  360 bu. is the least number of bu.

**101.** (1) Find the L. C. M. of 14, 21, 54, 56, 84.

3	<del>14</del>	<del>21</del>	54	56	84
2			18	56	<del>28</del>
			9	28	

$$\therefore \text{the L. C. M.} = 3 \times 2 \times 9 \times 28 = 1512.$$

14 is erased since it is a factor of 56, and 21 since it is a factor of 84. In the second line, 28 is erased since it is a factor of 56.

(2) Find the L. C. M. of 481 and 1665.

$$\begin{array}{r}
 481 \overline{)1665} (3 \\
 \underline{1443} \\
 222 \overline{)481} (2 \\
 \underline{444} \\
 37 \overline{)222} (6 \\
 \underline{222}
 \end{array}$$

$$481 \div 37 = 13; 1665 \div 37 = 45.$$

$\therefore$  the L. C. M. of 481 and 1665 =  $37 \times 13 \times 45 = 481 \times 45 = 21,645$ .

State how to find the L. C. M. of two numbers.

Show by division that 481 and 1665 are both factors of 21,645.

Show that the L. C. M. 21,645 is equal to the product of 481 and 1665 divided by their G. C. M. 37.

### Exercise 58

Solve the following problems and name the units of measurement:

1. How long is the L. C. M. of two lines, one 6 in. long, and the other 10 in. long? How many times is it measured by the 6-in. line? By the 10-in. line?

2. Find the L. C. M. of three lines respectively 12 in., 15 in., and 18 in.

3. Four bells toll at intervals of 3, 7, 12, and 14 sec. respectively, and begin to toll at the same instant. When will they next toll together?

4. If in two days A can build 28 rd. of fencing, B 50 rd., C 16 rd., and D 40 rd., find the least number of rd. that will furnish an exact number of days' work.

5. When is a number a common multiple of two or more numbers, and when the least common multiple?

6. What is the least number of A. that will admit of being divided into a number of farms containing 150, 200, or 250 A. each?

7. In each question, prove your answer correct.

**Exercise 59**

Find the L. C. M. of :

- |                             |                         |                     |
|-----------------------------|-------------------------|---------------------|
| 1. 4, 8, 16, 32.            | 4. 15, 18, 28, 36.      | 7. 65, 26, 56, 52.  |
| 2. 3, 6, 9, 12.             | 5. 12, 20, 21, 45.      | 8. 36, 48, 60, 54.  |
| 3. 24, 30, 36, 45.          | 6. 22, 33, 30, 44.      | 9. 33, 27, 55, 135. |
| 10. 30, 21, 40, 28, 24, 56. | 11. 56, 36, 63, 28, 72. |                     |

Find the L. C. M. of the denominators of these fractions :

- |                                                               |                                                                |                                                               |
|---------------------------------------------------------------|----------------------------------------------------------------|---------------------------------------------------------------|
| 12. $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$ .                 | 13. $\frac{6}{25}, \frac{7}{40}, \frac{19}{60}$ .              | 14. $\frac{1}{4}, \frac{3}{7}, \frac{7}{12}, \frac{11}{18}$ . |
| 15. $\frac{2}{5}, \frac{5}{7}, \frac{11}{21}, \frac{8}{15}$ . | 17. $\frac{1}{3}, \frac{8}{15}, \frac{5}{12}, \frac{13}{42}$ . |                                                               |
| 16. $\frac{4}{5}, \frac{2}{3}, \frac{1}{9}, \frac{14}{15}$ .  | 18. $\frac{1}{3}, \frac{2}{5}, \frac{7}{12}, \frac{11}{18}$ .  |                                                               |

**Exercise 60**

Find the L. C. M. of :

- |                |                |                    |
|----------------|----------------|--------------------|
| 1. 266, 703.   | 3. 3045, 4515. | 5. 8159, 14,227.   |
| 2. 1173, 1702. | 4. 3589, 2257. | 6. 10,959, 12,753. |

**Exercise 61**

1. Show by examples that the L. C. M. of two numbers can never exceed their product.

2. Find the L. C. M. of 90, 150, and 168 by resolving the numbers into their prime factors. When several numbers have been resolved into their prime factors, which of these factors must be taken to form by their product the L. C. M. of the numbers ?

3. A, B, C, and D start together, and travel the same way around an island which is 600 mi. in circuit. A goes 20 mi. per da., B 30, C 25, and D 40. How long must their journeying continue, in order that they may all come together again ?

4. A shepherd, on telling his sheep, found that when he told them out by twos, threes, fours, and fives, he had none left, and he knew his flock was above 300, but less than 400. What number had he ?

5. Three bodies move uniformly in similar orbits round the same centre in 87, 224, 365 da. respectively. Supposing all three in conjunction at a given time, find after how many days they will be in conjunction again.

6. An island is 48 mi. in circumference. A, B, and C have to walk round till they all arrive together at the starting-point. A walks 2, B 3, and C 4 mi. an hour. How many times must each go round before the task is accomplished?

7. Find the L. C. M. of 11, 14, 28, 22, 7, 56, 27, 81, 54, and 36.

8. If 1 lb. Avoirdupois contains 7000 gr., and 1 lb. Troy 5760 gr., find the least weight which can be expressed without fractions in both lb. Troy and lb. Avoirdupois.

9. Prove by example that the product of the H. C. F. and the L. C. M. of two numbers is equal to the product of the numbers, and state why this is so.

10. The L. C. M. of 391 and another number is 12,121, and the G. C. M. is 23. Find the other number.

11. Along a certain path 1600 yd. long, there is a house every 50 yd., and a tree every 20 yd. How many houses will have a tree in front?

12. The periods of three planets which move uniformly in circular orbits round the sun, are respectively 200, 250, and 300 da. Supposing their positions relatively to each other and the sun to be given at any moment, determine how many da. must elapse before they again have exactly the same relative positions.



## CHAPTER X

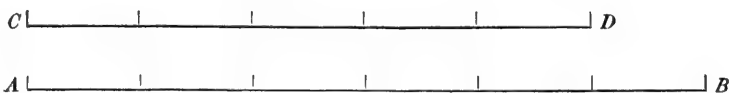
### FRACTIONS

**102.** The expression 5 ft. denotes a quantity measured by the number 5 and the unit 1 ft. The expression  $5 \times 2$  in. denotes a quantity measured by the number 5 and the unit 2 in. The expression  $5 \times \frac{1}{6}$  ft. denotes a quantity measured by the number 5 and the *unit one-sixth* of a ft.

**103.** The expression  $\frac{5}{6}$  ft. denotes that the quantity 1 ft. is conceived as made up of 6 equal parts or units, and that 5 of these parts or units have been taken to measure the quantity denoted by  $\frac{5}{6}$  ft.

The primary unit, 1 ft., has been divided into 6 equal parts to give the direct measuring unit, which is  $\frac{1}{6}$  ft., or 2 in. The number of these units in the given quantity is 5. The *ratio* of the given quantity to the direct measuring unit is 5.

**104.** The quantity represented by  $\frac{5}{6}$  ft. contains 5 direct measuring units, and the primary unit, 1 ft., contains 6 of these units. Hence the fraction  $\frac{5}{6}$  *expresses the ratio of the quantity denoted by  $\frac{5}{6}$  ft. to the primary unit, 1 ft.*



**105.** Draw a line,  $AB$ , 1 ft. long. Divide it into 6 equal parts, or units, each  $\frac{1}{6}$  of a ft. long. Draw a second line,  $CD$ , above the first, containing 5 of these units, and use these two lines to illustrate the preceding paragraph.

## Exercise 62

In the following questions give the number of direct measuring units that makes up the primary unit. Name the direct measuring unit in two ways. Give the ratio of the quantity to the direct measuring unit. Give the ratio of the quantity to the primary unit.

- |                          |                         |                           |                        |
|--------------------------|-------------------------|---------------------------|------------------------|
| 1. $\frac{3}{4}$ ft.     | 5. \$ $\frac{3}{4}$ .   | 9. $\frac{27}{8}$ sq. ft. | 13. $\frac{3}{8}$ da.  |
| 2. $\frac{2}{3}$ yd.     | 6. \$ $\frac{2}{5}$ .   | 10. $\frac{5}{9}$ cu. yd. | 14. $\frac{3}{4}$ da.  |
| 3. $\frac{7}{8}$ lb.     | 7. \$ $\frac{6}{10}$ .  | 11. $\frac{2}{7}$ wk.     | 15. $\frac{11}{5}$ hr. |
| 4. $\frac{5}{9}$ sq. yd. | 8. \$ $\frac{13}{20}$ . | 12. $\frac{24}{3}$ yr.    | 16. $\frac{5}{8}$ min. |

**106. A Fraction** is a number in which the unit of measure is a definite part of some primary unit of the same kind.

The **denominator** shows into how many parts the primary unit is divided to give the direct unit of measure; it also *names* this unit. The **numerator** shows the number of them that measures the quantity.

A **proper fraction**, as an expression of measured quantity, is one in which the numerator is less than the denominator. Select the proper fractions:  $\frac{2}{3}$ ,  $\frac{4}{7}$ ,  $\frac{9}{5}$ ,  $\frac{8}{8}$ ,  $\frac{4}{5}$ .



**107.** Let  $AB$  represent some quantity measured by 5 units, each equal to  $AC$ , and  $DE$  as measured by 6 units, each equal to  $AC$ . Then if we think of  $AB$  in relation to  $DE$ , we think of 5 units in relation to 6 units, and this relation or ratio is expressed by the fraction  $\frac{5}{6}$ .

Similarly, the fraction, or number  $\frac{5}{6}$ , expresses the ratio of \$5 to \$6, 5 hr. to 6 hr., 5 mi. to 6 mi., 5 (8 ft.) to 6 (8 ft.), 5 (12 lb.) to 6 (12 lb.), or, generally, 5 of any unit to 6 of the same unit.



It is evident from this diagram that  $\frac{2}{2}, \frac{3}{3}, \frac{4}{4}, \frac{5}{5}, \frac{6}{6}, \frac{8}{8}, \frac{10}{10}, \frac{12}{12}$ , of a quantity measure it, and are all equal. It is also evident that  $\frac{1}{2}, \frac{2}{4}, \frac{3}{6}, \frac{4}{8}, \frac{5}{10}, \frac{6}{12}, \frac{8}{16}$ , of a quantity measure one-half of it, and are all equal.

Similarly,  $\frac{1}{3}, \frac{2}{6}, \frac{4}{12}$ , of a quantity measure one-third of it, and are equal. Similarly,  $\frac{1}{5} = \frac{2}{10}$ .

#### Exercise 64

- Find  $\frac{1}{2}$  of \$8;  $\frac{2}{4}$  of \$8;  $\frac{4}{8}$  of \$8.
- Find  $\frac{1}{5}$  of \$30;  $\frac{2}{10}$  of \$30;  $\frac{3}{15}$  of \$30.
- Find  $\frac{1}{8}$  of 36 ft.; find also respectively  $\frac{2}{8}, \frac{3}{8}, \frac{4}{8}, \frac{6}{8}$ , and  $\frac{12}{8}$  of 36 ft.
- Find  $\frac{2}{8}$  of 24 dimes;  $\frac{4}{8}$  of 24 dimes;  $\frac{8}{12}$  of 24 dimes.
- Find  $\frac{1}{4}$  of 28 lb.;  $\frac{2}{8}$  of 28 lb.;  $\frac{3}{6}$  of 28 lb.
- Find  $\frac{2}{4}$  of 32 da.;  $\frac{4}{8}$  of 32 da.;  $\frac{6}{12}$  of 32 da.;  $\frac{8}{16}$  of 32 da.
- Name three fractions equal to  $\frac{1}{2}$ ; three equal to  $\frac{1}{5}$ ; three equal to  $\frac{3}{4}$ .
- Name two fractions each equal to  $\frac{1}{4}$ , and prove your results correct by taking each fraction of \$24.

#### Exercise 65

- If 1 da. is made up of 24 measuring units, find the number of such units respectively in  $\frac{1}{2}, \frac{1}{4}, \frac{1}{6}, \frac{1}{8}, \frac{1}{12}, \frac{1}{24}$ , of a da.
- If 1 da. is made up of 24 measuring units, find the number of such units respectively in  $\frac{1}{2}, \frac{2}{4}, \frac{3}{6}, \frac{4}{8}, \frac{6}{12}$ , and  $\frac{12}{24}$  of a da.
- If 1 hr. is made up of 60 measuring units, find the number of units in  $\frac{1}{3}, \frac{2}{10}, \frac{4}{20}$ , of an hr.
- If 1 wk. contains 168 units of time, how many units of time are there respectively in  $\frac{1}{7}, \frac{2}{14}, \frac{3}{21}$ , and  $\frac{4}{28}$  of a wk.?

5. If 1 sq. ft. contains 144 units of area, find the number of units of area in  $\frac{3}{4}$ ,  $\frac{6}{8}$ ,  $\frac{9}{12}$ ,  $\frac{12}{16}$ , and  $\frac{1}{2}$  of a sq. ft. respectively.

6. If \$1 contains 100 measuring units, find the number of units that measure  $\$ \frac{1}{5}$ ,  $\$ \frac{1}{10}$ ,  $\$ \frac{1}{20}$ , and  $\$ \frac{1}{50}$  respectively.

7. If 1 lb. is represented by a 16-unit quantity, what part of a lb. is a 1-unit quantity? a 2-unit quantity? a 4-unit quantity? an 8-unit quantity? What part of a pound is eight 1-unit quantities? four 2-unit quantities? two 4-unit quantities? one 8-unit quantity?

8. What actual coins and how many of each kind are equal respectively to  $\frac{1}{2}$ ,  $\frac{2}{4}$ ,  $\frac{5}{10}$ , and  $\frac{1}{20}$  of \$1?

9. From the above examples state what changes can be made in the numerator and denominator of a fraction without altering the value of the fraction.

**109.** In this exercise, the quantities 1 da., 1 hr., 1 wk., 1 sq. ft., \$1, and 1 lb. have been said to contain a certain number of units. Thus, 1 da. contains 24 units of 1 hr.; 1 hr., 60 units of 1 min.; 1 wk., 168 units of 1 hr. Any measured quantity may, however, be represented by *any* convenient number of units. Thus 1 da. may be measured by 75 units, \$1 by 24 units, a piece of work by 40 units, and so on.

**110.** (1) Express 9 yd. as eighths of a yd.



Let the line  $AB$  be drawn to represent 1 yd. Think of 1 yd. as containing 8 units of length, as shown in the diagram.

Then 9 yd. will contain  $9 \times 8$ , or 72 of these units.

Therefore 9 yd. is equal to  $\frac{72}{8}$  of 1 yd.

(2) Express  $\$ \frac{3}{4}$  as a fraction with 20 as a denominator.

Think of \$1 as containing 20 units of 5¢ each. Then  $\$ \frac{3}{4}$  contains  $\frac{3}{4}$  of 20, or 15 of these units. Therefore,  $\$ \frac{3}{4} = \$ \frac{15}{20}$ .

**Exercise 66**

Express as fractions :

- |                     |                     |
|---------------------|---------------------|
| 1. \$ 8 as 10ths.   | 7. 5 yr. as 12ths.  |
| 2. \$ 4 as 20ths.   | 8. 3 da. as 24ths.  |
| 3. 9 yd. as 3ds.    | 9. 4 min. as 60ths. |
| 4. 6 ft. as 12ths.  | 10. 6 hr. as 60ths. |
| 5. 14 gal. as 4ths. | 11. 8 pk. as 8ths.  |
| 6. 11 wk. as 7ths.  | 12. 12 bu. as 4ths. |

13. In each of these questions name the direct measuring unit in your fraction as an actual unit of measure in common use.

14. Express as fractions with 100 as denominator:  $\frac{1}{2}$ ,  $\frac{1}{4}$ ,  $\frac{1}{5}$ ,  $\frac{1}{10}$ ,  $\frac{1}{20}$ ,  $\frac{1}{25}$ ,  $\frac{3}{4}$ ,  $\frac{3}{5}$ ,  $\frac{2}{5}$ ,  $\frac{9}{10}$ .

Reduce, illustrating your work by diagrams :

- |                                  |                                 |
|----------------------------------|---------------------------------|
| 15. $\frac{3}{4}$ ft. to 12ths.  | 19. $\frac{7}{8}$ da. to 24ths. |
| 16. $\frac{2}{3}$ yd. to 33ds.   | 20. $\frac{2}{13}$ to 78ths.    |
| 17. $\frac{5}{8}$ lb. to 16ths.  | 21. $\frac{4}{15}$ to 45ths.    |
| 18. $\frac{7}{9}$ gal. to 28ths. | 22. $\frac{11}{18}$ to 54ths.   |

23. What are the new units of measure in your results? Where possible, identify them with actual units in common use.

**111.** (1) Express 8 in. as a fraction of 1 ft.

$$8 \text{ in.} = \frac{8}{12} \text{ or } \frac{2}{3} \text{ of a ft.} = \frac{2}{3} \text{ ft.}$$

(2) A man's capital is represented by 20 units of value. He invests  $\frac{1}{4}$  of it in land and  $\frac{1}{3}$  of the remainder in bank stock. How many units did he invest in bank stock, and what part is it of his entire capital?

The amount invested in land =  $\frac{1}{4}$  of 20 units, or 5 units.

The remainder =  $20 - 5$ , or 15 units.

The amount invested in stock =  $\frac{1}{3}$  of 15, or 5 units.

$\therefore$  the amount invested in stock =  $\frac{5}{20}$ , or  $\frac{1}{4}$  of his entire capital.

**Exercise 67**

1. Express as a fraction of a ft. :  
3 in., 4 in., 5 in., 6 in., 9 in., 10 in.
2. Express as a fraction of a lb. Avoir. :  
14 oz., 6 oz., 9 oz., 12 oz., 15 oz.
3. Express as a fraction of a mo. :  
12 da., 15 da., 18 da., 20 da., 25 da.
4. If 1 mi. is the measuring unit, what number measures each of the following :  
20 rd. ? 35 rd. ? 80 rd. ? 120 rd. ? 150 rd. ?
5. Express as a fraction of a da. :  
6 hr., 9 hr., 12 hr., 15 hr., 18 hr.
6. Express as fractions of a dollar :  
25, 50, 75, 60, 80, and 100 cents.
7. If \$4 is used as a measure of \$4, what is the number expressing the measurement ? If \$4 is used as a measure of \$3, what is the number ?
8. If cloth is 40¢ a yd., how much can I buy for 30¢ ?
9. If oranges cost 30¢ a doz., what part of a doz. can I buy for 25¢ ? How many oranges ?
10. A man walks a certain distance in 4 hr. What part of it does he walk in 1 hr. ? In 2 hr. ? In  $\frac{1}{2}$  an hr. ?
11. A can do a piece of work in 3 hr., B the same amount in 4 hr. If the work be measured by 12 units, how many units will A do in an hr. ? How many B ? How many both working together ? What part will their joint work for an hr. be of the whole work ?
12. A received a certain sum of money, B twice as much, and C as much as A and B together. How many units measure the entire amount ? B receives what part of the whole sum ?
13. Given that pure water contains 15 parts by weight of oxygen, and 2 parts of hydrogen, what part of the weight of a gallon of water is hydrogen ?

14. Six brothers join in paying a debt of \$700. The eldest pays  $\frac{1}{4}$  of it, and each of the others  $\frac{1}{4}$  of the remainder. How much does the eldest pay? How much does each of the five pay? This is what part of the whole debt?

15. If a pipe empties a tank at the rate of 12 gal. in 1 min., what is the rate per sec.?

16. \$40 is divided among A, B, and C, giving A  $\frac{1}{4}$  of it, B  $\frac{1}{4}$ , and C the remainder. What is the sum of A and B's shares? What is C's share? C's share is what part of the whole sum?

17. The value of a mine is represented by 10 units of money. A man who owns  $\frac{3}{4}$  of it sells  $\frac{1}{4}$  of his share. How many units did he own? How many did he sell? What part of the whole mine did he sell?

18. The length of an oblong is 8 ft., and the width 4 ft. What part of the perimeter is the length?

19. An oblong 6 ft. wide and 8 ft. long is divided into strips, each 1 ft. wide, made by drawing lines parallel to the length. What part of the area of the oblong is the area of one strip?

20. The area of an oblong is 16 sq. ft. What part of its area is that of a square whose side is 2 ft.?

21. If 20 units measure the cost of a lb. of tea sold at a gain of  $\frac{3}{10}$  of the cost, how many units are gained by selling? What is the selling price? The cost price is what part of the selling price? What is the ratio of the selling price to the cost? The gain is what part of the selling price?

22. A has 12 marbles, and B has 3. They play together, and A loses  $\frac{1}{4}$  of his marbles. How many has B now? What part are they of what A now has?

23. The value of a house is measured by 5 units. The lot on which it stands is worth  $\frac{1}{5}$  as much as the house. What is the measure of the value of the house and lot? The value of the lot is worth what part of both together?



24. A and B set out at the same time from places 42 mi. apart, and meet at the end of 6 hr. A travels at the rate of 3 mi. an hr. How far does B travel? What is B's rate? A's rate is what part of B's?

25. Apples are sold at the rate of 12 for a dime, and bananas at the rate of 8 for a dime. Compare the value of an apple with that of a banana.

NOTE. — Let the dime be measured by 24 units.

26. A crew can row 6 mi. an hour in still water. What is the rate of rowing up stream in a current running at the rate of 2 mi. an hr.? What is the rate down stream? What is the ratio of the rate down stream to that up stream?

### REDUCTION OF FRACTIONS

**112.** A fraction is in its lowest terms when its numerator and denominator have no common factor.

(1) Reduce  $\$ \frac{315}{495}$  to its lowest terms.

The object of reduction is to give a more definite idea of the value of the quantity by expressing the ratio in the smallest numbers.

$$\$ \frac{315}{495} = \$ \frac{105}{165} = \$ \frac{35}{55} = \$ \frac{7}{11}.$$

The common factors are 3, 3, and 5.

The effect of dividing each term of the first fraction by 3 is to make each measuring unit 3 times as large, and to reduce the number of these units to one-third as many. Similarly, with the second division by 3, and the third by 5.

(2) Reduce  $\frac{713}{992}$  to its lowest terms.

713	992
558	713
155	279
124	155
31	124
	124

$$\therefore \frac{713}{992} = \frac{713 \div 31}{992 \div 31} = \frac{23}{32}.$$

In this instance the direct measuring unit of the fraction  $\frac{23}{31}$  is 31 times as great as that of the fraction  $\frac{713}{992}$ , but only  $\frac{1}{31}$  as many units are needed to measure the quantity denoted by  $\frac{713}{992}$  or  $\frac{23}{31}$  of the primary unit.

(3) A real estate dealer bought a house for \$2545 and sold it for \$1679. Find the ratio of the selling price to the cost price.

The selling price =  $\frac{1679}{2545}$  of the cost price  
 =  $\frac{23}{31}$  of the cost price.

The G. C. M. of 1679 and 2545 is 73.

Dividing both numerator and denominator by 73, we have the required ratio equal to  $\frac{23}{31}$ .

### Exercise 68

Reduce to its lowest terms:

- |                         |                       |                        |
|-------------------------|-----------------------|------------------------|
| 1. $\$ \frac{16}{24}$ . | 6. $\frac{48}{64}$ .  | 11. $\frac{35}{91}$ .  |
| 2. $\$ \frac{16}{34}$ . | 7. $\frac{14}{83}$ .  | 12. $\frac{84}{96}$ .  |
| 3. $\$ \frac{14}{22}$ . | 8. $\frac{32}{56}$ .  | 13. $\frac{24}{80}$ .  |
| 4. $\$ \frac{39}{48}$ . | 9. $\frac{18}{99}$ .  | 14. $\frac{48}{156}$ . |
| 5. $\$ \frac{14}{18}$ . | 10. $\frac{54}{78}$ . |                        |

15. If I pay \$60 for a bicycle and afterward sell it for \$45, what part of the cost do I sell it for?

16. A merchant sells 55 yd. of cloth from a piece containing 66 yd. What part of the whole piece does he sell?

17. A man buys a horse for \$128 and sells it for \$112. Find the selling price as a fraction of the cost.

18. On an investment of \$88 a merchant gains \$33. Compare the gain with the cost.

19. Sixty days is what fraction of a year?

20. Out of a farm containing 135 A., 81 A. were sold. Find what part of the farm was sold.

## Exercise 69

Reduce to the lowest terms :

1.  $\frac{234}{815}$ .

3.  $\frac{459}{1139}$ .

5.  $\frac{3996}{5681}$ .

7.  $\frac{1218}{7647}$ .

2.  $\frac{559}{663}$ .

4.  $\frac{2124}{3195}$ .

6.  $\frac{1161}{1377}$ .

8.  $\frac{1617}{8083}$ .

9. A person made \$ 282 on an investment of \$ 329. What part of the cost did he gain ?

10. A speculator paid \$ 1071 for a lot and shortly after sold it at an advance of \$ 259. Find his gain as a fraction (in its lowest terms) of the cost price.

11. A man insured his house which cost \$ 2552 for \$ 1798. For what part of the cost did he insure it ?

12. I paid \$ 1449 for a house and insured it so that in case of fire I should lose only \$ 184. What fraction of the cost would I lose in case of fire ?

13. Property to the value of \$ 3341 was assessed for \$ 1542. Find the ratio of the assessed valuation to the real value of the property.

14. Having \$ 5943, I invested \$ 5094 in business. What part of my money did I invest in business ?

**113.** Such an expression (as  $\$ 2\frac{1}{3}$ , *e.g.*) denotes a quantity in which two units of measure of different values have been used : a primary unit and parts of this, a derived unit.

$\$ 2\frac{1}{3}$  is the number 7 in disguise. To make it *number* in the strict sense, we must express the quantity in the *smaller* unit of measure, *i.e.* as  $\$ \frac{7}{3}$ ; this as a quantity can be *counted*;  $\$ 2\frac{1}{3}$  *cannot* be *counted*.

An *improper fraction* as an expression of measured quantity is one whose numerator is equal to or greater than its denominator; as  $\frac{7}{6}$ ,  $\frac{8}{8}$ ,  $\frac{8}{8}$ ,  $\frac{10}{8}$ .

**(1) Reduce to an improper fraction  $75\frac{2}{3}$  yd.**

Let 1 yd. = 3 units of length of  $\frac{1}{3}$  yd. each.

Then 75 yd. = 225 units of length of  $\frac{1}{3}$  yd. each.

and  $\frac{2}{3}$  yd. = 2 units of length of  $\frac{1}{3}$  yd. each.

$\therefore 75\frac{2}{3}$  yd. = 227 units of length of  $\frac{1}{3}$  yd. each, or  $227\frac{1}{3}$  yd.

Hence if we count the smaller unit  $\frac{1}{3}$  yd. 227 times, we measure the whole quantity  $75\frac{2}{3}$  yd.

**Exercise 70**

Reduce to improper fractions:

- |                        |                         |                       |
|------------------------|-------------------------|-----------------------|
| 1. $\$4\frac{1}{4}$ .  | 8. $20\frac{3}{4}$ gal. | 15. $3\frac{4}{17}$ . |
| 2. $\$4\frac{3}{8}$ .  | 9. $8\frac{5}{8}$ pk.   | 16. $20\frac{3}{5}$ . |
| 3. $\$7\frac{3}{5}$ .  | 10. $5\frac{3}{16}$ lb. | 17. $21\frac{3}{4}$ . |
| 4. $\$16\frac{1}{2}$ . | 11. $12\frac{3}{7}$ .   | 18. $19\frac{4}{5}$ . |
| 5. $\$9\frac{3}{10}$ . | 12. $9\frac{5}{9}$ .    | 19. $7\frac{6}{13}$ . |
| 6. $8\frac{1}{3}$ yd.  | 13. $12\frac{2}{11}$ .  | 20. $40\frac{7}{8}$ . |
| 7. $5\frac{7}{12}$ ft. | 14. $6\frac{8}{15}$ .   |                       |

21. What are the two units of measurement in each question? What is the unit of measurement in the result? How many of these units must be counted to measure the quantities?

22. What is the quantity which is measured by the unit  $\frac{2}{3}$  ft. and the number 9? If the same quantity be measured by the number 3, what is the measuring unit?

23. If 9 boys receive  $\$ \frac{4}{5}$  each, what sum was divided? If the same sum had been divided among 4 boys, what would each have received?

**Exercise 71**

Reduce to improper fractions:

- |                        |                        |                        |                           |
|------------------------|------------------------|------------------------|---------------------------|
| 1. $15\frac{3}{11}$ .  | 5. $34\frac{5}{14}$ .  | 9. $64\frac{21}{47}$ . | 13. $29\frac{14}{35}$ .   |
| 2. $21\frac{9}{18}$ .  | 6. $48\frac{15}{19}$ . | 10. $152\frac{8}{9}$ . | 14. $87\frac{232}{301}$ . |
| 3. $29\frac{15}{16}$ . | 7. $51\frac{25}{33}$ . | 11. $98\frac{4}{15}$ . | 15. $341\frac{17}{19}$ .  |
| 4. $41\frac{3}{28}$ .  | 8. $36\frac{3}{4}$ .   | 12. $168\frac{5}{8}$ . | 16. $453\frac{41}{116}$ . |

**114.** (1) Which is greater,  $\frac{5}{8}$  ft. or  $\frac{6}{7}$  ft.?

Consider the ft. as having 42 measuring units, then  $\frac{5}{8}$  of this is 35, and  $\frac{6}{7}$  is 36 units.

$\therefore \frac{6}{7}$  ft. is greater than  $\frac{5}{8}$  ft. by 1 unit, or  $\frac{1}{42}$  ft.

(2) Compare the quantities  $\$ \frac{3}{4}$ ,  $\$ \frac{7}{12}$ ,  $\$ \frac{5}{8}$ .

Let \$1 be represented by 24 units of value. Then  $\$ \frac{3}{4}$ ,  $\$ \frac{7}{12}$ ,  $\$ \frac{5}{8}$ , are respectively equal to 18, 14, and 15 units. Hence the first fraction is the greatest and the second the least.

### Exercise 72

Find the greatest and least of the following:

1.  $\frac{2}{3}$  yd.,  $\frac{3}{4}$  yd.

4.  $\frac{5}{6}$  ft.,  $\frac{3}{4}$  ft.,  $\frac{11}{12}$  ft.

2.  $\$ \frac{7}{8}$ ,  $\$ \frac{8}{9}$ .

5.  $\frac{3}{4}$ ,  $\frac{8}{11}$ ,  $\frac{17}{22}$ .

3.  $\$ \frac{3}{10}$ ,  $\$ \frac{1}{3}$ ,  $\$ \frac{7}{20}$ .

6.  $\frac{2}{3}$ ,  $\frac{5}{8}$ ,  $\frac{11}{16}$ .

7. State how to find the greatest and least of a number of fractions.

**115.** Express  $\frac{23}{5}$  yd. in terms of the primary and derived units of measure.

The primary unit 1 yd. contains 5 derived units of  $\frac{1}{5}$  of a yd. each.

Hence  $\frac{23}{5}$  yd., i.e. 23 derived units = 4 primary and 3 derived units;

$$= 4\frac{3}{5} \text{ yd.}$$

Or, more simply,

$$\begin{array}{r} 5 \overline{)23} \\ \underline{45} \\ 4\frac{3}{5} \end{array}$$

### Exercise 73

Express in terms of the primary and direct measuring units:

1.  $\$ \frac{43}{5}$ .

5.  $\frac{187}{12}$  yr.

9.  $\frac{385}{63}$  mi.

2.  $\$ \frac{89}{7}$ .

6.  $\frac{293}{16}$  ft.

10.  $\frac{3708}{87}$  oz.

3.  $\frac{113}{8}$  da.

7.  $\frac{284}{42}$  yd.

11.  $\frac{4849}{125}$  lb.

4.  $\frac{124}{5}$  wk.

8.  $\frac{276}{45}$  rd.

12.  $\frac{8099}{343}$  lb.

13. What are the two units of measurement in the answers to the above problems?

## Exercise 74

Reduce to integers and proper fractions:

1.  $\frac{8048}{112}$ . 2.  $\frac{20835}{144}$ . 3.  $\frac{8943}{97}$ . 4.  $\frac{9947}{61}$ . 5.  $\frac{5793}{389}$ . 6.  $\frac{8679}{231}$ .

## ADDITION OF FRACTIONS

**116.** The sum of 3 dimes and 4 dimes = 7 dimes.

The sum of  $\$ \frac{3}{10}$  and  $\$ \frac{4}{10} = \$ \frac{7}{10}$ .

The sum of 5 oz. and 8 oz. = 13 oz.

The sum of  $\frac{5}{16}$  lb. and  $\frac{8}{16}$  lb. =  $\frac{13}{16}$  lb.

Here the direct measuring unit of value is  $\$ \frac{1}{10}$  or 1 dime, that of weight  $\frac{1}{16}$  lb. or 1 oz.

(1) Add  $\frac{1}{2}$  ft.,  $\frac{2}{3}$  ft.,  $\frac{3}{4}$  ft.

The L. C. M. of the denominators is seen to be 12; the ft. is considered as measured by 12 equal parts or units.

$\frac{1}{2}$  ft. = 6 units;  $\frac{2}{3}$  ft. = 8 units;  $\frac{3}{4}$  ft. = 9 units.

The sum = 6 + 8 + 9 or 23 units

=  $\frac{23}{12}$  ft. or  $1\frac{11}{12}$  ft.

$\therefore$  the sum =  $1\frac{11}{12}$  ft.

Prove the correctness of this result by drawing a line and measuring off  $\frac{1}{2}$  ft.,  $\frac{2}{3}$  ft., and  $\frac{3}{4}$  ft., and also  $1\frac{11}{12}$  ft.

(2) Find the sum of  $\$21\frac{3}{8}$ ,  $\$15\frac{5}{6}$ ,  $\$13\frac{4}{9}$ ,  $\$8\frac{7}{12}$ .

The L. C. D. = 72,  $\therefore$  the dollar is considered as measured by 72 units.

$\$ \frac{3}{8} + \$ \frac{5}{6} + \$ \frac{4}{9} + \$ \frac{7}{12} = 27 + 60 + 32 + 42$  or 161 units, i.e.  $\$ \frac{161}{72}$  or  $\$2\frac{17}{72}$ .

$\$21 + \$15 + \$13 + \$8 = \$57$ .

$\therefore$  the sum =  $\$57 + \$2\frac{17}{72} = \$59\frac{17}{72}$ .

## Exercise 75

Find the sum of:

1.  $\$ \frac{3}{4}$ ,  $\$ \frac{1}{4}$ . 3.  $\frac{7}{16}$  oz.,  $\frac{2}{16}$  oz. 5.  $\frac{4}{7}$  wk.,  $\frac{3}{7}$  wk.  
2.  $\frac{5}{12}$  ft.,  $\frac{4}{12}$  ft. 4.  $\frac{13}{24}$  da.,  $\frac{18}{24}$  da. 6.  $\frac{37}{60}$  min.,  $\frac{41}{60}$  min.

## Exercise 76

In the following exercise what is a *convenient* number of direct measuring units by which to represent the primary units \$1, 1 ft., 1 lb., 1 gal., and so on?

Find the sum of:

1.  $\$ \frac{1}{2}$ ,  $\$ \frac{1}{4}$ . 2.  $\$ \frac{1}{2}$ ,  $\$ \frac{3}{4}$ ,  $\$ \frac{5}{8}$ . 3.  $\frac{2}{3}$  ft.,  $\frac{3}{4}$  ft. 4.  $\frac{1}{3}$  ft.,  $\frac{3}{4}$  ft.,  $\frac{5}{8}$  ft.

Prove your answers to 3 and 4 correct by measuring with a ruler.

5.  $\frac{3}{4}$  lb.,  $\frac{5}{8}$  lb.,  $\frac{1}{16}$  lb. 6.  $\frac{1}{2}$  gal.,  $\frac{3}{4}$  gal. 7.  $\frac{3}{4}$  bu.,  $\frac{7}{8}$  bu.,  $\frac{1}{16}$  bu.  
8.  $\frac{2}{3}$  yr.,  $\frac{1}{8}$  yr. 9.  $\frac{2}{3}$  da.,  $\frac{5}{8}$  da.,  $\frac{3}{4}$  da.

Prove your results correct by reducing each fractional quantity to a lower denomination (as  $\$ \frac{1}{2}$  to ct.,  $\frac{2}{3}$  ft. to in.,  $\frac{3}{4}$  lb. to oz., and so on) and also your result. Then add the integers.

## Exercise 77

Find the sum of the following fractional parts of any unit:

1.  $\frac{3}{8}$ ,  $\frac{4}{5}$ ,  $\frac{2}{3}$ .
2.  $\frac{7}{12}$ ,  $\frac{16}{21}$ ,  $\frac{9}{28}$ .
3.  $\frac{1}{15}$ ,  $\frac{17}{21}$ ,  $\frac{12}{35}$ .
4.  $\frac{5}{12}$ ,  $\frac{8}{15}$ ,  $\frac{13}{20}$ .
5.  $\frac{3}{14}$ ,  $\frac{19}{24}$ ,  $\frac{11}{21}$ ,  $\frac{37}{42}$ .
6.  $\frac{8}{9}$ ,  $\frac{7}{9}$ ,  $\frac{13}{18}$ ,  $\frac{14}{18}$ ,  $\frac{15}{44}$ .
7.  $3\frac{2}{3}$ ,  $8\frac{4}{15}$ ,  $23\frac{7}{89}$ ,  $18\frac{22}{45}$ ,  $46\frac{6}{65}$ .
8.  $2\frac{3}{4}$ ,  $3\frac{5}{8}$ ,  $5\frac{2}{7}$ ,  $9\frac{5}{11}$ ,  $28\frac{1}{21}$ .
9.  $23\frac{7}{12}$ ,  $32\frac{1}{18}$ ,  $43\frac{29}{36}$ ,  $62\frac{1}{18}$ .
10.  $13\frac{17}{105}$ ,  $47\frac{24}{165}$ ,  $34\frac{81}{231}$ ,  $13\frac{27}{385}$ .

11. State how to find the sum of two or more (1) proper fractions, (2) mixed numbers.

12. Explain clearly the principles involved in finding the sum of two fractions.

## SUBTRACTION OF FRACTIONS

**117.** (1) Find the difference between  $\frac{5}{6}$  hr. and  $\frac{8}{15}$  hr.

Let 1 hr. = 30 units of time.

Then  $\frac{5}{6}$  hr. = 25 units and  $\frac{8}{15}$  hr. = 16 units.

$\therefore$  the difference =  $25 - 16$  or 9 units, i.e.  $\frac{9}{30}$  or  $\frac{3}{10}$  hr.

Prove this result by expressing  $\frac{5}{6}$ ,  $\frac{8}{15}$ , and  $\frac{3}{10}$  hr. in min., and taking the difference of the first two.

(2) Find the value of:

$$\$6\frac{29}{36} - \$4\frac{17}{28}.$$

$$\$6 - \$4 = \$2.$$

$$\text{The L. C. D.} = 144.$$

Let \$1 = 144 units.

Then  $\$ \frac{29}{36} = 116$  units;  $\$ \frac{17}{28} = 51$  units.

$$116 - 51 = 65.$$

$$\therefore \$6\frac{29}{36} - \$4\frac{17}{28} = \$2\frac{65}{144}.$$

(3) Subtract  $6\frac{7}{12}$  da. from  $9\frac{3}{8}$  da.

Let 1 da. = 24 units of time.

$$\begin{aligned} \text{Then } 9\frac{3}{8} \text{ da.} - 6\frac{7}{12} \text{ da.} &= 3 \text{ da.} + 9 \text{ units} - 14 \text{ units} \\ &= 2 \text{ da.} + 33 \text{ units} - 14 \text{ units} \\ &= 2 \text{ da.} + 19 \text{ units} \\ &= 2\frac{19}{12} \text{ da.} \end{aligned}$$

## Exercise 78

Find the value of:

1.  $\$ \frac{9}{10} - \$ \frac{5}{10}.$

4.  $\frac{3}{4} \text{ gal.} - \frac{1}{4} \text{ gal.}$

2.  $\frac{7}{12} \text{ ft.} - \frac{3}{12} \text{ ft.}$

5.  $\frac{31}{60} \text{ hr.} - \frac{16}{60} \text{ hr.}$

3.  $\frac{7}{8} \text{ pk.} - \frac{3}{8} \text{ pk.}$

6.  $\frac{15}{24} \text{ da.} - \frac{3}{24} \text{ da.}$

7. What is the direct unit of measure in each question? Express your answer in two ways.

8. Prove results by reducing each fraction to the next lower denomination and then subtracting.



**Exercise 79**

Find the difference as a fraction of any unit of measure :

1.  $\frac{3}{4} - \frac{2}{3}$ .

3.  $\frac{6}{7} - \frac{2}{3}$ .

5.  $1\frac{1}{2} - \frac{8}{9}$ .

2.  $\frac{7}{8} - \frac{5}{6}$ .

4.  $\frac{8}{15} - \frac{3}{10}$ .

6.  $\frac{13}{18} - \frac{5}{12}$ .

7. Give the three steps required in subtracting one proper fraction from another.

8.  $12 - \frac{4}{9}$ .

13.  $12 - 8\frac{5}{4}$ .

18.  $5\frac{1}{2} - 3\frac{2}{8}$ .

9.  $5 - \frac{7}{18}$ .

14.  $8\frac{9}{16} - 5\frac{5}{12}$ .

19.  $4\frac{5}{18} - 1\frac{3}{4}$ .

10.  $8 - \frac{5}{31}$ .

15.  $9\frac{2}{3} - 4\frac{1}{8}$ .

20.  $46\frac{1}{9} - 39\frac{7}{9}$ .

11.  $7 - 4\frac{3}{8}$ .

16.  $18\frac{1}{15} - 12\frac{3}{20}$ .

21.  $95\frac{2}{3} - 84\frac{1}{4}$ .

12.  $9 - 6\frac{1}{11}$ .

17.  $4\frac{1}{2} - 2\frac{1}{8}$ .

**Exercise 80**

1.  $\frac{2}{5}$  of the value of a horse = \$ 60.

$\frac{1}{5}$  of the value of a horse = .

$\frac{3}{5}$  of the value of a horse = .

$\therefore$  the horse is worth \$ .

Fill out the blanks.

2. If  $\frac{3}{4}$  of the value of a farm is \$ 9000, what is the value of  $\frac{1}{4}$  of the farm ? What is the farm worth ?

3. A person sold a cow, gaining  $\frac{2}{7}$  of the cost price. If he gained \$ 12, what did the cow cost him ?

4. A man lost in business  $\frac{5}{8}$  of his property. His loss was \$ 4500; what was his property worth ?

5. A boy lost  $\frac{3}{8}$  of his marbles, and then had 60 marbles left. What fraction of his marbles did he have left ? How many marbles had he at first ?

6. If  $\frac{5}{4}$  of the cost of a gal. of a wine is \$ 3, what was the cost ?

7. A merchant sold potatoes at 75¢ a bu., gaining  $\frac{1}{4}$  of the cost. The selling price was what fraction of the cost? What was the cost price per bu. of the potatoes?

8. A merchant sold cloth for 80¢ a yd., thereby losing  $\frac{1}{8}$  of the cost. Find the cost.

## MULTIPLICATION OF FRACTIONS

**118.** (1) Find the cost of 12 yd. of cloth at  $\$ \frac{3}{4}$  per yd.

We are required to find the quantity measured by the number 12 and the measuring unit  $\$ \frac{3}{4}$ .

$$12 \text{ yd. cost } 12 \times \$ \frac{3}{4} = \$9.$$

(2) Find the cost of  $\frac{5}{6}$  yd. at \$12 a yd.

The cost is measured by the number  $\frac{5}{6}$  and the unit \$12.

$$\frac{5}{6} \text{ yd. costs } \frac{5}{6} \text{ of } \$12 = \$10.$$

Explain the solution  $\frac{5}{6}$  yd. costs  $12 \times \$ \frac{5}{6} = \$10$ .

(3) Find the area of a floor 12 ft. long and  $9\frac{3}{4}$  ft. wide.

The area is measured by the unit  $9\frac{3}{4}$  sq. ft., which is the area of 1 strip, and the number 12.

The area of 1 strip =  $9\frac{3}{4}$  or  $\frac{39}{4}$  sq. ft.

$$\therefore \text{the total area} = 12 \times \frac{39}{4} \text{ sq. ft.} = 117 \text{ sq. ft.}$$

(4) Reduce  $\frac{8}{9}$  ft. to in.

$$\frac{8}{9} \text{ ft.} = \frac{8}{9} \times \frac{12}{1} \text{ in.} = \frac{32}{3} \text{ in.} = 10\frac{2}{3} \text{ in.}$$

**Exercise 81**

Find the cost of:

1. 10 yd. of cloth at  $\$ \frac{3}{5}$  per yd.

2. 12 yd. at  $\$ 1\frac{3}{4}$  per yd.

3. 9 yd. at  $\$ 2\frac{1}{2}$  a yd.

4.  $\frac{4}{7}$  yd. at \$6 a yd.

5. If the cost price is measured by the number  $\frac{2}{3}$  and the unit \$5, find the cost.

Reduce to in.:

6.  $\frac{5}{8}$  ft.

7.  $2\frac{3}{4}$  ft.

8.  $5\frac{3}{7}$  ft.

Reduce to yd.:

9. 8 rd.

10. 12 rd.

11. 17 rd.

Reduce to qt.:

12.  $3\frac{5}{8}$  gal.

13.  $12\frac{11}{2}$  gal.

Find the areas of the following rooms:

14. Length 24 ft., width 14 ft. 10 in.

15. Length 14 ft., width 12 ft. 6 in.

16. Length 18 ft., width 14 ft. 10 in.

17. Length 19 ft., width 16 ft. 4 in.

18. Length 22 ft., width 16 ft. 9 in.

19. Length 28 ft., width 20 ft. 11 in.

Find the area of the walls of a room whose:

20. Perimeter is 80 ft., height 9 ft. 6 in.

21. Perimeter is 63 ft., height 10 ft. 8 in.

22. Perimeter is 67 ft., height 8 ft. 9 in.

**119.** (1) Find the cost of  $12\frac{3}{4}$  yd. of cloth at  $\$1\frac{3}{8}$  a yd.

$$12\frac{3}{4} \text{ yd. cost } 12\frac{3}{4} \times \$1\frac{3}{8} = \frac{51}{4} \times \$\frac{11}{8} = \$\frac{561}{32} = \$17\frac{17}{32}.$$

(2) Find the area of the four walls of a room whose perimeter is 62 ft. 8 in., and height 8 ft. 9 in.

$$\text{The perimeter} = 62 \text{ ft. } 8 \text{ in.} = 62\frac{2}{3} \text{ ft.} = \frac{188}{3} \text{ ft.}$$

$$\text{The height} = 8 \text{ ft. } 9 \text{ in.} = 8\frac{3}{4} \text{ ft.} = \frac{35}{4} \text{ ft.}$$

$$\therefore \text{ the area} = \frac{47}{3} \times \frac{35}{4} \text{ sq. ft.} = 548\frac{1}{3} \text{ sq. ft.}$$

(3) Reduce  $\frac{8}{9}$  rd. to yd.

$$\frac{8}{9} \text{ rd.} = \frac{8}{9} \times 5\frac{1}{2} \text{ yd.} = \frac{8}{9} \times \frac{11}{2} = \frac{44}{9} \text{ or } 4\frac{8}{9} \text{ yd.}$$

## Exercise 82

Reduce to yd.:

1.  $\frac{7}{8}$  rd.,  $2\frac{1}{8}$  rd.,  $5\frac{6}{11}$  rd.,  $3\frac{6}{13}$  rd.

Find the cost of the following:

2.  $8\frac{1}{2}$  yd. at \$ $3\frac{1}{2}$  a yd.;  $4\frac{3}{4}$  yd. at \$ $2\frac{3}{4}$  a yd.;  
 $20\frac{3}{4}$  yd. at \$ $7\frac{1}{2}$  a yd.;  $5\frac{1}{4}$  yd. at \$ $2\frac{3}{4}$  a yd.
3.  $21\frac{3}{8}$  lb. of sugar at  $5\frac{1}{4}$ ¢ a lb.;  
 $13\frac{3}{8}$  lb. of sugar at  $5\frac{1}{2}$ ¢ a lb.
4.  $17\frac{3}{8}$  yd. of cotton at  $11\frac{1}{4}$ ¢ a yd.
5.  $15\frac{1}{2}$  doz. of eggs at  $14\frac{3}{4}$ ¢ a doz.
6.  $8\frac{1}{4}$  T. of hay at \$ $11\frac{5}{8}$  a T.
7. Find the area of the floor of a room whose dimensions are:  
(1) 12 ft. 6 in., 10 ft. 8 in.; (2) 16 ft. 4 in., 11 ft. 3 in.; (3) 18 ft. 8 in., 15 ft. 3 in.
8. Find the area of the four walls of a room whose perimeter and height are respectively: (1) 52 ft. 6 in., 9 ft. 4 in.; (2) 63 ft. 4 in., 10 ft. 6 in.
9. On  $\frac{3}{4}$  of a field I planted potatoes; on  $\frac{2}{3}$  of the remainder I sowed wheat. What part of the field did I sow with wheat?
10. I withdrew from the bank  $\frac{5}{8}$  of my deposit and then  $\frac{4}{9}$  of the remainder. What part of the original deposit did I take out the second time?
11. A man who owns  $\frac{9}{16}$  of a ship sells  $\frac{1}{8}$  of his share. What fraction of his former share does he still own? What fraction of the ship? If he had sold  $\frac{2}{3}$  of his share, what part of the ship would he have still owned?
12. A grain dealer invested  $\frac{3}{8}$  of his money in wheat, and  $\frac{4}{5}$  of the remainder in oats. What part of his money did he invest in oats? If he invested \$3000 in oats, how much did he have at first?
13. The owner of a farm valued at \$12,000 sells  $\frac{3}{8}$  of it to one man, and  $\frac{1}{2}$  of the remainder to another. What part of the

farm does he sell to the second man, and what should he get for it?

14. Four brothers enter into partnership; the eldest puts in  $\frac{1}{4}$  of the capital and the others the remainder in equal shares. What part of the entire capital does each of the younger brothers put in? If they each put in \$2000, what is the entire capital?

15. If I own  $\frac{3}{8}$  of  $\frac{5}{6}$  of a business, what part do I own? If I sell  $\frac{1}{4}$  of my share, what part of the business do I still own?

16. A man left his farm to be divided among his three sons; the oldest got 80 A., the second  $\frac{1}{3}$  of the farm, and the youngest  $\frac{2}{5}$  as much as the other two. Prove that the farm contained 210 A.

17. If the loss is measured by the number  $\frac{2}{3}$  and the unit \$17 $\frac{1}{4}$  tons, what is the loss?

If the gain is measured by the number  $\frac{2}{7}$  and the unit \$11 $\frac{2}{3}$ , what is the gain?

120. A owns a farm containing 81 $\frac{3}{8}$  A., B owns 96 $\frac{7}{12}$  A., and C 64 $\frac{1}{5}$  A. How many A. do they own altogether?

Here we are required to find the whole quantity measured by the parts, 81 $\frac{3}{8}$  A., 96 $\frac{7}{12}$  A., 64 $\frac{1}{5}$  A.

Let 1 A. = 120 units.

Then  $\frac{3}{8}$  A. = 45 units;  $\frac{7}{12}$  A. = 70 units;  $\frac{1}{5}$  A. = 24 units.

$\therefore \frac{3}{8}$  A. +  $\frac{7}{12}$  A. +  $\frac{1}{5}$  A. = 45 + 70 + 24, or 203 units =  $\frac{203}{120}$  A. = 1 $\frac{83}{120}$  A.

81 A. + 96 A. + 64 A. = 241 A.

$\therefore$  the sum = 241 A. + 1 $\frac{83}{120}$  A. = 242 $\frac{83}{120}$  A.

121. A sum of money is divided among 4 persons. The first receives  $\frac{1}{3}$ , the second  $\frac{1}{4}$ , the third  $\frac{1}{5}$ , and the fourth the remainder. It is found that the first received \$700 more than the fourth. Find the sum received by each.

Consider the sum of money as made up of 60 units.

The first receives  $\frac{1}{3}$  of 60 or 20 units; the second 15, and the third 12 units.

The three receive  $20 + 15 + 12$  or 47 units.

The fourth receives  $60 - 47$  or 13 units.

The first receives  $20 - 13$  or 7 units more than the fourth.

$$7 \text{ units} = \$700.$$

$$1 \text{ unit} = \$100.$$

$\therefore$  the first received 20 units or \$2000, the second \$1500, the third \$1200, and the fourth \$1300.

### Exercise 83

1. What is the combined weight of three men, the first of whom weighs  $125\frac{1}{4}$  lb., the second  $147\frac{3}{8}$  lb., and the third  $175\frac{3}{4}$  lb.?

2. A grocer has three bbl. of oil; the first contains  $18\frac{3}{8}$  gal., the second  $24\frac{5}{8}$ , and the third  $16\frac{3}{4}$ . Find how much there was in the three bbl.

3. If three crocks contain respectively  $8\frac{1}{4}$ ,  $12\frac{3}{8}$ , and  $141\frac{5}{8}$  lb. of butter, how much do the first two contain more than the third?

4. Four farms join each other; the first contains  $125\frac{3}{8}$  A., the second  $78\frac{3}{8}$  A., the third  $96\frac{1}{2}$  A., and the fourth  $110\frac{3}{4}$  A. Find the total area.

5. A person paid \$165 $\frac{3}{4}$  for a horse, and \$23 $\frac{1}{2}$  more than that for a carriage, and shortly after sold them at a loss of \$46 $\frac{5}{8}$ . What was the selling price?

6. What fraction subtracted from the sum of  $\frac{3}{4}$  and  $\frac{5}{7}$  will have unity for remainder?

7. If  $\frac{2}{3}$  of a school term exceed  $\frac{1}{2}$  of it by  $13\frac{1}{8}$  da., how many da. are there in the whole term?

8. I am the owner of  $\frac{1}{5}$  of a ship worth \$30,000, and sell  $\frac{1}{6}$  of the ship. What part of it will then belong to me, and what will it be worth?

9. Add together the greatest and least of the fractions  $\frac{3}{4}$ ,  $\frac{7}{8}$ ,  $\frac{11}{12}$ ,  $\frac{13}{16}$ , and subtract this sum from the sum of the other two fractions.

10. A man invested  $\frac{1}{2}$  his fortune in land,  $\frac{1}{3}$  of it in bank stock,  $\frac{1}{6}$  in provincial debentures, and lost the remainder, \$8000, in speculation. What was his fortune at first?

11. Explain why fractions having different denominators must be altered in form before their sum or difference can be expressed by one fraction.

12. A gentleman sold  $\frac{3}{10}$  of an estate to one person, and  $\frac{5}{17}$  to a second. What part of the estate did he still retain? If this is worth \$8346, what is the value of the estate?

13. If a man fills  $\frac{1}{3}$  of a cask with brandy,  $\frac{1}{4}$  with wine, and  $\frac{1}{5}$  with water, and it lacks  $21\frac{2}{3}$  gal. of being full, how many gal. will it contain?

14. Show that the fraction  $\frac{10+12}{12+14}$  is greater than  $\frac{5}{6}$  and less than  $\frac{7}{8}$ .

15. One person expends \$5 for coal at \$7 per T.; and another \$6 at \$9 per T. Which of them obtains the greater quantity of coal?

16. An estate worth \$10,000 is left to A, B, and C;  $\frac{3}{8}$  to A,  $\frac{2}{5}$  to B, and the remainder to C. Find C's portion and its value.

17. If during the day I pay out  $\frac{1}{2}$ , then  $\frac{1}{3}$ , next  $\frac{1}{12}$ , and lastly  $\frac{1}{15}$  of the money I had in the morning, what fraction of it have I left? If the sum left amounts to \$1.54, what sum had I at first?

18. (a) How much must be added to the denominator of  $\frac{5}{7}$ , that the resulting fraction may be equal to  $\frac{7}{12}$ ?

(b) How much must be subtracted from the denominator of  $\frac{7}{12}$  that the resulting fraction may be equal to  $\frac{5}{7}$ ?

19. In a certain subscription list,  $\frac{1}{3}$  of the number of subscriptions are for \$5 each,  $\frac{1}{6}$  are for \$4 each,  $\frac{1}{3}$  are for \$2 each,  $\frac{1}{6}$  are for \$1 each, and the remaining subscriptions, amounting to \$10.50, are for 50¢ each. Find the whole number of subscribers, and the total amount of their subscriptions.

20. A man lost  $\frac{1}{4}$  of his property in speculation; he afterwards purchased a partnership in business for \$16,000, and had still \$6000 left. What was he worth at first?

21. The sum paid for 494 gal. of oil, including a duty on each gal. which amounts to  $\frac{1}{5}$  of the cost price of a gal., is \$1719.12. Find the duty on each gal.

22. A house and lot cost \$3600; the value of the lot is  $\frac{1}{5}$  that of the house. Find the value of each.

23. What must be the length of a plot of ground, if the breadth is  $15\frac{3}{4}$  ft., that its area may contain 46 sq. yd.?

24. A boy gives  $\frac{1}{2}$  of his marbles to A,  $\frac{1}{3}$  to B, and the rest to C. C loses 20, and has then 70 less than A. How many had each at first?

25. A has \$3 more than  $\frac{1}{3}$  of the whole of a sum of money; B has \$4 more than  $\frac{1}{4}$  of the whole; and C has \$5 less than  $\frac{1}{5}$  of the whole. Find the sum divided.

26. After spending \$10 less than  $\frac{3}{5}$  of my money, I had \$15 more than  $\frac{3}{10}$  of it left. How much had I at first?

**122.** (1) Find the product of  $\frac{12}{35} \times 6\frac{2}{9} \times 7\frac{1}{2}$ .

Reducing to improper fractions and cancelling, the product

$$= \frac{12}{35} \times \frac{56}{9} \times \frac{95}{12} = \frac{152}{9} = 16\frac{8}{9}.$$

(2) Find the volume of a solid whose dimensions are  $2\frac{3}{8}$  in.,  $2\frac{3}{8}$  in., and  $4\frac{1}{2}$  in.

$$\begin{aligned} \text{The volume} &= 2\frac{3}{8} \times 2\frac{3}{8} \times 4\frac{1}{2} \text{ cu. in.} \\ &= 1\frac{3}{8} \times \frac{3}{8} \times \frac{9}{2} \text{ cu. in.} \\ &= 1\frac{5}{8} \text{ or } 31\frac{1}{8} \text{ cu. in.} \end{aligned}$$

The smallest unit of volume is 1 cu. in.

The next larger unit of volume is  $4\frac{1}{2}$  cu. in.

The largest unit of volume is  $2\frac{3}{8} \times 4\frac{1}{2}$  cu. in. = 12 cu. in.

How many units of each magnitude measure the solid?



## Exercise 84

Find the volume of the solids whose dimensions are:

1. 2 in., 4 in.,  $6\frac{1}{2}$  in.
2.  $1\frac{1}{2}$  in.,  $2\frac{1}{2}$  in.,  $2\frac{2}{3}$  in.
3.  $2\frac{1}{2}$  in.,  $2\frac{2}{3}$  in.,  $4\frac{2}{3}$  in.
4.  $\frac{3}{16}$  ft.,  $2\frac{2}{3}$  ft.,  $4\frac{1}{3}$  ft.
5.  $\frac{2}{3}$  ft.,  $\frac{4}{7}$  ft.,  $\frac{5}{12}$  ft.
6.  $\frac{6}{35}$  yd.,  $2\frac{6}{11}$  yd.,  $8\frac{1}{4}$  yd.
7. What are the units of volume in the first and second questions?
8. How many units of each magnitude measure the solids in the first and second questions?

Find the product of:

9.  $\frac{2}{3} \times \frac{6}{7} \times \frac{5}{12}$ .
10.  $\frac{3}{13} \times \frac{5}{6} \times \frac{26}{33}$ .
11.  $\frac{10}{21} \times \frac{1}{8} \times \frac{28}{15}$ .
12.  $\frac{17}{56} \times 2\frac{3}{8} \times 13\frac{1}{2}$ .
13.  $\frac{4}{27} \times 2\frac{1}{7} \times 31\frac{5}{8}$ .
14.  $16\frac{1}{4} \times 4\frac{4}{15} \times 2\frac{3}{26}$ .

**123.** (1) A owns  $\frac{2}{5}$  of a ship and B the remainder, and  $\frac{3}{4}$  of the difference between their shares is \$1500. What is the vessel worth?

Represent the value of the ship by 20 units of money.

A owns  $\frac{2}{5}$  of the ship or 8 units, B  $\frac{3}{5}$  of the ship or 12 units.

The difference between their shares = 12 - 8 or 4 units.  $\frac{3}{4}$  of the difference between their shares = 3 units.

$$3 \text{ units} = \$1500.$$

$$1 \text{ unit} = 500.$$

$$20 \text{ units} = 10,000.$$

$$\therefore \text{the vessel is worth } \$10,000.$$

(2) A man lost  $\frac{2}{5}$  of the value of his horse by selling it for \$60. For what should he have sold it to gain  $\frac{2}{5}$  of its value?

Let the value of the horse = 5 units of money.

Then the loss on selling = 2 units of money.

The first selling price = 3 units of money.

The second selling price = 7 units of money.

$$\therefore \text{the second selling price} = \frac{7}{3} \text{ of } \$60 = \$140.$$

(3) A person who has  $\frac{2}{5}$  of a mine sells  $\frac{3}{4}$  of his share for \$6000. What is the value of the whole mine?

Let the value of the mine be measured by 20 units of money.

$$\frac{2}{5} \text{ of the mine} = 8 \text{ units.}$$

$$\text{The amount sold} = \frac{3}{4} \text{ of 8 units} = 6 \text{ units.}$$

$$6 \text{ units} = \$6000.$$

$$1 \text{ unit} = 1000.$$

$$20 \text{ units} = 20,000.$$

$$\therefore \text{the value of the mine} = \$20,000.$$

### Exercise 85

1. A grocer buys tea at 64¢ a lb., and sells it so as to gain  $6\frac{1}{2}\%$  of the cost price. Find his receipts on 6043 lb.

2. A man bought a horse for \$120, which was \$30 less than  $\frac{5}{7}$  of one and a half times what he sold him for. How much did he make on the sale of the horse?

3. If I own  $\frac{3}{4}$  of  $\frac{4}{5}$  of  $\frac{2}{3}$  of a ship worth \$20,000, and sell  $\frac{1}{4}$  of the ship, what will the part I have left be worth?

4. The owner of a ship which was valued at \$10,000 sells  $\frac{2}{3}$  of it for \$3800, and then  $\frac{1}{3}$  of the remainder for \$1800. What did he gain or lose by the transaction?

5. A man has \$4000 in the bank. He drew out  $\frac{3}{20}$  of it, and then  $\frac{1}{5}$  of the remainder, and afterwards deposited  $\frac{1}{8}$  of what he had drawn out. How much had he then in the bank?

6. A man divided a farm among three sons; to the first he gave 80 A., to the second  $\frac{4}{9}$  of the whole, and to the third  $\frac{3}{4}$  as much as to both the others. How many A. did the farm contain?

7. Divide \$65.80 between two persons, so that one shall receive one-third as much again as the other.

8. Five brothers join in paying a sum of money; the eldest pays  $\frac{2}{7}$  of it, and the others pay the remainder in equal shares.

It is found that the eldest brother pays \$300 $\frac{2}{176}$  more than a younger brother's share. Find the sum of money.

9. Show clearly that both terms of a fraction can be multiplied by the same number, without changing the value of the fraction.

10. A man commenced business with a capital of \$8000; the first year he gained \$40 for every \$100 invested, adding his gain to his capital; the second year he gained \$25 for every \$100 invested, adding his gain as before; the third year he lost  $\frac{1}{7}$  of his accumulated capital. How much did he make in the three years?

11. If  $\frac{2}{3}$  of  $\frac{3}{7}$  of an A. produces 43 bu. of potatoes, how many bu. will an A. produce?

12. A paid \$60 per A. for his farm, which was  $\frac{5}{9}$  as much as B paid per A. for his farm of 150 A. Find the entire cost of B's farm.

13. If the sum paid for 247 gal. of spirits amounts, together with the duty, to \$859.56, and the duty on 1 gal. be  $\frac{1}{5}$  part of its original cost, what is the duty per gal.?

14. A piece of cloth, when measured with a yd. measure which is  $\frac{2}{3}$  of an in. too short, appears to be 10 $\frac{1}{2}$  yd. long. What is its true length?

15. I had a sum of money, of which I paid away  $\frac{1}{5}$ , then  $\frac{1}{2}$  of the remainder, then  $\frac{2}{3}$  of what was still left, and found that I had still left half a dollar less than  $\frac{1}{5}$  of  $\frac{3}{4}$  of the whole. What sum had I at first?

16. A man who owns  $\frac{10}{11}$  of a mill sells  $\frac{2}{5}$  of his share. What fraction of the mill does he still own? Had he sold  $\frac{2}{5}$  of the mill, what fraction of the mill would he still have owned?

17. Find the sum of the greatest and least of the fractions  $\frac{3}{8}$ ,  $\frac{5}{12}$ ,  $\frac{4}{9}$ , and  $\frac{7}{20}$ ; the sum of the other two; and the difference of these sums.

18. If when  $\frac{7}{12}$  of a certain time has elapsed, and then 1 hr., and then  $\frac{3}{5}$  of the remainder of the time, it is found that 16 min. of the time still remains, what was the whole time?

19. A man sold  $\frac{5}{9}$  of his wheat and then  $\frac{7}{8}$  of the remainder, and next  $\frac{1}{10}$  of what then remained, and had 18 bu. more than  $\frac{3}{25}$  of his wheat left. How many bu. had he at first?

20. A man sold  $\frac{1}{2}$  of his farm, then  $\frac{1}{3}$  of the remainder, then  $\frac{1}{4}$  of what remained, then  $\frac{1}{5}$  of what still remained, and he then found that he had sold altogether 72 A. more than he had remaining. How many A. had he at first?

## DIVISION OF FRACTIONS

124. Divide  $3\frac{1}{3}$  ft. by  $\frac{5}{6}$  ft.

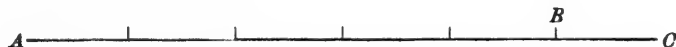
Here we are given the whole measured quantity  $3\frac{1}{3}$  ft. and the unit of measure  $\frac{5}{6}$  ft., and we are required to find the number which expresses the ratio of the whole quantity to the unit. Consider the ft. as measured by 6 units of length. Then  $3\frac{1}{3}$  ft. contains 20 units, and  $\frac{5}{6}$  ft. contains 5 units.

$\therefore$  the number or ratio = 4.

Prove that  $\frac{5}{6}$  ft. divides  $3\frac{1}{3}$  ft. 4 times by drawing a line  $3\frac{1}{3}$  ft. or 40 in. long, and measuring it with a line  $\frac{5}{6}$  ft. or 10 in. long. Prove that the quotient 4 is correct by multiplying  $\frac{5}{6}$  ft. by 4 and finding the product to be  $3\frac{1}{3}$  ft.

125. If we represent any quantity by  $\frac{5}{6}$  of itself, we think of the quantity as made up of 5 units, and the unit as equal to  $\frac{1}{6}$  of the quantity. Hence we may think of any quantity as equal to  $5 \times \frac{1}{6}$  (*i.e.* five times one-sixth) of itself.

What, then, is the meaning of the terms 5 and  $\frac{1}{6}$  considered separately? 5 shows the relation or the ratio of the quantity to the *unit of measure*, and  $\frac{1}{6}$  shows the ratio of the unit of measure to the quantity. Numbers thus mutually related are said to be *reciprocal*.



Again, let the line  $AB$ , which is divided into 5 equal parts, represent the primary unit, \$1. Then  $AC$  represents the quantity denoted by  $\$ \frac{2}{3}$ . Hence it is evident from the diagram that  $\frac{2}{3}$  is the ratio of  $AC$  to  $AB$ , *i.e.* of the quantity denoted by  $\$ \frac{2}{3}$  to the primary unit, \$1; also, that its reciprocal  $\frac{3}{2}$  is the ratio of  $AB$  to  $AC$ , *i.e.* of the primary unit, \$1, to the quantity denoted by  $\$ \frac{2}{3}$ . Similarly, the ratio of \$1 (which contains 2 units) to the quantity  $\$ \frac{2}{3}$  (which contains 3 units) is equal to  $\frac{3}{2}$ , *i.e.* to the reciprocal of  $\frac{2}{3}$ .

### Exercise 86

1. How many units of money are there in \$1 and also in  $\$ \frac{3}{2}$ , the unit being one-half dollar?

How many in \$1 and also in  $\$ \frac{4}{3}$ , the unit being one-third of a dollar?

How many in \$1 and also in  $\$ \frac{5}{3}$ , the unit being one-fifth of a dollar?

2. What is the ratio of \$1 to  $\$ \frac{3}{2}$ ?  $\$ \frac{4}{3}$ ?  $\$ \frac{5}{3}$ ?  $\$ 2\frac{1}{3}$ ?  $\$ \frac{1}{2}$ ?  $\$ \frac{1}{4}$ ?  $\$ \frac{4}{7}$ ?

3. What part is \$1 of each of these quantities: \$2?  $\$ \frac{12}{5}$ ?  $\$ \frac{20}{9}$ ?  $\$ 3\frac{1}{8}$ ?  $\$ 4\frac{2}{11}$ ?

4. What is the ratio of 1 lb. to  $\frac{3}{4}$  lb.?  $\frac{5}{8}$  lb.?  $2\frac{3}{8}$  lb.?  $2\frac{3}{8}$  lb.?

5. What is the ratio of 2 lb. to each quantity given in question 4? Of 3 lb.? Of 4 lb.?

6. What is the ratio of  $1\frac{1}{3}$  lb. to each quantity given in question 4?

7. If  $\frac{4}{5}$  of a yd. cost \$1, what part of \$1 will 1 yd. cost?

8. If  $\frac{5}{6}$  of a yd. cost \$1, what part of \$1 will 1 yd. cost?

9. If  $\frac{3}{2}$  of a yd. cost  $\$ 2\frac{1}{2}$ , what part of  $\$ 2\frac{1}{2}$  will 1 yd. cost? How much will 1 yd. cost?

10. If  $3\frac{1}{2}$  yd. of cloth cost  $\$ 1\frac{3}{4}$ , what will 1 yd. cost?

11. If  $4\frac{1}{8}$  lb. of sugar cost 26¢, what will 1 lb. cost?

12. If  $\frac{3}{8}$  of a yd. of cloth costs 32¢, what will 1 yd. cost? What will 3 yd. cost?  $4\frac{1}{2}$  yd.?  $\frac{9}{16}$  yd.?

13. If  $7\frac{1}{2}$  lb. of raisins cost 85¢, what will  $4\frac{1}{2}$  lb. cost?

**126.** (1) At \$5 a yd., how much lace can be bought for \$ $\frac{7}{10}$ ?

Here we are given the whole quantity or \$ $\frac{7}{10}$ , and the measuring unit \$5, and we are required to find the number.

Consider \$1 as measured by 10 units.

Then \$ $\frac{7}{10}$  contains 7 units and \$5 contains 50 units. Therefore the number of yd. = \$ $\frac{7}{10} \div \$5 = 7 \text{ units} \div 50 \text{ units} = \frac{7}{50}$ .

(2) Find the value of \$ $\frac{6}{7} \div \$\frac{5}{4}$ .

Consider \$1 as made up of 28 units. Then \$ $\frac{6}{7}$  is equal to 24 of these units and \$ $\frac{5}{4}$  to 35 of these units. Therefore, \$ $\frac{6}{7} \div \$\frac{5}{4} = 24 \text{ units} \div 35 \text{ units} = \frac{24}{35}$ . Hence, to divide one fractional quantity by another, we first reduce them to the same unit.

**127.** It may be observed that  $\frac{24}{35}$  is equal to  $\frac{6}{7}$  multiplied by  $\frac{4}{5}$ , the reciprocal of  $\frac{5}{4}$ . Hence, to divide one fraction by another, multiply the first fraction by the reciprocal of the second.

### Exercise 87

In the following exercise, solve (1) by reducing the quantities to the same unit, (2) by multiplying by the reciprocal:

1. \$ $\frac{2}{5} \div \$\frac{3}{4}$ .

3.  $\frac{5}{9}$  pk.  $\div \frac{1}{3}$  pk.

5.  $\frac{7}{8} \div \frac{5}{12}$ .

2.  $\frac{2}{7}$  yd.  $\div \frac{1}{3}$  yd.

4.  $2\frac{1}{2} \div \frac{3}{8}$ .

6.  $\frac{8}{27} \div \frac{4}{15}$ .

**128.** (1) Reduce  $2\frac{1}{2}$  ft. to a fraction of 1 yd.

$$1 \text{ ft.} = \frac{1}{3} \text{ yd.}$$

$$2\frac{1}{2} \text{ ft.} = 2\frac{1}{2} \times \frac{1}{3} \text{ yd.} = \frac{5}{2} \times \frac{1}{3} \text{ yd.} = \frac{5}{6} \text{ yd.}$$

Give the solution which follows from the fact that  $2\frac{1}{2}$  ft. is the whole measured quantity, 3 ft. the unit of measurement, and that we are required to find the number.

(2) Find the number of strips required to carpet a room 12 ft. wide with carpet  $2\frac{1}{2}$  ft. wide.

The number of strips =  $12 \text{ ft.} \div 2\frac{1}{2} \text{ ft.} = \frac{12}{2\frac{1}{2}} \times \frac{2}{2} = 4\frac{4}{5}$ , i.e. 5.

$\therefore$  5 strips are required.

### Exercise 88

1. Reduce to the fraction of a ft. :  $1\frac{1}{5}$  in.,  $2\frac{1}{4}$  in.,  $4\frac{1}{2}$  in.,  $2\frac{3}{8}$  in.
2. Reduce to the fraction of a yd. :  $1\frac{1}{2}$  ft.,  $2\frac{2}{3}$  ft.,  $1\frac{1}{5}$  ft.,  $2\frac{1}{4}$  ft.
3. Reduce to fractions of \$100 : \$12 $\frac{1}{2}$ , \$37 $\frac{1}{2}$ , \$62 $\frac{1}{2}$ , \$87 $\frac{1}{2}$ , \$6 $\frac{1}{4}$ , \$6 $\frac{3}{8}$ .
4. If I paid \$ $\frac{9}{10}$  for 6 yd. of cloth, what will 1 yd. cost?
5. If I paid \$4 $\frac{1}{5}$  for 14 yd. of cloth, what will 1 yd. cost?
6. If \$ $\frac{3}{4}$  is considered as measured by 5 units, what is the value of 1 unit?
7. If \$8 $\frac{3}{4}$  is measured by the unit \$5, what is the number expressing the measurement?
8. At \$2 $\frac{1}{4}$  a yd., how much lace can be bought for \$ $\frac{9}{18}$ ?
9. At \$3 $\frac{5}{9}$  a yd., how much lace can be bought for \$ $\frac{8}{15}$ ?
10. Find the number of strips of carpet required to carpet a room 23 ft. 4 in. long, with carpet 2 ft. wide. With carpet 3 ft. wide.
11. Find the lengths of the following rooms :
  - Area 466 $\frac{2}{3}$  sq. ft., width 20 ft.
  - Area 242 $\frac{1}{2}$  sq. ft., width 15 ft.
  - Area 681 $\frac{1}{4}$  sq. ft., width 25 ft.
  - Area 877 $\frac{1}{2}$  sq. ft., width 27 ft.

129. (1) Divide  $1\frac{3}{4}$  ft. by  $\frac{3}{8}$  ft.

$$1\frac{3}{4} \text{ ft.} \div \frac{3}{8} \text{ ft.} = \frac{7}{4} \div \frac{3}{8} = \frac{7}{4} \times \frac{8}{3} = 4\frac{2}{3}.$$

Prove this by drawing a line  $1\frac{3}{4}$  ft. long, and dividing it into parts  $\frac{3}{8}$  ft. long.

(2) If I paid  $\$ \frac{2}{3}$  for  $\frac{3}{5}$  of a yd. of cloth, what is the price per yd.? Here we are given the whole measured quantity, that is,  $\$ \frac{2}{3}$ , and the number, or  $\frac{3}{5}$ , and we are required to find the measuring unit.

$\frac{3}{5}$  of a yd. costs  $\$ \frac{2}{3}$ .

$\therefore$  1 yd. costs  $\$ \frac{2}{3} \div \frac{3}{5} = \$ \frac{2}{3} \times \frac{5}{3} = \$ 1 \frac{1}{3}$ .

### Exercise 89

1. Reduce to the fraction of a rd. :  $2\frac{3}{4}$  yd.,  $1\frac{3}{8}$  yd.,  $4\frac{1}{2}$  yd.,  $6\frac{7}{8}$  yd.,  $4\frac{3}{17}$  yd.

2. Find and prove by actual measurement the following:  
 $2\frac{1}{8}$  ft.  $\div \frac{1}{8}$  ft.;  $1\frac{1}{4}$  ft.  $\div \frac{1}{8}$  ft.;  $2\frac{1}{4}$  ft.  $\div \frac{1}{8}$  ft.;  $1\frac{7}{8}$  ft.  $\div \frac{1}{4}$  ft.;  $4\frac{1}{8}$  ft.  $\div \frac{5}{12}$  ft.;  
 $6\frac{1}{8}$  ft.  $\div \frac{7}{12}$  ft.

3. At  $\$ 3\frac{1}{4}$  a bbl., how much flour can I buy for  $\$ 17\frac{1}{2}$ ? At  $\$ 9\frac{7}{8}$  a yd., how much lace can be bought for  $\$ 24\frac{1}{5}$ ?

4. Find the number of strips of carpet required to carpet each of the following rooms:

Width, 22 ft. 6 in.; width of carpet,  $2\frac{1}{2}$  ft. ( $22\frac{1}{2}$  ft.  $\div 2\frac{1}{2}$  ft.)

Width, 24 ft. 3 in.; width of carpet,  $2\frac{1}{2}$  ft.

Width, 8 yds. 2 ft.; width of carpet,  $\frac{5}{8}$  yd.

Width, 9 yds. 1 ft.; width of carpet,  $\frac{5}{8}$  yd.

5. Find the length of each of the following rooms:

Area of floor,  $126\frac{3}{4}$  sq. yd.; width, 8 yd. 2 ft.

Area of floor,  $133\frac{1}{2}$  sq. yd.; width,  $9\frac{3}{4}$  yd.

Area of floor,  $334\frac{1}{8}$  sq. ft.; width, 15 ft. 9 in.

Area of floor,  $253\frac{1}{8}$  sq. ft.; width, 10 ft. 8 in.

6. Find the perimeter of each of the following rooms:

AREA OF THE WALLS

HEIGHT

$533\frac{1}{8}$  sq. ft.

8 ft. 4 in.

$704\frac{3}{8}$  sq. ft.

9 ft. 4 in.

$877\frac{1}{8}$  sq. ft.

10 ft. 9 in.

$65\frac{7}{8}$  sq. yd.

2 yd. 2 ft.



7. If  $\frac{5}{7}$  of the sum a man paid for a horse is  $\$83\frac{1}{3}$ , what did the horse cost?

8. If  $\frac{2}{3}$  of a yd. of cloth cost  $\$ \frac{3}{5}$ , what will 1 yd. cost?

**130.** What is the ratio of  $8\frac{4}{7}$  da. to  $1\frac{1}{4}$  da.?

The ratio of  $8\frac{4}{7}$  da. to  $1\frac{1}{4}$  da.  $= 8\frac{4}{7} \div 1\frac{1}{4} = \frac{60}{7} \div \frac{5}{4} = \frac{60}{7} \times \frac{4}{5} = 4\frac{4}{7}$ .

### Exercise 90

1. What is the ratio of  $\$5\frac{1}{7}$  to  $\$3\frac{3}{8}$ ?
2. What is the ratio of  $1\frac{1}{3}$  T. to  $6\frac{2}{3}$  T.?
3. If  $6\frac{2}{3}$  T. of hay cost  $\$42$ , what part of  $\$42$  will  $1\frac{1}{3}$  T. cost? How much will  $1\frac{1}{3}$  T. cost?
4. If  $\$ \frac{2}{25}$  is represented by unity, what number will represent  $\$ \frac{42}{85}$ ?
5. Find the value of (1)  $5\frac{1}{7} \div 3\frac{3}{8}$ ; (2)  $7\frac{7}{8} \div 4\frac{1}{12}$ ; (3)  $2\frac{5}{14} \div 5\frac{1}{27}$ ; (4)  $1\frac{3}{8} \div 2\frac{1}{7}$ .

**131.** Simplify  $\frac{4\frac{2}{3}}{5\frac{1}{4}}$ .

$$\frac{4\frac{2}{3}}{5\frac{1}{4}} = 4\frac{2}{3} \div 5\frac{1}{4} = \frac{14}{3} \times \frac{4}{21} = \frac{8}{9}.$$

Or thus, multiplying the numerator and denominator by 12, the L. C. M. of 3 and 4, we have

$$\frac{4\frac{2}{3}}{5\frac{1}{4}} = \frac{56}{63} = \frac{8}{9}.$$

### Exercise 91

Simplify, using both methods:

1.  $\frac{\frac{2}{3}}{\frac{5}{6}}$

3.  $\frac{2\frac{3}{4}}{5\frac{1}{2}}$

5.  $\frac{14\frac{2}{3}}{30\frac{1}{4}}$

7.  $\frac{6\frac{1}{4}}{17\frac{1}{4}}$

2.  $\frac{\frac{3}{4}}{1\frac{3}{5}}$

4.  $\frac{5\frac{5}{8}}{3\frac{3}{8}}$

6.  $\frac{4\frac{2}{18}}{1\frac{6}{6}}$

8.  $\frac{1\frac{3}{4}}{25\frac{2}{3}}$

**132.** (1) A man earns \$4 $\frac{1}{5}$  a da., and his daily expenses are \$1 $\frac{7}{8}$ . How many da. will it take him to save enough money to buy a bicycle costing \$58 $\frac{1}{8}$ ?

$$\text{The sum saved each day} = \$4\frac{1}{5} - \$1\frac{7}{8} = \$2\frac{13}{40}.$$

$$\therefore \text{the number of days} = 58\frac{1}{8} \div 2\frac{13}{40} = 25.$$

(2) A farm of 340 A. was divided between two sons, so that  $\frac{3}{4}$  of the youngest son's share was equal to  $\frac{2}{3}$  of the eldest son's share. Find the size of each farm.

$$\frac{3}{4} \text{ of the youngest son's share} = \frac{2}{3} \text{ of the eldest son's share.}$$

$$\text{The youngest son's share} = \frac{2}{3} \div \frac{3}{4}, \text{ or } \frac{8}{9} \text{ of the eldest son's share.}$$

$$\therefore \frac{8}{9} + \frac{8}{9}, \text{ or } \frac{16}{9} \text{ of the eldest son's share} = 340 \text{ A.}$$

$$\therefore \text{the eldest son's share} = 340 \text{ A.} \div \frac{16}{9} = 180 \text{ A.}$$

$$\therefore \text{the youngest son's share} = \frac{8}{9} \text{ of } 180 \text{ A.} = 160 \text{ A.}$$

Complete the following solution of question (2):

$$\text{Let } \frac{3}{4} \text{ of the youngest son's share} = 6 \text{ units.}$$

(3) Sold tea at 90¢ per lb., having gained  $\frac{3}{20}$  of the cost. Find the selling price per lb. if he had lost  $\frac{3}{20}$ .

$$\text{Let the cost of 1 lb. be measured by 20 units.}$$

$$\text{Then } \frac{3}{20} \text{ of the cost price} = 3 \text{ units.}$$

$$\text{The first selling price} = 23 \text{ units.}$$

$$\text{The second selling price} = 17 \text{ units.}$$

$$\therefore \text{the second selling price} = \frac{17}{23} \text{ of the first}$$

$$= \frac{17}{23} \text{ of } 90¢ = 66\frac{2}{3}¢.$$

### Exercise 92

1. A vessel holds  $2\frac{4}{13}$  qt. How many times can it be filled from a barrel containing  $31\frac{1}{2}$  gal. of oil? After filling the vessel as often as possible, how much oil will remain in the barrel? What fraction of a vesselful will this remaining quantity be?

2. Divide the sum of the fractions  $\frac{3}{4}$  and  $\frac{1}{13}$  by the product of  $\frac{9}{11}$  and  $\frac{1}{4}$ , and reduce the result to its lowest terms.

3. The bottom of a cistern measures 7 ft. 6 in. by 3 ft. 2 in. How deep must it be to contain 76 cu. ft. of water?

4. A man sold 24 horses for \$150 each; on ~~some~~ of them he gained  $\frac{3}{10}$  of what they cost; and on the remainder he lost  $\frac{1}{5}$  of what they cost. Find his whole gain or loss.

5. By selling cigars at the rate of \$2.60 for 4 doz., it was found that  $\frac{5}{8}$  of the cost was gained. Find the price at which each cigar ought to have been sold in order to gain  $\frac{7}{10}$  of the original cost.

6. James received a present of some money. He gave  $\frac{1}{3}$  of it to his sister, and  $\frac{3}{8}$  of the remainder to his brother, and kept the rest, \$4, for himself. How much did he receive, and how much did his brother get?

7. A tree of 140 ft. in length was broken into two pieces by falling, and  $\frac{8}{11}$  of the longer piece was equal to  $\frac{4}{7}$  of the shorter. Find the length of each piece.

8. A owns  $\frac{2}{5}$  of a ship and B the remainder, and  $\frac{3}{4}$  of the difference between their shares is \$1500. What is the vessel worth?

9. A person who has  $\frac{2}{5}$  of a mine sells  $\frac{3}{4}$  of his share for \$6000. What is the value of the whole mine?

10. Divide \$2380 between A and B so that  $\frac{2}{3}$  of A's share will be equal to  $\frac{3}{4}$  of B's.

11. If 13 were added to a certain number,  $\frac{5}{8}$  of  $\frac{4}{9}$  of the sum would be 40. Find the number.

12. Three partners, A, B, and C, gain \$17,100; A's gain and C's are together \$11,000, and  $\frac{2}{5}$  of A's is equal to  $\frac{8}{25}$  of C's. Find each man's gain.

13. A grocer in selling goods sells  $15\frac{3}{4}$  oz. for 1 lb. How much does he cheat a customer who buys to the amount of \$40?

14. The numerator of a certain fraction is  $\frac{1}{5}$  as much again as its denominator, and the sum of the numerator and denominator is 352. Find the fraction.

15. A cannon ball travels at the rate of 1500 ft. in  $1\frac{1}{2}$  sec. How far will it have gone in  $\frac{1}{3}$  of a min.?

16. A man divides the value of his estate equally among his three sons. The first son gains an amount equal to  $\frac{1}{3}$  of what he has; the second loses  $\frac{1}{3}$  of what he has, and the third gains  $\frac{1}{3}$  of what he has, and then loses  $\frac{1}{3}$  of what he has after his gain; and now the sons together have \$300 less than the value of the estate. What was its value?

17. Given, that pure water is composed of oxygen and hydrogen in the proportion by weight of 15 to 2, find the weight of each in a cu. ft. of water. (A cu. ft. of water weighs 1000 oz.)

18. A line  $A$  is half as long again as  $B$ , and  $B$  is one-quarter as long again as  $C$ . What fraction of the length of  $A$  is equal to  $\frac{1}{3}$  of the length of  $C$ ?

19. A and B go into business with equal sums of money. A gains a sum equal to  $\frac{3}{4}$  of what he had at first, and B loses \$60. A then has  $1\frac{1}{4}$  times as much as B. What sums had they at first?

20. A and B sit down to play. A has  $5\frac{1}{2}$  times as much money as B. At the first game B wins  $\frac{4}{5}$  of A's money. What fraction of B's money must A win back so that they may have equal shares?

21. A young man's salary increased  $\frac{1}{3}$  every year; his expenses each year were  $\frac{1}{3}$  of his salary, and at the end of 4 years he had saved \$1050. Find his last year's salary.

22. Find a fraction equivalent to  $\frac{5}{13}$ , and having its numerator 44 less than its denominator.

23. A trader spent the first year \$40, and added to his capital  $\frac{1}{4}$  of what he had left; the second year he spent \$50, and added to his capital  $\frac{1}{5}$  of what he had left; the third year he spent \$60, and added to his capital  $\frac{1}{6}$  of what he had left; he finds that his capital is now  $1\frac{1}{2}$  of what it was at first. Find the original capital.

24. In a field in which cows and sheep were grazing,  $\frac{1}{7}$  of the total number were cows; but when 3 cows more were driven in,

the latter numbered  $\frac{2}{11}$  of the whole. How many sheep were there?

25. Two persons, A and B, finish a work in 20 da., which B by himself could do in 50 da. In what time could A finish it by himself? How much more of the work is done by A than by B?

Let the work be represented by 100 units of work.

A and B do 100 units  $\div$  20 or 5 units in 1 da.

B does 100 units  $\div$  50 or 2 units in 1 da.

A does 5 - 2 or 3 units in 1 da.

$\therefore$  A does the work in  $100 \div 3$  or  $33\frac{1}{3}$  da.

Again,

A does 3 - 2 or 1 unit more than B in 1 day.

A does 20 units more than B in 20 days.

$\therefore$  A does  $\frac{20}{100}$  or  $\frac{1}{5}$  of the work more than B.

26. If A can do a piece of work in 3 da., and B in 4 da., in what time can both working together do the work?

27. A alone can do a piece of work in 11 da., and B alone can do it in 17 da. How long would they take to do it together?

28. A and B can do a piece of work together in 6 da., and B can do  $\frac{1}{3}$  of the same in  $1\frac{1}{2}$  da. How long would each be in doing it alone?

29. A can do a piece of work in 5, B in 6, and C in 8 da. If A and B work at it 2 da. each, how long will it take B and C to finish it?

30. A and B can do a piece of work in 3 da., B and C in 6 da., and A and C in 4 da. If \$16 be paid for the work, what is each man worth per da.?

31. A and B can do a piece of work alone in 15 and 18 da. respectively; they work together at it for 3 da., when B leaves, but A continues, and after 3 da. is joined by C, and they finish it together in 4 da. In what time would C do the work by himself?

## G. C. M. AND L. C. M. OF FRACTIONS

**133.** To find the G. C. M. of  $5\frac{1}{3}$  and  $2\frac{2}{5}$  of any unit.

Consider the unit as divided into 15 units, then  $5\frac{1}{3}$  is equal to 80, and  $2\frac{2}{5}$  to 36 of these units. Here the G. C. M. is equal to the G. C. M. of 80 and 36, or to 4 of these units, *i.e.* to  $\frac{4}{15}$  of the primary unit.

Hence we have the following rule:

*To find the G. C. M. of a number of fractions, reduce the fractions to a common unit, and then find the G. C. M. of the resulting numbers. Express the result as a fraction of the primary unit.*

**134.** To find the L. C. M. of  $5\frac{1}{3}$  and  $2\frac{2}{5}$ .

Reduce the fractions, as in the preceding paragraph, to a common unit. The L. C. M. of 80 and 36 is 720, and the L. C. M. is equal to  $\frac{720}{15}$ , or 48 of the primary unit.

*Hence, to find the L. C. M. of a number of fractions, reduce them to a common unit, and then find the L. C. M. of the resulting numbers. Express this result as a fraction of the primary unit.*

## Exercise 93

Find the G. C. M. of:

1.  $\frac{15}{4}$  and  $\frac{10}{8}$ .      2.  $\frac{2}{3}$  and  $1\frac{3}{5}$ .      3.  $2\frac{2}{3}$  and  $7\frac{7}{27}$ .

Find the L. C. M. of:

4.  $\frac{2}{3}$ ,  $\frac{5}{8}$ , and  $\frac{7}{12}$ .      5.  $1\frac{1}{3}$ ,  $2\frac{1}{2}$ , and  $3\frac{3}{4}$ .      6.  $4\frac{1}{5}$ ,  $\frac{1}{2}$ , and  $4\frac{2}{3}$ .

7. A man has a triangular field, of which the sides are  $115\frac{1}{2}$  ft.,  $128\frac{1}{2}$  ft., and  $134\frac{3}{4}$  ft. Find the length of the longest boards of equal length that can be used in fencing it without cutting a board.

8. The driving wheels of a locomotive are  $17\frac{1}{2}$  ft. in circumference, and the trucks  $10\frac{5}{8}$ . What distance must the train move to bring wheel and truck into same relative positions as at starting?

9. The side of a field is  $19\frac{1}{5}$  rd. in length, and the end  $16\frac{8}{15}$  rd. What is the longest board that can be used in fencing both side and end of the field, so that no fraction (of a board) may be left?

10. The distance between the post-office and schoolhouse is half a mile. A carriage stands on the pavement in front of the schoolhouse. The fore wheels are  $10\frac{4}{15}$  ft. in circumference, the hind wheels  $15\frac{5}{8}$  ft. A chalk mark is made upon the upper side of each wheel. How often will the 4 chalk marks be all up together while the carriage drives to the post-office?

11. Show how to find the least common multiple of two or more fractions. A, B, and C start at a given place to travel round an island 120 mi. in circumference; A's rate is  $5\frac{1}{2}$  mi. a da., B's  $8\frac{1}{4}$ , C's  $9\frac{5}{8}$ . In what time will they all be together again?

## CHAPTER XI

### DECIMALS

**135.** The standard unit of money is \$1. One dime is one tenth of \$1, and 1¢ is one tenth of 1 dime. Hence we write \$1, 1 dime, and 1¢ thus, using the dollar sign: \$1.11.

Write in terms of \$1, the sum of:

- (1) \$1, 2 dimes, and 4¢.
- (2) 1 ten-dollar bill, \$1, 1 dime, and 1¢.
- (3) 1 one-hundred dollar bill, 1 ten-dollar bill, \$1, 1 dime, and 1¢.

**136.** In the metric system of measures, the standard unit is 1 meter.\*

One decimeter is one tenth of 1 meter, 1 centimeter is one tenth of 1 decimeter, and 1 millimeter is one tenth of 1 centimeter. Since the same relation holds between these units as between 1 dime and \$1, 1¢ and 1 dime, we can express the results of measurements in terms of 1 meter, as we express money in terms of \$1.

Hence if we measure with the metric stick a distance equal to 1 meter, 1 decimeter, and 1 centimeter, we can express the distance thus: 1.11 meters.

Measure the following distances and express them in meters:

\* See Chapter XVIII. If the teacher has not a metric stick, the work in decimals may be based on dollars, cents, and mills.



(1) 2 meters, 3 decimeters, and 4 centimeters.

(2) 1 meter, 2 decimeters, 3 centimeters, and 5 millimeters.

(3) 1 meter, 1 decimeter, 1 centimeter, and 1 millimeter.

**137.** Again, 1 decameter is equal to 10 meters, and 1 hectometer to 10 decameters. Hence we can write 1 hectometer, 1 decameter, 1 meter, 1 decimeter, 1 centimeter, and 1 millimeter, thus: 111.111 meters.

Write in terms of the meter, the sum of:

(1) 1 hectometer, 2 decameters, 9 meters, 7 decimeters, 6 centimeters, and 8 millimeters.

(2) 3 hectometers, 1 decameter, 5 meters, 2 decimeters, 4 centimeters, and 6 millimeters.

**138. NOTATION AND NUMERATION.** — Consider the number 111: the first 1, beginning at the right, denotes one unit; the second, *one* ten or *ten* units; the third, *one* hundred or *one hundred* units.

The third 1 is equivalent to one hundred times the first 1, and to ten times the second 1; the second is equivalent to ten times the first 1, and to one tenth of the third 1; the first 1 is equivalent to one tenth of the second 1, and to one hundredth of the third 1.

Now rewrite the number 111, place a point after the first 1 to indicate that this 1 is to be regarded as representing the standard unit, and then place after the point three 1's, so that we have

111.111.

We may ask what each of these 1's should mean, if the same relation is to hold among successive digits that we

have supposed hitherto to hold. The 1 after the point would naturally mean one tenth. The next 1 to the right would naturally mean one hundredth. It is one tenth of the preceding one — that is, one tenth of one tenth.

Similarly, the next 1 would signify one thousandth, and would equal one hundredth of the one tenth or one tenth of the one hundredth. Thus, the number 111.111 may be written as follows: One hundred, one ten, one unit, one tenth, one hundredth, and one thousandth.

Again, the 1 to the extreme right is 1 thousandth; the next 1 is, from its position, equivalent to 10 thousandths; and the next 1 is 100 thousandths. So that to the right of the point we have 111 thousandths. The whole number may now be read, one hundred and eleven, and one hundred and eleven thousandths.

**139.** A **Decimal Fraction** or a decimal is one which has for its denominator 10, 100, 1000, or some power of 10.

The **Power** of a number is the product found by multiplying the number by itself one or more times; thus, 100 or  $10^2$  is the second power of 10; 1000 or  $10^3$ , the third power of 10.

The denominator of a decimal fraction is never expressed; thus  $\frac{5}{10}$  and  $\frac{57}{100}$  are written as decimals, .5 and .57.

The point placed to the right of the one-unit and between it and the tenth-unit is called the *decimal* point.

**140.** *To change a decimal to a vulgar fraction in its lowest terms,*

$$.425 = \frac{425}{1000} = \frac{85}{200} = \frac{17}{40}.$$

$$6.0364 = 6\frac{364}{10000} = 6\frac{91}{2500}.$$

Conversely,

$$5\frac{293}{1000} = 5.293.$$

$$92\frac{25}{1000} = 92.025.$$

**Exercise 94**

1. Read the numbers in Exercises 95 and 96, expressing them in terms of different units, as 1 meter, 1 mi., 1 lb.

2. Read .5, .05, .005, 0005.

3. What is the ratio of .5 to .05? To .005? To .0005?

4. .0005 is what part of .005? Of .05? Of .5?

5. Explain how it is that the insertion of a zero between the point and the 5 in the decimal .5 changes the value of the decimal.

6. Read .5, .50, .500, .5000.

7. What is the ratio of .5 to .50? To .500? To .5000?

8. .5000 is what part of .500? Of .50? Of .5?

9. Explain why the addition of zeros to the right does not change the value.

10. Name the decimal consisting of one digit which lies nearest to .54, .66, .92, .78, and .85.

11. Reduce the following decimals to vulgar fractions in their lowest terms:

.8, .45, .06, .0004, .375, .0625, .006, .00600.

12. Express the following fractions as decimals:

$\frac{7}{10}$ ,  $\frac{48}{100}$ ,  $\frac{6}{100}$ ,  $\frac{248}{1000}$ ,  $\frac{29}{1000}$ ,  $9\frac{8}{100}$ ,  $269\frac{54}{10000}$ ,  $17\frac{9}{100000}$ .

Express in figures:

13. Four tenths; two, and five hundredths; six thousandths.

14. Six hundred and five, and twenty-eight-thousandths; three thousand and twenty-nine, and sixty-five ten-thousandths; two, and four hundred and nine millionths.

15. Eighteen, and two ten-thousandths; nine hundred, and twenty-nine ten-millionths; one hundred, and one ten-thousandth.

## ADDITION OF DECIMALS

**141.** What is the sum of 4.9, 6.084, 24.32, and .8976?

$$\begin{array}{r} 4.9 \\ 6.084 \\ 24.32 \\ .8976 \\ \hline 36.2016 \end{array}$$

In arranging the numbers, be careful to put the decimal points directly under each other, thus bringing units under units, tenths under tenths, etc. Then begin at the lowest order and add as if the figures were integers, putting the decimal between the unit and the tenths' place.

**Exercise 95**

Find the sum and prove your answers correct by adding down the columns:

$$\begin{array}{r} 1. \quad 53.67 \\ \quad 4.009 \\ 821.64 \\ \quad 2.182 \\ \hline \end{array}$$

$$\begin{array}{r} 2. \quad 9.7 \\ \quad 20.492 \\ \quad .0487 \\ \hline 918.0006 \end{array}$$

$$\begin{array}{r} 3. \quad .8592 \\ \quad 913.74 \\ \quad 21.0106 \\ \hline 47.9 \end{array}$$

4. Explain step by step the process of addition in the first three examples.

Write in columns and add:

5.  $6.5 + 32.47 + 2.048 + 59.$

6.  $.452 + 4.08 + .646 + .06 + 49.027.$

7.  $4.0406 + 213.939 + 2.91 + 3.04.$

8.  $89432.1 + 7.65439 + .0084 + 8400.$

9.  $27.064 + .0012 + 394.2001 + .819.$

10. How many yd. are there in five pieces of cloth, the first of which contains 37.5 yd., the second 26.75, the third 14.375, the fourth 36.5, and the fifth 63.125?

11. Four sections of land contain the following areas: 24.729 sq. mi., 92.04 sq. mi., 8.007 sq. mi., and 36.429 sq. mi. Find the total area.

12. Find the sum of sixty-one ten-thousandths; eight, and seven thousand six hundred and ninety-five ten-thousandths; nine thousand seven hundred and eighty-six.

13. Find the sum of the following: twenty-four ten-thousandths; nine, and three thousand and four ten-thousandths; two hundred and seventy-seven, and six hundred thousandths; nine, and nine thousandths.

14. Find the sum of 4 of the tenths' unit, 6 of the thousandths' unit, and 8 of the millionths' unit.

15. What is the sum of 4, 6, 9, 7, 5, 8, 3, 6, and 7, when the unit of value is \$1? \$.01? \$.001? \$.0001?

### SUBTRACTION OF DECIMALS

**142.** From 29.364 take 3.87049.

$$\begin{array}{r} 29.36400 \\ 3.87049 \\ \hline \end{array}$$

$$\begin{array}{r} 29.364 \\ 3.87049 \\ \hline \end{array}$$

As in addition of decimals, place the decimal points under each other, thus placing units under units, tenths under tenths, etc.

As the value of the decimal is not changed by annexing zeros to the right of the decimal, annex in this case two zeros. Subtract as in whole numbers, and place the decimal point in the remainder between the unit and the tenths' place.

### Exercise 96

From	1. 8.43	2. 13.47016	3. .503	4. .52
Take	<u>2.95</u>	<u>2.0984</u>	<u>.28914</u>	<u>.13064</u>

5. Explain step by step the process of subtraction in the first three examples.

Find the difference and prove your answers correct:

- |                 |                      |                      |
|-----------------|----------------------|----------------------|
| 6. .62 — .47.   | 10. .07 — .059.      | 13. .7304 — .67.     |
| 7. .73 — .35.   | 11. 8.9 — 3.4265.    | 14. 4.8295 — 3.9998. |
| 8. .894 — .406. | 12. 39.42 — 15.9879. | 15. 2.03 — .00428.   |
| 9. .74 — .365.  |                      |                      |

16. Explain whether .067 or .068 is nearer .06748, and express in words the difference in each case.

17. The length of a seconds pendulum is 39.1392 in., and that of a meter is 39.371 in. Find the difference in their lengths.

18. Find the difference between the length of 1 meter and 1 yd.

19. From a piece of cloth containing 35.5 yd., a merchant sold 12.75 yd. How much was left?

20. Take 1 millionth from 1 thousandth.

21. Find the difference between sixty-four ten-thousandths and seventy-five tenths. Add this difference to their sum.

22. Find the difference between \$  $11\frac{33}{100}$  and 35¢.

23. The mercury in a barometer rose .121 in., .073 in., and .019 in. in three successive days; it fell .054 in. and .065 in. during the two following days, rose .053 in. on the sixth day, and fell .028 in. on the seventh day. If its height at the beginning of the first day was 30.078 in., what was its height at the close of the seventh day?

### MULTIPLICATION OF DECIMALS

**143.** (1)  $\frac{7}{10}$  multiplied by 10 = 7, and therefore .7 multiplied by 10 = 7.  
 $\frac{627}{100}$  multiplied by 10 =  $\frac{6270}{100}$ , or 62.7, and therefore 6.27 multiplied by 10 = 62.7.

(2)  $\frac{75}{100}$  multiplied by 100 = 75, and therefore .75 multiplied by 100 = 75.  
 $\frac{6275}{1000}$  multiplied by 100 =  $\frac{627500}{1000}$  = 627.5, and therefore 6.275 multiplied by 100 = 627.5.

(3)  $\frac{62758}{10000}$  multiplied by 1000 =  $\frac{62758000}{10000}$  = 6275.8, and therefore 6.2758 multiplied by 1000 = 6275.8.

### Exercise 97

1. Multiply each of the following numbers by 10:

.6, .8, .84, .95, .842, .763.

2. Multiply by 10: .06, .04, .005, .0123.

3. Multiply by 10: 42.3, 5.69, .478.
4. State how to multiply a decimal by 10, without actually doing the work of multiplication.
5. Multiply by 100: .84, 9.65, .763, .003, .04, .246.
6. State how to multiply a decimal by 100, without actually doing the work of multiplication.
7. Multiply by 1000: .982, .0642, .0009, .008, .0123.
8. State how to multiply a decimal by 1000. By 10,000. By 100,000.
9. Multiply by 100: .86, 8.6, .9, .060, 9.8, .4.
10. Multiply by 1000: .594, 5.94, 59.4, .007, .07, .7, 3.14, 2.5.
11. State how to divide a decimal by 10. By 100. By 1000. By 10,000.
12. Divide by 10: 27, 82.19, 4.8, 52.93, .4, .06, .009.
13. Divide by 100: 482, 76, 415.62, 8.1, .78, .4, .09, 789.46.
14. Divide by 1000: 643, 2459.7, .69, 2.31, .03, .009.

**144.** (1) Multiply 6.24 by 46.

$$\begin{array}{r}
 6.24 \\
 46 \\
 \hline
 37.44 \\
 249.6 \\
 \hline
 287.04
 \end{array}$$

Here we multiply 4 hundredths by 6, and the product is 24 hundredths, or 2 tenths and 4 hundredths. Again, we multiply 2 tenths by 6, and, adding in the 2 tenths, the result is 14 tenths, or 1 unit and 4 tenths, and so on. Next, multiplying by 4, we must write the results one place to the left, as in the multiplication of integers. In multiplying, it is as well to omit the decimal points from the partial products 37.44 and 249.6.

(2) Multiply 6.24 by 4.6.

$$\begin{array}{r} 6.24 \\ 4.6 \\ \hline 3744 \\ 2496 \\ \hline 28.704 \end{array}$$

This differs from the former question only in that the multiplier 4.6 is one-tenth of 46, and therefore the product is one-tenth as large, or 28.704.

**145.** From this and other similar problems the rule can be deduced:

*To multiply two decimals, proceed as if they were integers, and mark off in the product as many places as there are in both multiplier and multiplicand.*

**146.** (1) Multiply 2.56 by .94.

$$\begin{array}{r} 2.56 \\ .94 \\ \hline 1024 \\ 2304 \\ \hline 2.4064 \end{array}$$

(2) Multiply .249 by .035.

$$\begin{array}{r} .249 \\ .035 \\ \hline 1245 \\ 747 \\ \hline .008715 \end{array}$$

EXPLANATION. —

$$.249 \times .035 = \frac{249}{1000} \times \frac{35}{1000} = \frac{8715}{1000000} = .008715.$$

### Exercise 98

- Find .05 of \$ 26; .06 of \$ 248; .04 of \$ 16.
- Find .08 of \$ 46.50; .03 of \$ 894.75; .07 of \$ 2389.20.
- Find .3 of \$ 250; .9 of \$ 64; .8 of \$ 76.
- Find .025 of \$ 324; .035 of \$ 704.60; .045 of \$ 852.94.



5. Find .375 of 45 mi.; .0625 of 640 A.; .0875 of \$415.60.

Multiply :

6.  $4.8 \times 5.12$ ;  $.21 \times 4.67$ .
7.  $3.1416 \times .02$ ;  $1.46 \times .39$ .
8.  $.004 \times .99$ ;  $.004 \times .005$ .
9.  $\$249 \times 1.04$ .
10.  $.84 \times .251$ ;  $2.04 \times .0037$ .
11.  $.8 \times .8$ ;  $.09 \times .09$ .
12.  $.1 \times .1$ ;  $.01 \times .01$ .
13.  $.2 \times .2$ ;  $.7 \times .7$ .
14.  $.375 \times 2.15$ ;  $.0375 \times 2.15$ .
15.  $.051 \times .042$ ;  $.014 \times .0038$ .
16. Find the area of a rectangle 6.2 ft. long by 4.7 ft. wide.
17. Find the area of a rectangle 7.5 ft. wide by 3.4 ft. long.
18. Find the value of a rectangular solid whose dimensions are 2.5, 4.6, and 5.8 in.
19. Find the volume of a solid whose dimensions are 8.8, 6.6, and 4.4 in.
20. If my gain on selling an article is measured by the number .23 and the cost \$265, find the gain.
21. If my loss on selling an article that cost \$226.50 is measured by the number .34 and the cost, find the loss.
22. Multiply \$240 by 1.05 twice in succession.
23. Multiply \$325 by 1.06 three times in succession.
24. Multiply \$415.80 by 1.04 twice and the result by 1.02.
25. Multiply \$75 by 1.045 three times in succession.
26. Multiply \$975 by 1.065 three times in succession.
27. Multiply 3.1416 by 8; by 7.5; by .04.
28. Multiply 7.48 by 9.1; by .04; by .006.
29. Multiply the square of 5 by 3.1416.

30. Measure in inches and decimals of an inch the diameters of several circles and multiply the result in each case by 3.1416. Then measure the circumference and note which are greater, the products or the lengths of the circumferences.

**Exercise 99**

1. The length of a wall, according to the French metric system, is 9.48 meters. Find its length in in., the length of 1 meter being 39.371 in.

2. Multiply the sum of 2.616, .00132, and 1.0448 by .62639.

3. A piece of land is 63.5 rd. long, and 27.75 rd. wide. What will it cost to fence it at \$.875 per rd.?

4. Multiply 10.5 by 1.05 and reduce the result to a fraction in its lowest terms.

5. The specific gravity of atmospheric air compared with water is .0012. I ask for the specific gravity of common gas, and am told it is .45 compared with air. Find its specific gravity compared with water.

6. A lumber merchant bought 106,250 ft. of lumber at \$14.375 per M., and retailed it at \$1.75 per C. Find his gain.

7. Water is composed of two gases, oxygen and hydrogen, in the proportion of 88.9 to 11.1. What weight is there of each in a cubic yard of water? (1 cu. ft. of water weighs 1000 oz.)

8. The chain for measuring land is 66 ft. long. What is the length in yards of a fence that measures 2456 ch., and how much would it cost at \$8.86 per yd.?

9. Find the area of a rectangular field whose breadth is 78.23 ch., and length 85.40 ch., there being 10 sq. ch. in 1 A.

10. A farmer bought 48.125 T. of hay; for 20.25 T. of it he paid \$16 per T., and for the rest \$18.2625 per T.; he sold the whole at the average price of \$.945 per cwt. How much did he gain or lose?

11. A person sold  $.15$  of an estate to one person, and then  $\frac{5}{17}$  of the remainder to another person. What part of the estate did he still retain?

12. If a business produces an annual return of \$12,000, and of three partners one has  $.465$  and another  $.28$  share of the profits, how much money falls to the share of the third partner?

13. A merchant sells 28.5 yd. of cloth which cost him 25¢ a yd., for 37.5¢ a yd. What was his gain?

## DIVISION OF DECIMALS

$$\begin{array}{r} 8 \overline{)24} \\ 3 \end{array}$$

$$\begin{array}{r} 8 \overline{)2.4} \\ .3 \end{array}$$

$$\begin{array}{r} 8 \overline{).24} \\ .03 \end{array}$$

That is, 24 divided by  $8 = 3$ .

24 tenths divided by  $8 = 3$  tenths  $= .3$ .

24 hundredths divided by  $8 = 3$  hundredths  $= .03$ .

$$\begin{array}{r} 6 \overline{).0018} \\ .0003 \end{array}$$

$$\begin{array}{r} 46 \overline{)251.62} (5.47 \\ \underline{230} \\ 216 \\ \underline{184} \\ 322 \\ \underline{322} \end{array}$$

That is, 18 ten-thousandths  $\div 6 = 3$  ten-thousandths  $= .0003$ ; and 25,162 hundredths  $\div 46 = 547$  hundredths  $= 5.47$ .

$$\text{147. } \begin{array}{r} 5 \overline{)15} \\ 3 \end{array}$$

$$\begin{array}{r} 50 \overline{)150} \\ 3 \end{array}$$

$$\begin{array}{r} 500 \overline{)1500} \\ 3 \end{array}$$

$$\begin{array}{r} 5000 \overline{)15000} \\ 3 \end{array}$$

Therefore, if we multiply the divisor by 10, 100, 1000, and so on, and at the same time multiply the dividend by 10, 100, or 1000, and so on, the quotient remains unchanged.

(1) Find the value of  $4.1262 \div .69$ .

The quotient of  $4.1262 \div .69$  is the same as that of  $412.62 \div 69$ . Here we multiplied each number by 100.

$$69 \overline{)412.62(5.98}$$

$$\begin{array}{r} 345 \\ 676 \\ 621 \\ 552 \\ 552 \end{array}$$

$$\therefore 4.1262 \div .69 = 5.98.$$

In this division the 412 is 412 units, and the quotient 5 is therefore 5 units; the first remainder, 676, is 676 tenths, and the quotient 9 is therefore 9 tenths; the second remainder, 552, is 552 hundredths, and the quotient 8 is therefore 8 hundredths.

(2) Find correct to the third decimal place the quotient of  $8.94 \div 3.1416$ .

$$\begin{array}{r} 3.1416 \overline{)8.94(} \\ 31416 \overline{)89400(2.845} \end{array}$$

$$\begin{array}{r} 62832 \\ 265680 \\ 251328 \\ 143520 \\ 125664 \\ 17856 \\ 15708 \\ 2148 \end{array}$$

$$\therefore 8.94 \div 3.1416 = 2.845, \text{ correct to three decimal places.}$$

### Exercise 100

Divide, proving your answer correct to every third question:

1.  $25.68 \div 3.21$ .
2.  $10.836 \div 5.16$ .
3.  $8.54 \div .07$ .
4.  $\$49.92 \div .065$ .
5.  $\$54.75 \div .98$ , correct to three decimal places.
6.  $\$75.60 \div .99$ , correct to three decimal places.
7.  $\$64.26 \div 1.02$ .
8.  $\$84.3648 \div 1.04$  twice in succession and the result by 1.02.

9.  $\$16989.7728 \div 1.04$  twice in succession and the result by 1.02.
10.  $2450.90 \div .998$ , correct to three decimal places.
11.  $\$23881 \div 1.06$  three times in succession, correct to three decimal places.
12.  $\$11.19195 \div \$4.8665$ .
13.  $.00081 \div 27$ , and  $1.77089$  by  $4.735$ .
14.  $1 \div .1$ , by  $.01$ , and  $1 \div .0001$ .
15.  $31.5 \div .126$ ;  $5.2 \div .32$ .
16.  $12.6 \div .0012$ , and  $.065341 \div .000475$ .
17.  $3.012 \div .0006$ .
18.  $130.4 \div .0004$  and  $.004$ , and  $46.634205 \div 4807.65$ .
19.  $1.69 \div 1.3$ , by  $.13$ , by  $13$ , and by  $.013$ .
20.  $816 \div .0004$ .
21.  $.00005 \div 2.5$ , by  $25$ , and by  $.0000025$ .
22.  $32.5 \div 8.7$ ;  $.02 \div 1.7$ , correct to four decimal places.
23.  $.009384 \div .0063$ , correct to four decimal places.
24.  $37.24 \div 2.9$ ;  $.0719 \div 27.53$ , correct to four decimal places.
25. Measure in inches and decimals of an inch the diameter of any circle, and divide the result into the length of the circumference. Do this until you find the quotient to be nearly 3.1416.

#### Exercise 101

1. If a gal. contains 231 cu. in. and a cu. ft. 1728 cu. in., find correct to two decimal places the number of gal. in a cu. ft. of water.
2. Using the result obtained in question 1, find the number of gal. of water that a rectangular tin 2 ft., by 3 ft., by 4 ft. will hold.
3. If a bu. contain 2150.42 cu. in., find the number of cu. in. in a dry qt.

4. Find correct to three decimal places the number of cu. ft. in a bu.

5. By what decimal part of 1 in. does .0009 of 1 ft. exceed .00003 of 1 yd. ?

6. Find cost of 7225 lb. coal at \$ 7.25 per ton of 2000 lb.

7. Find the value of

$$\frac{3.0005 \times .006}{.0009}$$

8. A man paid \$ 2,896,863.50 for land and sold 56.25 A. at \$ 31 an A.; the remainder then stood him at \$ 20.05 an A. How many A. did he buy ?

9. Water expands when freezing so that a cu. ft. of water becomes 1.089 cu. ft. of ice. Find how many cu. ft. of water there are in an iceberg which is 900 ft. long, 88 ft. broad, and 220 ft. high.

10. Show by examples that a decimal is divided by 10,000 by removing the decimal point in the dividend four places towards the left.

11. A creditor receives \$ 1.50 for every \$ 4 of what was due to him, and thereby loses \$ 301.05. What was the sum due ?

12. Divide .0075 by 6.4, and explain the reason for fixing the position of the decimal point in the quotient.

13. A person expended \$ 55.92 in tea at \$.875 per lb., coffee at \$.1875, and sugar at \$.1025, buying an equal quantity of each. How many lb. of each did he buy ?

14. A gentleman whose real property is .834 of his personal property leaves the former, amounting to \$ 10,008, to his eldest son; and the latter to be equally shared by him and two others. Find the amount received by each.

15. A merchant expended \$ 280.60 in purchasing cloth at 95¢ a yd., at \$ 1.37 a yd., and at 73¢ a yd., buying the same quantity of each. Find the entire number of yd. purchased.

16. Suppose unity to represent .0012, what number represents .0001 ?

17. Find the earth's equatorial diameter in miles, supposing the sun's diameter, which is 111.454 times as great as the equatorial diameter of the earth, to be 883,345 mi.

18. The French meter is 39.371 in. in length. Express the length of 25 meters as a decimal of an English mi., there being 5280 ft. in 1 mi. and 12 in. in 1 ft.

19. The total Indian population on the reservations in 1880 was 255,327, while the area of the Indian reservations, in the United States, was 241,800 sq. mi. What average quantity of land was occupied by 100 Indians ?

20. The total Indian population on the reservations in 1893 was 249,366, while the area of the Indian reservations was 134,176 sq. mi. What average quantity of land was occupied by 100 Indians ?

21. The area of the reservations in 1893 was how many thousandths of that in 1880 ?

22. The Indian population on the reservations in 1893 was how many thousandths of that in 1880 ?

23. Find the quantity of coal consumed by a steamer for a voyage of 4043 mi., supposing her rate per hr. to be 16.172 mi. and her consumption of coal 87 T. per da.

#### REDUCTION OF DECIMALS

148. (1) Reduce .275 to a common fraction.

$$.275 = \frac{275}{1000} = \frac{55}{200} = \frac{11}{40}.$$

(2) Reduce to a common fraction .08 $\frac{1}{3}$ .

$$.08\frac{1}{3} = \frac{8\frac{1}{3}}{100} = \frac{25}{300} = \frac{1}{12}.$$

## Exercise 102

Reduce to common fractions in their lowest terms:

- |          |                        |                        |                          |
|----------|------------------------|------------------------|--------------------------|
| 1. .5.   | 6. .625.               | 11. $.33\frac{1}{3}$ . | 16. 8.9375.              |
| 2. .25.  | 7. .125.               | 12. $.03\frac{1}{3}$ . | 17. 29.975.              |
| 3. .75.  | 8. .0625.              | 13. $.06\frac{1}{4}$ . | 18. $18.06\frac{2}{3}$ . |
| 4. .60.  | 9. .875.               | 14. $.66\frac{2}{3}$ . | 19. $6.00\frac{1}{2}$ .  |
| 5. .375. | 10. $.16\frac{2}{3}$ . | 15. $.14\frac{2}{7}$ . | 20. 249.00075.           |

21. If my gain on selling a lb. of tea is measured by the number .125, and the cost 72¢, what was my gain on 1 lb.?

22. If a crate of berries which cost \$1.35 was sold at a gain of  $.22\frac{2}{3}$  of the cost, find the gain.

23. I bought a farm for \$4800, and sold it at a loss of .375 of the cost price. Find the selling price.

24. A merchant sold coffee at a gain of  $.33\frac{1}{3}$  of the cost. If his gain on a quantity of coffee was \$12, what did it cost him?

25. A grain merchant sold wheat for \$3400, gaining  $.06\frac{1}{4}$  of the cost. Find the cost price.

149. (1) Divide 7 by 8, expressing the result as a decimal.

$$\begin{array}{r} 8 \overline{)7.000} \\ \underline{.875} \end{array}$$

$$\text{Now } 7 \div 8 = \frac{7}{8}. \quad \therefore \frac{7}{8} = .875.$$

PROOF

$$.875 = \frac{875}{1000} = \frac{175}{200} = \frac{7}{8}.$$

(2) Reduce  $\frac{15}{16}$  to a decimal.

$$\begin{array}{r} 16 \overline{)150.9375} \\ \underline{144} \phantom{00} \\ 60 \phantom{00} \\ \underline{48} \phantom{00} \\ 120 \phantom{00} \\ \underline{112} \phantom{00} \\ 80 \phantom{00} \\ \underline{80} \phantom{00} \end{array}$$

$$\therefore \frac{15}{16} = .9375.$$



(3) Reduce  $9\frac{18}{23}$  to a decimal correct to four decimal places.

$$\begin{array}{r}
 23 \overline{)180} (.7826 \\
 \underline{161} \phantom{00} \\
 190 \phantom{00} \\
 \underline{184} \phantom{00} \\
 60 \phantom{00} \\
 \underline{46} \phantom{00} \\
 140 \phantom{00} \\
 \underline{138} \phantom{00} \\
 2
 \end{array}$$

$\therefore 9\frac{18}{23} = 9.7826$ , correct to four decimal places.

### Exercise 103

Reduce to decimals and prove results:

1.  $\frac{1}{4}, \frac{1}{2}, \frac{3}{4}$ .
2.  $\frac{1}{8}, \frac{3}{8}, \frac{5}{8}, \frac{7}{8}$ .
3.  $\frac{1}{16}, \frac{5}{16}, \frac{9}{16}$ .
4.  $\frac{9}{25}, \frac{19}{25}$ .
5.  $\frac{66}{128}, \frac{54}{128}, \frac{1}{16}$ .
6.  $6\frac{11}{32}, 3\frac{13}{16}$ .
7.  $\frac{5}{4}, \frac{9}{8}, \frac{32}{25}, \frac{31}{25}$ .
8. Find correct to four decimal places the value of  $\frac{4}{18}, \frac{15}{17}, \frac{14}{29}, \frac{16}{21}$ .

9. Butter bought for 25¢ a lb. was sold for 28¢ a lb. Express the gain as a decimal of the cost.

10. I bought a store for \$6912, and sold it for \$5184. Express the selling price as a decimal of the cost.

11. A real estate agent sold land which cost him \$6240 at a loss of \$2340. What was his loss on each \$1000 invested?

12. A ch. contains 66 ft., and a mi. 5280 ft. What decimal part of a mi. is a ch.?

## CHAPTER XII

### COMPOUND QUANTITIES

**150.** Quantities like 4 yd.,  $3\frac{1}{2}$  lb., and  $6\frac{1}{4}$  gal. are called *simple quantities*, because they are expressed in terms of a single unit of measurement.

Quantities like 3 lb. 8 oz., 6 gal. 1 qt., are called *compound quantities*, because they are expressed in terms of two or more units of measurement.

**151.** The units of money are the units which are used to measure the *value* of things. The one-dollar gold piece is at present (July, 1896) the prime unit or standard of value in the United States and Canada.

**152.**

#### UNITS OF VALUE

##### UNITED STATES MONEY

10 mills (m.)	= 1 cent (ct. or ¢)
10 cents	= 1 dime (d.)
10 dimes	= 1 dollar (\$)
10 dollars	= 1 eagle (E.)

The coins of the United States are :

Bronze : the cent.

Nickel : the five-cent piece.

Silver : the dime, quarter-dollar, half-dollar, and dollar.

Gold : the quarter-eagle, half-eagle, eagle, and double-eagle.

**153.** **Sterling Money** is the money of Great Britain and Ireland.

The prime unit is 1 pound, whose value is \$4.8665.

The pound, when coined, is called the sovereign.

BRITISH OR STERLING MONEY

4 farthings (far.)	= 1 penny (d.)
12 pence	= 1 shilling (s.)
20 shillings	= 1 pound (£)
5 shillings	= 1 crown
21 shillings	= 1 guinea

**154.** The unit of *French Money* is 1 franc, which is worth 19.3¢.

The unit of *German Money* is 1 mark, which is worth 23.85¢.

Exercise 104

- How many mills are there in 2¢? 3¢?  $\frac{1}{2}$ ¢?  $1\frac{1}{2}$ ¢?  $2\frac{1}{2}$ ¢?
- How many cents are there in 40 mills? 60 mills? 15 mills? 5 mills? 25 mills?
- State orally the table of English Money.
- Reduce to farthings: 3d.; 6d. 2 far.; 9d. 3 far.
- Reduce to pence: 4s.; 8s. 5d.; 12s. 6d.
- Reduce to shillings: £7; £2 12s.; £9 7s.
- How many pence are there in  $\frac{1}{2}$ s.?  $\frac{3}{4}$ s.?  $\frac{2}{3}$ s.?  $\frac{5}{8}$ s.?  $\frac{3}{5}$ s.?
- How many shillings and pence are there in £ $\frac{3}{5}$ ? £ $\frac{1}{3}$ ? £ $\frac{5}{8}$ ? £ $\frac{7}{8}$ ?
- How many shillings are there in £.3? £.7? £.25? £.33 $\frac{1}{3}$ ?
- What fraction of a shilling is 3d.? 4d.? 8d.? 9d.? 10d.?
- What is the value of £1 in American money? Of £10? Of £100?
- How many shillings and pence are there in 60d.? 84d.? 39d.? 58d.? 112d.?

13. How many pounds and shillings are there in 80 s. ? 65 s. ?  
120 s. ? 48 s. ?

14. How many pounds and shillings in 1 guinea ? 4 guineas ?  
6 guineas ? 9 guineas ?

15. What decimal of a pound is 10 s. ? 12 s. ? 17 s. ? 24 s. ?

16. What part of a crown is 1 s. ? How many crowns in 10 s. ?  
20 s. ?

17. Express 2 guineas in sovereigns and shillings.

18. What is the value of 10 francs in U. S. money ?

19. What is the value of 100 marks in U. S. money ?

20. What is the cost in cents of 3 books at 1 franc each ?

21. What is the difference in value between 100 marks and  
100 francs ?

#### UNITS OF WEIGHT

**155. Avoirdupois Weight** is used for weighing everything except jewels, precious metals, and medicines when dispensed.

The prime unit of weight is 1 pound Avoirdupois.

#### AVOIRDUPOIS WEIGHT

16 ounces (oz.)	= 1 pound (lb.)
100 pounds	= 1 hundredweight (cwt.)
20 hundredweight	= 1 ton (T.)

In the United States Custom House, and in weighing iron and coal at the mines, the long hundredweight and the long ton are used.

112 pounds = 1 long hundredweight

2240 pounds = 1 long ton

One pound Avoirdupois = 7000 grains

One ounce Avoirdupois =  $437\frac{1}{2}$  grains

#### Exercise 105

1. State orally the table of Avoirdupois Weight.
2. Reduce to oz. : 1 lb. 8 oz. ; 2 lb. 4 oz.

3. Express 1 lb. 8 oz. as a fraction of 2 lb. 4 oz.
4. Express 1 lb. 8 oz. as a decimal of 2 lb. 4 oz.
5. What part of 1 lb. is 4 oz. ? 12 oz. ? 2 oz. ? 8 oz. ?  
Express your results also as decimals.
6. What part of 1 T. is 400 lb. ? 800 lb. ? 1500 lb. ?
7. A coal dealer buys coal by the car-load at the mines. How many more lb. of coal does he get for 1 T. than he gives ?
8. Show by dividing 7000 gr. by 16, that 1 oz. Avoir. is equal to  $437\frac{1}{2}$  gr.
9. One oz. is what part of 1 lb. ?  $\frac{1}{4}$  of an oz. is what part of 1 lb. ?
10. A farmer sells 3 cows whose united weight is 1 T. 5 cwt. What is the average weight of the cows ?

**156. Troy Weight** is chiefly used for weighing gold, silver, and jewels.

#### TROY WEIGHT

24 grains (gr.)	= 1 pennyweight (pwt.)
20 pennyweights	= 1 ounce (oz.)
12 ounces	= 1 pound (lb.)
One pound Troy	= 5760 grains
One ounce Troy	= 480 grains

#### Exercise 106

1. State orally the table of Troy Weight.
2. Reduce to gr. : 1 pwt. 16 gr. ; 2 pwt. 12 gr.
3. What is the ratio of 2 pwt. 12 gr. to 1 pwt. 16 gr. ?
4. Express 1 pwt. 16 gr. as a decimal of 2 pwt. 12 gr.
5. Reduce 1 oz. to gr.
6. Divide 5760 gr. by 12, and verify your result in question 5.

7. By how many gr. is 1 lb. Avoir. heavier than 1 lb. Troy?
8. By how many gr. is 1 oz. Troy heavier than 1 oz. Avoir.?
9. How many oz. and pwt. are there in  $\frac{1}{4}$  lb.?  $\frac{3}{8}$  lb.?  $\frac{5}{8}$  lb.?  
 $\frac{3}{8}$  lb.?
10. How many pwt. are there in .25 oz.? .4 oz.? .35 oz.?
11. What part of a lb. is 8 oz.? 1 oz.?  $\frac{2}{3}$  oz.?  $\frac{4}{5}$  oz.?
12. If coal is worth \$6 a T., how many lb. can be bought for \$3? \$2?
13. A cu. ft. of water contains 1000 oz. How many lb. does a cu. ft. of water weigh?

**157.** Druggists buy their medicines by Avoirdupois Weight, but use Apothecaries' Weight in mixing and in selling medicines.

#### APOTHECARIES' WEIGHT

20 grains (gr.)	= 1 scruple ( $\mathfrak{D}$ )
3 scruples	= 1 dram ( $\mathfrak{z}$ )
8 drams	= 1 ounce ( $\mathfrak{z}$ )
12 ounces	= 1 pound (lb.)

One pound Apothecaries' weight = 5760 grains

One ounce Apothecaries' weight = 480 grains

#### Exercise 107

1. State orally the table of Apothecaries' Weight.
2. How many ounces in 16  $\mathfrak{z}$ ? 40  $\mathfrak{z}$ ? 72  $\mathfrak{z}$ ?
3. How many drams in 9  $\mathfrak{D}$ ? 18  $\mathfrak{D}$ ? 54  $\mathfrak{D}$ ?
4. What part of a pound is 4  $\mathfrak{z}$ ? 9  $\mathfrak{z}$ ? 10  $\mathfrak{z}$ ?
5. How many scruples in 3  $\mathfrak{z}$  and 2  $\mathfrak{D}$ ? In 40 gr.? In 120 gr.?

## UNITS OF LENGTH

**158.** The prime or standard unit of length is 1 yard.

## LONG MEASURE

12 inches (in.)	= 1 foot (ft.)
3 feet	= 1 yard (yd.)
$5\frac{1}{2}$ yards or $16\frac{1}{2}$ feet	= 1 rod (rd.)
320 rods	= 1 mile (mi.)
1 mi. = 320 rd. = 1760 yd. = 5280 ft.	

A hand, used in measuring the height of horses, = 4 in. ; a knot, used in navigation, = 6086 ft. or 1.15 mi.

A fathom, used in measuring depth at sea, = 6 ft.

**159.** **Surveyor's Linear Measure** is used by surveyors in measuring land. The prime unit is 1 chain, called *Gunter's Chain*.

## SURVEYOR'S LINEAR MEASURE

100 links (l.)	= 1 chain (ch.)
80 chains	= 1 mile (mi.)
1 ch. = 4 rd. = 22 yd. = 66 ft. = 792 in.	
1 link = 7.92 in.	

**160.** Mark off in the schoolroom 1 ft., 1 yd., and 1 rd. Locate two points exactly 1 mi. apart.

## Exercise 108

- How many in. are there in 1 yd. ?  $\frac{1}{4}$  yd. ?  $\frac{3}{4}$  yd. ?  $\frac{1}{8}$  yd. ?  $\frac{5}{8}$  yd. ?
- Reduce to yd. : 4 rd. ; 8 rd. ; 32 rd. ; 320 rd. ; 1 mi.
- Reduce to ft. : 18 yd. ; 76 yd. ; 176 yd. ; 1760 yd. ; 1 mi.
- Show that 1 mi. = 320 rd. = 1760 yd. = 5280 ft. = 63,360 in.
- How many yd. are there in .3 mi. ? .8 mi. ? .25 mi. ?  $.06\frac{1}{4}$  mi. ?
- What part of a ft. is 3 in. ? 6 in. ? 8 in. ? 10 in. ?
- What part of a yd. is 12 in. ? 18 in. ? 24 in. ? 27 in. ?

8. What part of a rd. is 1 yd. ? 3 yd. ? 5 yd. ?  $5\frac{1}{2}$  yd. ?  
 9. What part of a mi. is 40 rd. ? 200 yd. ? 280 yd. ?  
 10. How many yd. are there in 1 rd. ? How many ft. ? How many in. ?  
 11. How many ch. are there in 320 rd. ? How many rd. are there in 1 ch. ?  
 12. Show that 1 ch. = 4 rd. = 22 yd. = 66 ft. = 792 in.  
 13. How many in. are there in 100 links ? In 1 link ?

## UNITS OF SURFACE OR SQUARE MEASURE

**161.** Surface has two dimensions, — length and breadth.

**162.** The prime unit of area is 1 square yard, which, like 1 square inch, 1 square foot, 1 square rod, and 1 square mile, is derived from the corresponding unit of linear measure.

The measure of 1 ft. is 12, the unit being 1 in.

The measure of 1 sq. ft. is 144, the unit being 1 sq. in.

$\therefore$  1 sq. ft. = 144 sq. in.

The measure of 1 yd. is 3, the unit being 1 ft.

The measure of 1 sq. yd. is 9, the unit being 1 sq. ft.

$\therefore$  1 sq. yd. = 9 sq. ft.

The measure of 1 rd. is  $5\frac{1}{2}$ , the unit being 1 yd.

The measure of 1 sq. rd. is  $(5\frac{1}{2} \times 5\frac{1}{2})$ , or  $30\frac{1}{4}$ , the unit being 1 sq. yd.

$\therefore$  1 sq. rd. =  $30\frac{1}{4}$  sq. yd.

Illustrate the above by drawing 1 sq. ft., 1 sq. yd., and 1 sq. rd., and dividing each into the next lower units of area.

In the case of 1 sq. rd. draw according to the scale of 4 in. to 1 rd.

## SURFACE OR SQUARE MEASURE

144 square inches (sq. in.)	= 1 square foot (sq. ft.)
9 square feet	= 1 square yard (sq. yd.)
$30\frac{1}{4}$ square yards	= 1 square rod (sq. rd.)
160 square rods	= 1 acre (A.)
640 acres	= 1 square mile (sq. mi.)
10 square chains = 1 acre ; 1 acre = 4840 square yards	



**163.** A township is 6 mi. square, and is divided, as in the accompanying figure, into 36 sections, each 1 mi. square.

6	5	4	3	2	1
7	8	9	10	11	12
18	17	16	15	14	13
19	20	21	22	23	24
30	29	28	27	26	25
31	32	33	34	35	36

TOWNSHIP.

W. $\frac{1}{2}$ 320 ACRES	N. E. $\frac{1}{4}$ 160 ACRES	
	N. $\frac{1}{2}$ OF S.E. $\frac{1}{4}$ 80 ACRES	
	S. W. $\frac{1}{4}$ OF S. E. $\frac{1}{4}$ 40 ACRES	S. E. $\frac{1}{4}$ OF S. E. $\frac{1}{4}$ 40 ACRES

SECTION.

Locate sections 8, 22, and 36 in the drawing. Draw a township on a scale of 1 in. to 1 mi., and divide it into 36 sections, numbering each section. Divide one of these sections into 4 square farms of 160 A. each, and name each farm according to its position in the section. Divide a second section into 8 rectangular farms of 80 A., and a third into 16 square farms of 40 A., and locate each farm as before.

### Exercise 109

- How many sq. in. are there in 2 sq. ft. ? 4 sq. ft. ? 9 sq. ft. ?
- How many sq. ft. in 5 sq. yd. ?  $\frac{1}{2}$  sq. yd. ?  $\frac{1}{4}$  sq. yd. ?
- One sq. rd. is equal to how many sq. yd. ? 4 sq. rd. ? 16 sq. rd. ? 160 sq. rd. ? 1 A. ?
- What part of a sq. rd. is 1 sq. yd. ?
- Reduce to sq. rd. : 484 sq. yd. ;  $151\frac{1}{4}$  sq. yd.
- What part of an A. is 80 sq. rd. ? 120 sq. rd. ?
- How many sq. ch. are there in 160 sq. rd. ? One sq. ch. equals how many sq. rd. ?

8. Reduce to A. : 5 sq. mi. ; 8 sq. mi.
9. If 1 sq. mi. is the unit of area, find the number which expresses the measure of 2560 A. Of 4 townships.
10. Into how many townships can a county be divided which contains 324 sq. mi. ?
11. What is the area of a square 6 ft. in length ?
12. What is the difference in area between two figures, one 6 in. sq. and the other containing 6 sq. in. ?  
Illustrate by a drawing.

## UNITS OF VOLUME

**164.** A volume has three dimensions, — length, breadth, and thickness.

**165.** The prime unit of volume is 1 cubic yard, which, like 1 cubic inch and 1 cubic foot, is derived from the corresponding unit of linear measure.

The measure of the volume of 1 cu. ft. =  $12 \times 12 \times 12 = 1728$ , the unit of volume being 1 cu. in.

The measure of the volume of 1 cu. yd. =  $3 \times 3 \times 3 = 27$ , the unit of volume being 1 cu. ft.

## CUBIC OR VOLUME MEASURE

1728 cubic inches (cu. in.) = 1 cubic foot (cu. ft.)

27 cubic feet = 1 cubic yard (cu. yd.)

**166.** Fire wood and rough stone are measured by the *cord* (cd.). The cord is a pile 8 ft. long, 4 ft. wide, and 4 ft. high. It contains 128 cu. ft. One cord foot (cd. ft.) is 1 ft. in length of the cord. Its volume is 16 cu. ft.

A cu. yd. of earth is called a *load*.

How many loads of dirt are there in a pile 15 ft. long, 12 ft. wide, and 6 ft. deep ?

**167.** Mark off in one corner 1 cu. ft., 1 cu. yd., and 1 cd. Divide the cd. into cd. ft.

## UNITS OF CAPACITY

**168.** The prime unit of capacity is 1 gallon.

## LIQUID MEASURE

4 gills (gi.)	= 1 pint (pt.)
2 pints	= 1 quart (qt.)
4 quarts	= 1 gallon (gal.)

**169.** The capacity of cisterns, reservoirs, and the like is often expressed in barrels (bbl.) of  $31\frac{1}{2}$  gal. each, or in hogsheads (hhd.) of 63 gal. each. A gal. contains 231 cu. in. Have a tin box made 11 in. long, 7 in. wide, and 3 in. deep, and note that 1 gal. of water will just fill it.

## DRY MEASURE

2 pints (pt.)	= 1 quart (qt.)
8 quarts	= 1 peck (pk.)
4 pecks	= 1 bushel (bu.)

One bushel contains 2150.42 cubic inches

**170.** Apothecaries' Fluid Measure is used by druggists in mixing medicines.

## APOTHECARIES' FLUID MEASURE

60 minims (℥)	= 1 fluid dram (f 3)
8 fluid drams	= 1 fluid ounce (f 3)
16 fluid ounces	= 1 pint (O.)
8 pints	= 1 gallon (Cong.)
One minim is about equal to 1 drop	

## Exercise 110

1. What part of 1 gal. is 1 qt.? 1 pt.?
2. What is the number of cu. in. in 1 gal.? In 1 qt., liquid measure? In 1 pt., liquid measure?
3. What part of 1 bu. is 1 qt.? 1 pt.?
4. What is the number of cu. in. in 1 bu.? In 1 qt., dry measure? In 1 pt., dry measure?
5. How many more cu. in. are contained in 1 qt., dry measure, than in 1 qt., liquid measure?

6. Show that 1 bu. is nearly equal to 9.31 gal.
7. Find the number of cu. ft. in 1 cd.
8. Find the number of cd. of wood in a pile 30 ft. long, 6 ft. high, and 4 ft. wide.
9. Find the cost of a pile of wood 24 ft. long,  $5\frac{1}{2}$  ft. high, and 4 ft. wide, at \$6 a cd.

## UNITS OF TIME

**171.** The prime unit of time is 1 day.

## MEASURE OF TIME

60 seconds (sec.)	= 1 minute (min.)
60 minutes	= 1 hour (hr.)
24 hours	= 1 day (da.)
7 days	= 1 week (wk.)
365 days	= 1 common year (yr.)
366 days	= 1 leap year (l. yr.)
100 years	= 1 century (C.)

The year is divided into 12 calendar months:

January (Jan.) . . . . .	31 da.	July . . . . .	31 da.
February (Feb.) . . . . .	28 or 29 "	August (Aug.) . . . . .	31 "
March . . . . .	31 "	September (Sept.) . . . . .	30 "
April . . . . .	30 "	October (Oct.) . . . . .	31 "
May . . . . .	31 "	November (Nov.) . . . . .	30 "
June . . . . .	30 "	December (Dec.) . . . . .	31 "

In business transactions, 1 mo. is generally taken as equal to 30 da., and 1 yr. as equal to 360 da.

The following lines are useful in enabling one to remember the number of days in a month:

"Thirty days hath September,  
April, June, and November."

A year is the period of the earth's revolution about the sun. It consists of 365 da. 5 hr. 48 min. 50 sec.

A common year lacks 11 min. 10 sec. of being 365 da. 6 hr., or  $365\frac{1}{4}$  da. Hence when we take 365 da. to a common year, and 366 da. to a leap year, we increase each year by 11 min. 10 sec. In 400 years this amounts to a little over 3 da. For that reason three out of four centennial years are

counted as common years, *i.e.* the centennial years that do not divide equally by 400 have only 365 da.

**Exercise 111**

1. Express 9 hr. as a fraction of a wk.
2. Express 12 sec. as a decimal of a min.
3. Express 146 da. as a fraction of a yr.
4. Express as a fraction of a mo.: 10 da.; 15 da.; 18 da.
5. Express as a decimal of a mo.: 21 da.; 18 da.; 27 da.
6. Find the number of da. between Jan. 3 and Feb. 4; March 27 and April 30; Oct. 24 and Dec. 11.
7. How many da. are there in Nov.? Jan.? Dec.? April? Feb.?
8. State the number of da. in each of the following, and also what part of a yr. each is, there being 30 da. to the mo. and 360 da. to the yr.: 1 mo. 10 da.; 2 mo. 12 da.; 7 mo. 6 da.

**Exercise 112**

In the questions in the following exercise add 3 days to the given time (called *days of grace*) to find the day on which the note is due.

Find the date on which a note falls due, which I promise to pay :

1. Three months after March 3, 1894.
2. Four months after June 13, 1896.
3. Ninety days after May 13, 1890.
4. Sixty days after Sept. 16, 1895.
5. Ninety days after June 4, 1895.

Find the exact number of days between the day on which each of the following notes is discounted and the day on which it is due :

6. Day of discount, May 7, 1894; due June 6, 1894.
7. Day of discount, June 27, 1894; due Oct. 16, 1894.

8. Day of discount, Sept. 4, 1894; due Oct. 30, 1894.
9. Day of discount, Dec. 23, 1894; due Feb. 20, 1894.
10. Day of discount, Jan. 15, 1892; due May 1, 1892.

## CIRCULAR OR ANGULAR MEASURE

**172. Angular Measure** is used to measure arcs, angles, and in determining latitude, longitude, direction, the position of vessels at sea, and the like.

**173. A Circle** is a plane figure contained by one line called the **circumference**, all points of which are equally distant from a point within it called the **centre**.

One-half of the circumference is called the **semicircumference**, and one-fourth a **quadrant**.

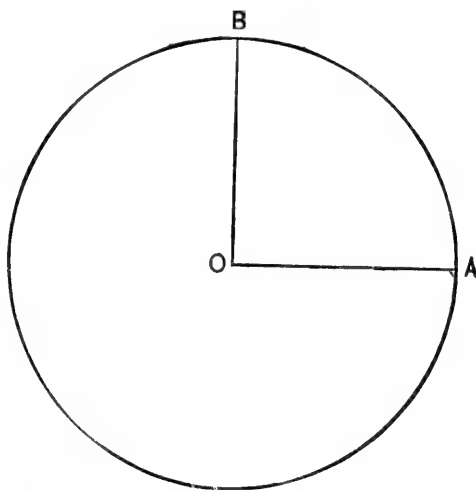
An **arc** is any portion of the circumference.

A line drawn through the centre and terminated at both extremities by the circumference is called the **diameter**.

The line drawn from the centre and terminated by the circumference is called the **radius**.

In the figure the line  $OB$  has revolved from  $OA$  through one-fourth of a *revolution*. The angle  $AOB$  is called a right angle and contains  $90^\circ$ .

$OA$  and  $OB$  are said to be *perpendicular* to each other.



## ANGULAR MEASURE

- 60 seconds ( $''$ ) = 1 minute ( $'$ )  
 60 minutes = 1 degree ( $^\circ$ )  
 360 degrees = 1 circumference (C.)

The circumference of the earth at the equator = 24,902 mi.

The length of a degree at the equator =  $24,902 \text{ mi.} \div 360 = 69.17 \text{ mi.}$

**Exercise 113**

1. What part of a revolution is 1 right angle? 2 right angles? 4 right angles?
2. What part of a revolution is  $60^\circ$ ?  $45^\circ$ ?  $225^\circ$ ?
3. What is the length of the arc of  $1^\circ$  in a circle whose circumference is 360 yd.?
4. If an arc of  $3^\circ$  is 6 ft. long, what is the length of the circumference of the circle?

**MISCELLANEOUS UNITS****174. NUMBERS**

12 units = 1 dozen (doz.)  
 12 dozen = 1 gross  
 12 gross = 1 great gross  
 20 units = 1 score

**PAPER**

24 sheets = 1 quire  
 20 quires = 1 ream  
 2 reams = 1 bundle  
 5 bundles = 1 bale

**MISCELLANEOUS WEIGHTS****175.**

A bushel of wheat = 60 lb.  
 A bushel of beans = 60 lb.  
 A bushel of clover seed = 60 lb.  
 A bushel of shelled corn = 56 lb.  
 A bushel of rye = 56 lb.  
 A bushel of barley = 48 lb.  
 A bushel of oats = 32 lb.  
 A bushel of potatoes = 60 lb.  
 A bushel of coarse salt (domestic) = 56 lb.

These are the legal number of pounds per bushel in Michigan, Indiana, Illinois, Wisconsin, Iowa, Missouri, and New York.

On the Chicago Board of Trade seeds are sold by the cental.

A barrel of flour = 196 lb.  
 A barrel of pork or beef = 200 lb.  
 A cental of grain = 100 lb.

**Exercise 114**

1. Pens are sold in boxes containing 1 gross. How many pens are there in a box?

2. Lead pencils are sold in boxes containing  $\frac{1}{2}$  gross. How many pencils are there in a box?
3. How many packages of lead pencils of 1 doz. each are there in a box?
4. Eggs are packed in crates holding 30 doz. How many eggs in a crate?
5. How many sheets of paper in 20 quires?
6. What articles of food weigh 60 lb. to the bu.?
7. How many bu. of wheat weigh as much as 10 bu. of barley?
8. What is the ratio of the weight of 1 bu. of barley to that of 1 bu. of oats?

**176.** A certain room is 8 yd. long. Here the unit of measurement is 1 yd., and the measure of the length of the room is the number 8.

The area of the floor of a room is 192 sq. yd. 6 sq. ft. Here the measure of the area is the sum of 192 units of 1 sq. yd. and 6 units of 1 sq. ft.

A pitcher holds  $\frac{3}{4}$  of a gal. of water.

Here the measure of the capacity of the pitcher is  $\frac{3}{4}$  and the unit of measurement is 1 gal.

#### Exercise 115

Name the measures of the following quantities, and the units of measurement:

1. The volume of a cistern which holds 450 cu. ft.
2. The volume of a cistern which holds 1200 gal.
3. The area of a field which contains  $8\frac{1}{2}$  A.
4. The value of a house worth \$2800. What are the measures of the value of the house with the following units: \$5, \$10, \$50, \$100?



5. The weight of 200 lb. of sugar. What are the measures of the weight with 1 oz., 1 cwt., and 1 T. as units?

6. The weight of a quantity of tea weighing 336 lb. If 1 long cwt. is used as the unit, what is the measure?

7. The weight of 8 oz. of gold. If 1 pwt. is the unit of measurement, what number expresses the measure? What number measures the weight when 1 lb. is the unit?

8. The weight of 40 gr. of quinine. What is the measure when 1  $\mathfrak{D}$  is the unit?

9. What are the measures of 1 lb. of gold, of lead, and of quinine, when the unit of measurement is 1 oz.? When 1 gr. is the unit?

10. The length of the circumference of a circle found to be 22 yd. long. What are the measures of the circumference when 1 ft. and 1 rd. are the measures?

11. The length between two points which measures 4 ch. What are the measures of the length when the units are 1 mi. and 1 link?

12. The area of a field which contains 80 sq. rd. What are the measures of the field when the units are 1 A. and 1 sq. yd.?

13. The capacity of a pitcher which contains  $\frac{3}{4}$  of a gal. of water. What is the measure when the unit is 1 qt.?

14. The capacity of a basket which holds  $6\frac{1}{2}$  qt. What are the measures, the units being 1 pt. and 1 pk.?

15. The time of a rainstorm, which lasted 2 hr. What are the measures, the units being 1 min. and 1 da.?

16. The weight of a silver cup which is 15 oz. 12 pwt. 12 gr.

177. The fundamental units used in the measurement of value are 1 cent, 1 dime, 1 dollar, and 1 eagle.

The value of a postage-stamp used to mail a letter to any part of the United States or Canada is measured by the number 2 and the unit 1 ct.

The cost of a quart of berries worth 15 ct. is measured by the number 3 and the unit 1 nickel, or by the number 1 and the unit 1 dime plus the number 1 and the unit the nickel.

It may also be measured by the number 15 and the unit 1 ct. It may also be measured by the number 1 and the unit 1 quarter-dollar less the number 1 and the unit 1 dime.

The unit for measuring oil is 1 gal. That for buying spice by retail is 1 oz. Frequently a quantity is expressed with reference to two or more units. Thus, the length of a table being 1 yd. 2 ft. 6 in., the units are 1 yd., 1 ft., and 1 in.

#### Exercise 116

1. Name instances in which \$1 is the unit of value; 1 ct.; 1 nickel; 1 dime. What unit of value is most commonly used?

2. Name instances in which the units of weight used are 1 oz.; 1 lb.; 1 cwt.; 1 T.

3. Name quantities whose weight is measured by these units: 1 gr., 1 pwt., 1 oz., and 1 lb.

4. Name quantities which are measured by the units: 1 gr., 1  $\text{D}$ , 1  $\text{Z}$ , 1  $\text{S}$ , 1 lb.

5. What quantities are expressed in terms of these units: 1 in.? 1 ft.? 1 yd.? 1 rd.? 1 mi.?

6. Name things whose measurement is given in terms of the units 1 pt., 1 qt., 1 gal.

7. Name articles whose measurement is expressed in terms of the unit 1 bu.; 1 pk.; 1 qt.

8. In measuring time give instances in which you use as the unit 1 century; 1 yr.; 1 mo.; 1 wk.; 1 da.; 1 hr.; 1 min.; 1 sec.

9. Name articles whose quantity is expressed in terms of the unit 1 doz.; 1 gross; 1 great gross; 1 score.

10. In measuring paper the following units are used: 1 quire, 1 ream. Give instances when 1 quire is used as the unit, and also when 1 ream is used.

11. What unit of weight connects Avoirdupois, Troy, and Apothecaries' weight? What number expresses the measure of 1 lb. of each kind in terms of the common unit? Of 1 oz.?

12. What units are common to Apothecaries' and Troy weight and of equal value?

13. What units of length connect Surveyors' Long Measure with Linear Measure?

14. Name five units of area which are derived from corresponding units of length. Why is 1 A. chosen as a unit of area? Give instances in which 1 A. is used as the unit of area, and also when 1 sq. mi. is the unit.

15. Name a unit of volume larger than 1 cu. yd. Why have we no units of volume corresponding to the linear units, 1 rd. and 1 mi.?

16. Find the number of cu. in. in 1 qt., dry measure, and find how much greater it is than 1 qt., liquid measure.

17. State the number of days in the years 1500, 1600, 1700, 1800, 1900, 2000.

18. The circumference of a circle is 1,296,000 in. in length. Find the length of  $1^{\circ}$ ; of  $1'$ ; of  $1''$ .

## REDUCTION

**178. Reduction Descending** is the process of reducing a quantity expressed in terms of a unit or units of measurement to a quantity expressed in terms of a smaller unit, or of smaller units of measurement.

Reduce 2 mi. 36 rd. 5 yd. 2 ft. to feet.

$$\begin{array}{r}
 320 \\
 \hline
 640 \\
 36 \\
 \hline
 676 \text{ rd.} \\
 5\frac{1}{2} \\
 \hline
 338 \\
 3380 \\
 5 \\
 \hline
 3723 \text{ yd.} \\
 3 \\
 \hline
 11169 \\
 2 \\
 \hline
 11171 \text{ ft.}
 \end{array}$$

NOTE.—In this reduction we are to think of the operation as signifying  $2 \times 320$  rd., or by the law of commutation as  $320 \times 2$  rd., and not as representing  $320 \times 2$  mi.

$$2 \text{ mi.} = 2 \times 320 \text{ rd.} = 640 \text{ rd.}$$

$$2 \text{ mi. } 36 \text{ rd.} = 640 \text{ rd.} + 36 \text{ rd.} = 676 \text{ rd.}$$

$$676 \text{ rd.} = 676 \times 5\frac{1}{2} \text{ yd.} = 3718 \text{ yd.}$$

$$2 \text{ mi. } 36 \text{ rd. } 5 \text{ yd.} = 3718 \text{ yd.} + 5 \text{ yd.} = 3723 \text{ yd.}$$

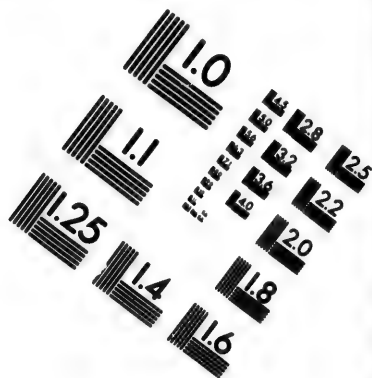
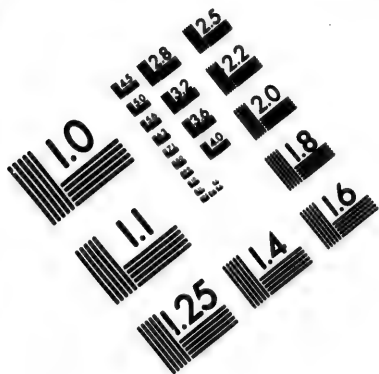
$$3723 \text{ yd.} = 3723 \times 3 \text{ ft.} = 11,169 \text{ ft.}$$

$$\therefore 2 \text{ mi. } 36 \text{ rd. } 5 \text{ yd. } 2 \text{ ft.} = 11,169 \text{ ft.} + 2 \text{ ft.} = 11,171 \text{ ft.}$$

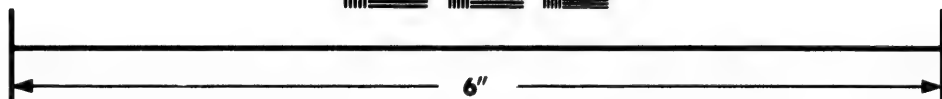
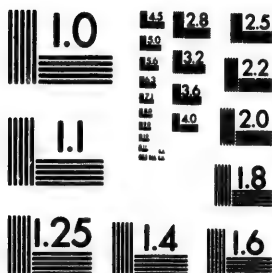
## Exercise 117

1. Reduce 5 gal. 3 qt. 1 pt. 2 gi. to gills.
2. Reduce 18 bu. 6 pk. 3 qt. 1 pt. to pints.
3. Reduce £5 12s. 9d. to pence.
4. Reduce 16 lb. 8 oz. 15 pwt. 17 gr. to grains.
5. Reduce 7 T. 18 cwt. 14 lb. 12 oz. to ounces.
6. Reduce 2 yr. 15 da. 17 min. to minutes.
7. Reduce 18 rd. 4 yd. 2 ft. 6 in. to inches.
8. Reduce 12 lb. 5 oz. 5 dr. 2 sc. 16 gr. to grains.
9. Reduce 2 A. 4 sq. rd. 2 sq. yd. 8 sq. ft. to square feet.





# IMAGE EVALUATION TEST TARGET (MT-3)



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16 24 32 36 40 44 48

101

10. Reduce 16 cu. ft. 1374 cu. in. to cubic inches.
11. Reduce 1 mi. 18 rd. 2 yd. 2 ft. 6 in. to inches.
12. State how to reduce a quantity from higher to lower denominations.
13. Find the number of A. in a township.
14. Find the number of cu. in. in a vessel containing 25 gal.
15. Reduce 59 lb. 7 oz. 14 pwt. 19 gr. to grains.
16. Reduce 7 T. 15 cwt. 56 lb. to ounces.
17. Reduce 17 lb. 2  $\frac{3}{4}$  2  $\frac{1}{2}$  to grains.
18. Reduce 17 cu. yd. 1001 cu. in. to cubic inches.
19. Reduce 760 bu. 3 pk. to quarts.
20. Reduce 56 reams 19 quires 18 sheets to sheets.
21. Reduce 36 wk. 5 da. 17 hr. to seconds; and 1 mo. of 30 da. 23 hr. 59 sec. to seconds.
22. Reduce 35 A. 80 sq. rd. 28 sq. yd. to square yards.

**179. Reduction Ascending** is the process of reducing a quantity expressed in terms of a unit, or of units, to a quantity expressed in terms of a larger unit, or of larger units.

(1) Reduce 242,337 in. to higher denominations.

12	242337	
3	20194	ft. 9 in.
	6731	yd. 1 ft.
	2	
11	13462	half yd.
320	1223	rd. 9 half yd., i.e. 4 yd. 1 ft. 6 in.
	3	mi. 263 rd.

$$\begin{aligned}
 \therefore 242,337 \text{ in.} &= 3 \text{ mi. } 263 \text{ rd. } 4 \text{ yd. } 1 \text{ ft. } 9 \text{ in. } \} \\
 &\qquad\qquad\qquad 1 \text{ ft. } 6 \text{ in. } \} \\
 &= 3 \text{ mi. } 263 \text{ rd. } 5 \text{ yd. } 0 \text{ ft. } 3 \text{ in.}
 \end{aligned}$$



$$\begin{aligned}
 242,337 \text{ in.} &= 20,194 \text{ ft. } 9 \text{ in.} \\
 20,194 \text{ ft. } 9 \text{ in.} &= 6731 \text{ yd. } 1 \text{ ft. } 9 \text{ in.} \\
 6731 \text{ yd. } 1 \text{ ft. } 9 \text{ in.} &= 1223 \text{ rd. } 4\frac{1}{2} \text{ yd. } 1 \text{ ft. } 9 \text{ in.} \\
 1223 \text{ rd. } 4\frac{1}{2} \text{ yd. } 1 \text{ ft. } 9 \text{ in.} &= 3 \text{ mi. } 263 \text{ rd. } 4\frac{1}{2} \text{ yd. } 1 \text{ ft. } 9 \text{ in.} \\
 &= 3 \text{ mi. } 263 \text{ rd. } 5 \text{ yd. } 0 \text{ ft. } 3 \text{ in.} \\
 \therefore 242,337 \text{ in.} &= 3 \text{ mi. } 263 \text{ rd. } 5 \text{ yd. } 3 \text{ in.}
 \end{aligned}$$

(2) To prove this answer correct, reduce 3 mi. 263 rd. 5 yd. 3 in. to in., by the method of the preceding exercise.

**180.** We reduce a quantity expressed in terms of a smaller unit to larger units in order to get a more definite idea of its value.

Thus we form no definite idea of a distance between two points when we are told that it is 242,337 in.; but we have a definite conception of the same distance when we are told that it is 3 mi. 263 rd. 5 yd. 3 in.

### Exercise 118

Reduce to higher denominations, and prove every third answer correct:

- |                                                                                       |                               |
|---------------------------------------------------------------------------------------|-------------------------------|
| 1. 678 pt.                                                                            | 4. 4728 cu. ft. to cd.        |
| 2. 4622 pt. of dry measure.                                                           | 5. 18,425 lb. of wheat to bu. |
| 3. 483,197 sec.                                                                       | 6. 21,489 d.                  |
| 7. 93,742 oz.                                                                         |                               |
| 8. State how to reduce a quantity expressed in terms of a lower unit to higher units. |                               |
| 9. Reduce 5420 gr. to oz., Avoir.                                                     |                               |
| 10. 141,728 gr.                                                                       | 14. 1,364,428 in.             |
| 11. 57,893 cu. in.                                                                    | 15. 273,460 sq. yd.           |
| 12. 56,735 d.                                                                         | 16. 6,188,724 sq. in.         |
| 13. 5,838,297 oz.                                                                     | 17. 429,678 in.               |
| 18. 73,940 m.                                                                         |                               |
| 19. 89,673 gr., Apothecaries' weight.                                                 |                               |
| 20. 7493 units.                                                                       |                               |
| 21. Reduce 37,921 in. to ch., rd., etc.                                               |                               |

22. Reduce 121,838 A. to townships, etc.
23. The Imperial gal. of Great Britain contains 277.274 cu. in. Find, correct to three decimal places, the number which measures the Imperial gal., the gal., liquid measure, being the unit.
24. Express in bu. and cu. in. the volume of a bin 8 ft. by 5 ft. by 4 ft.
25. How many cu. in. are there in 4 qt., dry measure? Show that this is .163 greater than the gal. measure.

## COMPOUND ADDITION AND SUBTRACTION

181. Add:

mi.	rd.	yd.	ft.	in.
2	27	1	2	8
1	146	2	1	6
8	91	2	0	4
7	152	1	2	9
<hr/>				
19	97	$2\frac{1}{2}$	1	3
		$\frac{1}{2} = 1$	1	6
<hr/>				
19	97	2	2	9

The sum of the in. column is 27 in., or 2 ft. 3 in.

The sum of the ft. column, increased by 2 ft., is 7 ft., or 2 yd. 1 ft.

The sum of the yd. column, increased by 2 yd., is 8 yd., or 1 rd.  $2\frac{1}{2}$  yd.

The sum of the rd. column, increased by 1 rd., is 417 rd., or 1 mi. 97 rd.

The sum of the mi. column, increased by 1 mi., is 19 mi.

Changing  $\frac{1}{2}$  yd. to 1 ft. 6 in. and adding, we have the sum = 19 mi. 97 rd. 2 yd. 2 ft. 9 in.

As in the problems in addition in Chapter III., we are required in the preceding question to find the whole quantity measured by the four given parts, of which the first is 2 mi. 27 rd. 1 yd. 2 ft. 8 in. What are the other three measured parts?

In this question how would the work of writing and adding be diminished if our units of length were arranged according to the *decimal* system?

**182.** Subtract 53 lb. 5 oz. 18 pwt. from 72 lb. 4 oz. 7 pwt.

lb.	oz.	pwt.
72	4	7
53	5	18
<hr/>		
18	10	9

Since we cannot take 18 pwt. from 7 pwt., take 1 oz. or 20 pwt. from 4 oz. and add it to the 7 pwt., making 27 pwt. 18 pwt. from 27 pwt. leaves 9 pwt. Since we cannot take 5 oz., from 3 oz., take 1 lb. or 12 oz. from 72 lb. and add it to the 3 oz., making 15 oz. 5 oz. from 15 oz. leaves 10 oz. 53 lb. from 71 lb. leaves 18 lb.

Hence the difference = 18 lb. 10 oz. 9 pwt.

As in the problems in subtraction in Chapter IV., we are given in the preceding question the whole quantity measured by 72 lb. 4 oz. 7 pwt., and one part, viz. 53 lb. 5 oz. 18 pwt., and are required to find the part measured by their difference.

**Exercise 119**

Add:

	bu.	pk.	qt.	pt.
1.	3	5	6	1
	8	4	1	0
	7	3	5	1
	9	4	3	1

	lb.	oz.	pwt.
2.	18	11	16
	16	9	22
	23	8	6
	17	6	13

	£	s.	d.
3.	5	17	10
	36	0	11
	7	3	4
	73	19	8
	30	14	5

	T.	cwt.	lb.
4.	16	17	74
	13	10	20
	17	15	19
	84	0	87
	11	11	36

	3	3	⊖	gr.
5.	22	3	2	19
	56	0	1	10
	3	2	2	11
	15	6	1	9
	79	4	1	10

	cu. yd.	cu. ft.	cu. in.
6.	3	23	171
	17	17	31
	28	26	1000
	34	23	1101

7. State how to add compound quantities.

Subtract:

	lb.	oz.	pwt.
8.	144	8	14
	106	11	16

	lb	$\frac{3}{4}$	$\frac{3}{8}$	$\frac{3}{16}$
10.	144	9	4	1
	129	0	7	3

	yd.	ft.	in.
9.	15	1	5
	13	2	7

	lb.	oz.	pwt.	gr.
11.	5836	0	0	0
	4976	7	13	19

	cu. yd.	cu. ft.	cu. in.
12.	37	18	857
	35	24	1280

13. State how to subtract one compound quantity from another.

14. State in what respects addition and subtraction of compound quantities are the same as addition and subtraction of numbers, and state how they differ.

### COMPOUND MULTIPLICATION AND DIVISION

**183.** Multiply 5 wk. 6 da. 18 hr. by 11.

We are here required to find the whole quantity measured by the unit 5 wk. 6 da. 18 hr. and the number 11.

wk.	da.	hr.
5	6	18
		11
65	4	6

We multiply 18 hr. by 11 and obtain the product 198 hr. or 8 da. 6 hr. Then we multiply 6 da. by 11 and obtain the product 66 da., which, increased by 8 da. is 74 da., or 10 wk. 4 da. Multiplying 5 wk. by 11, and adding 10 wk., we have 65 wk. Hence the product is 65 wk. 6 da. 18 hr.

What is the ratio of 65 wk. 4 da. 6 hr. to 5 wk. 6 da. 18 hr. ?

**184.** (1) Divide 88 rd. 3 yd. 1 ft. by 34.

$$\begin{array}{r}
 \text{rd. yd. ft. rd. yd. ft.} \\
 34) 88 \ 3 \ 1 \ (2 \ 3 \ 1 \\
 \underline{68} \\
 20 \text{ rd.} \\
 \underline{5\frac{1}{2}} \\
 110 \text{ yd.} \\
 \underline{3} \\
 113 \text{ yd.} \\
 \underline{102} \\
 11 \text{ yd.} \\
 \underline{3} \\
 33 \text{ ft.} \\
 \underline{1} \\
 34 \text{ ft.} \\
 \underline{34}
 \end{array}$$

Hence the quotient or unit of measure is 2 rd. 3 yd. 1 ft.

The remainder on dividing 88 rd. by 34 is 20 rd. 20 rd. 3 yd. = 113 yd.

The remainder on dividing 113 yd. by 34 is 11 yd. 11 yd. 1 ft. = 34 ft.

On dividing 34 ft. by 34, there is no remainder. Hence the quotient is 2 rd. 3 yd. 1 ft. What part of the dividend is the quotient ?

(2) Divide 73 gal. 1 pt. by 16 gal. 1 qt.

We are here required to find the number which is the ratio of the quantity 73 gal. 1 pt. to the unit 16 gal. 1 qt.

$$73 \text{ gal. 1 pt.} = 585 \text{ pt. ; } 16 \text{ gal. 1 qt.} = 130 \text{ pt.}$$

$$585 \text{ pt.} \div 130 \text{ pt.} = 4\frac{1}{2}.$$

$$\therefore \text{ the quotient or ratio is } 4\frac{1}{2}.$$

### Exercise 120

1. Multiply 7 gal. 3 qt. 1 pt. by 9.
2. Multiply 8 da. 12 hr. 25 min. by 28.
3. Divide £199 6s. 8d. by 13.
4. Divide 459 lb. 4 oz. 5 pwt. 22 gr. by 29.
5. Multiply 86 lb. 7 oz. 16 pwt. 11 gr. by 8.
6. Multiply 5 wk. 6 da. 18 hr. 14 min. by 11.

7. Divide 1738 cu. yd. 1236 cu. in. by 798.
8. Divide 684 da. 8 hr. 9 min. by 47.
9. Multiply 70 yd. 2 ft. 10 in. by 7.
10. Divide 1 mi. 54 rd. 1 ft. 2 in. by 29.
11. Multiply 2 hr. 8 min. 9 sec. by 15.
12. Divide  $13^{\circ} 26'$  by 15.
13. Divide 14 yd. 1 ft. 8 in. by 3 yd. 4 in.
14. Divide 16 bu. 1 pk. 2 qt. by 10 bu. 3 pk. 4 qt.
15. (a) State how to multiply a compound quantity by a given number. (b) State how to divide a compound quantity by a given number.
16. State in what respects multiplication and division of compound quantities are like multiplication and division of numbers, and how they differ.

## FRACTIONS OF SIMPLE AND COMPOUND QUANTITIES

**185.** (1) Find the value of  $\frac{5}{8}$  of a mi.

$$\frac{5}{8} \text{ mi.} = \frac{5}{8} \text{ of } 320 \text{ rd.} = 266\frac{2}{3} \text{ rd.}$$

$$\frac{2}{3} \text{ rd.} = \frac{2}{3} \text{ of } 5\frac{1}{2} \text{ yd.} = 3\frac{2}{3} \text{ yd.}$$

$$\frac{2}{3} \text{ yd.} = \frac{2}{3} \text{ of } 3 \text{ ft.} = 2 \text{ ft.}$$

$$\therefore \frac{5}{8} \text{ of a mi.} = 266 \text{ rd. } 3 \text{ yd. } 2 \text{ ft.}$$

(2) Find the value of  $\frac{5}{9}$  bu. —  $\frac{5}{7}$  pk.

$$\frac{5}{9} \text{ bu.} = \frac{5}{9} \text{ of } 4 \text{ pk.} = 2\frac{2}{3} \text{ pk.}$$

$$2\frac{2}{3} \text{ pk.} - \frac{5}{7} \text{ pk.} = 1\frac{3}{7} \text{ pk.}$$

$$\frac{3}{7} \text{ pk.} = \frac{3}{7} \text{ of } 8 \text{ qt.} = 4\frac{4}{7} \text{ qt.}$$

$$\therefore \frac{5}{9} \text{ bu.} - \frac{5}{7} \text{ pk.} = 1 \text{ pk. } 4\frac{4}{7} \text{ qt.}$$

**Exercise 121**

Find the value of:

1.  $\frac{7}{8}$  of £1;  $\frac{2}{3}$  of £2.
2.  $\frac{3}{10}$  of a da.;  $\frac{5}{11}$  of a mi.
3.  $\frac{5}{8}$  of 3 T.;  $7\frac{2}{3}$  lb. Avoir.

4.  $\frac{5}{8}$  bu.  $- \frac{7}{8}$  pk.;  $\frac{3}{8}$  lb. Troy  $+ \frac{5}{8}$  lb. Troy  $- \frac{8}{9}$  oz. Troy.
5.  $\frac{3}{8}$  A.;  $\frac{5}{8}$  A.  $- \frac{2}{8}$  sq. rd.;  $\frac{5}{8}$  sq. mi.
6.  $\frac{3}{4}$  of 5 lb. 8 oz. 6 pwt.
7.  $\frac{4}{5}$  of 2 mi. 38 rd. 4 yd. 2 ft. 2 in.
8. State how to express a fraction of a unit of measure in terms of smaller units.

Express .854 of an A. in lower denominations.

$$\begin{array}{r}
 .854 \text{ A.} \\
 \underline{160} \\
 51240 \\
 \underline{854} \\
 136.640 \text{ sq. rd.} \\
 \underline{30\frac{1}{2}} \\
 16 \\
 \underline{1920} \\
 19.36 \text{ sq. yd.} \\
 \underline{9} \\
 3.24 \text{ sq. ft.} \\
 \underline{144} \\
 96 \\
 \underline{96} \\
 24 \\
 \underline{24} \\
 34.56 \text{ sq. in.}
 \end{array}$$

$\therefore .854 \text{ A.} = 136 \text{ sq. rd. } 19 \text{ sq. yd. } 3 \text{ sq. ft. } 34.56 \text{ sq. in.}$   
 Adapt note, § 178, to this problem.

### Exercise 122

Find the value of:

1. .84 of a da.
2. .045 of a mi.
3. .6 of a lb. Troy.
4. 5.923 mi.  $- 75.18$  rd.
5. £ 75.43  $- 16.76$  s.
6. 4.7 A.  $- 2.93$  sq. rd.

**186.** Reduce 213 rd. 5 ft. 6 in. to the fraction of 3 mi.

$$\begin{aligned}
 213 \text{ rd. } 5 \text{ ft. } 6 \text{ in.} &= 42,240 \text{ in.} \\
 3 \text{ mi.} &= 3 \times 63,360 \text{ in.} = 190,080. \\
 \therefore 213 \text{ rd. } 5 \text{ ft. } 6 \text{ in.} &= \frac{42,240}{190,080}, \text{ or } \frac{2}{9} \text{ of } 3 \text{ mi.}
 \end{aligned}$$

The G. C. M. of 42,240 and 190,080 is 21,120, which divides the numerator twice and the denominator 9 times.

**Exercise 123**

1. Reduce £ 1 7s. 6d. to the fraction of £ 2.
2. Reduce 22 rd. 5 yd. 2 ft. 6 in. to the fraction of a mi.
3. Reduce 8 hr. 3 min. to the fraction of a da.
4. Reduce 4 mo. 3 da. to the fraction of a yr. (30 da. to a mo. and 360 da. to a yr.).
5. Reduce 11 mo. 18 da. to the fraction of a yr.
6. Reduce  $1\frac{1}{2}$  in. to the fraction of  $1\frac{1}{2}$  yd.
7. Reduce  $\frac{3}{4}$  lb. Avoir. to the fraction of 2 lb. Troy.
8. Express 213 rd. 1 yd. 2 ft. 6 in. as a fraction of 145 rd. 2 yd. 1 ft. 6 in.

**187.** Express 58 rd. 2 yd. 7.2 in. as a decimal of a mi.

12	7.2 in.
3	.6 ft.
5.5	2.2 yd.
320	58.4 rd.
	.1825 mi.

$$\begin{aligned}
 7.2 \text{ in.} &= .6 \text{ ft.} = .2 \text{ yd.} \\
 2 \text{ yd.} + .2 \text{ yd.} &= 2.2 \text{ yd.} = .4 \text{ rd.} \\
 58.4 \text{ rd.} &= .1825 \text{ mi.} \\
 \therefore 58 \text{ rd. } 2 \text{ yd. } 7.2 \text{ in.} &= .1825 \text{ mi.}
 \end{aligned}$$

Prove this answer correct by reducing .1825 mi. to lower denominations.

**Exercise 124**

1. Reduce 8 oz. 15.2 pwt. to the decimal of a lb.
2. Reduce 21 hr. 57 min. 36 sec. to the decimal of a da.
3. Reduce 147 rd. 1 yd. 3.6 in. to the decimal of a mi.
4. 25 sq. mi. 128 A. is what decimal of a township?
5. Reduce 67 sq. rd. 6 sq. yd. 64.8 sq. in. to the decimal of an A.



## BOARD MEASURE

**188.** Boards 1 inch or less in thickness are sold by the square foot.

Thus a board 18 ft. long, 14 in. wide, and 1 in. thick or less contains  $18 \times \frac{1}{4}$ , or 21 ft., board measure.

To find the number of feet, board measure, in lumber more than 1 inch thick, we find the number of square feet in the surface of the board and multiply this result by the number of inches that the lumber is thick.

Thus a board 15 ft. long, 8 in. wide, and  $2\frac{1}{2}$  in. thick contains  $15 \times \frac{8}{12} \times \frac{5}{2}$ , or 25 board feet.

## Exercise 125

How many feet, board measure, in :

1. A board 20 ft. long, 9 in. wide, and 1 in. thick ?  $\frac{3}{4}$  in. thick ?
2. A board 18 ft. long, 8 in. wide, and  $2\frac{1}{2}$  in. thick ?
3. A scantling 16 ft. long, 3 in. wide, and 4 in. thick ?
4. Twenty scantlings, 24 ft. long, 5 in. wide, and 7 in. thick ?
5. A stick of timber 33 ft. long, and 14 in. square ?
6. One cu. ft. ?
7. What is the cost of 25 joists each 6 in. by 4 in. by 15 ft. at \$22 per M. ?
8. What is the cost of 24 joists each 5 in. by 7 in. by 10 ft. at \$21 per M. ?
9. How much will it cost to enclose a rectangular lot 50 ft. wide and 100 ft. deep with a tight board fence 16 ft. high with boards that cost \$18 per M. ?

## LONGITUDE AND TIME

**189.** Turn to your Geography and find several meridian lines. Find the prime meridian which passes through Greenwich, England.

The imaginary lines drawn on the earth's surface from pole to pole are called *meridians*. The meridian passing through Greenwich, a town near London, England, having the royal observatory, is called the *prime* or *standard* meridian.

Places west of the prime meridian are in west longitude, and places east of the prime meridian are in east longitude. Thus, Washington is  $77^{\circ} 7'$  west longitude, and Paris  $2^{\circ} 20'$  east longitude.

**190.** Find from the maps in your geographies to the nearest degree the longitude of these cities: New York, Pittsburg, Richmond, Atlanta, Chicago, Denver, Salt Lake City, San Francisco.

Find also the longitude of Rome, Stockholm, Athens, Constantinople, St. Petersburg, and Moscow.

**191.** The difference in longitude between Philadelphia, which is  $75^{\circ} 9'$  west longitude, and Portland, which is  $70^{\circ} 15'$  west longitude, is  $4^{\circ} 54'$ .

The difference between the longitude of Philadelphia and that of Paris, which is  $2^{\circ} 20'$  east longitude, is  $77^{\circ} 29'$ , and is obtained by finding the sum of the longitudes.

**192.** Find on the map of the United States and name the meridians that denote Eastern time, Central time, Mountain time, and Pacific time. How many degrees are there between these meridian lines? What is the difference in time between places situated on these meridians?

**193.** As the sun rises in the east, it is sunrise in New York earlier than in Chicago; consequently at any time during the day the clock time in New York is later than in Chicago.

Similarly, clock time in San Francisco, which is west of Chicago, is earlier than in the latter city.

**194.** Since the sun appears to move in a circle about the earth, *i.e.* through  $360^\circ$  in 24 hr., we have the following:

In 24 hr. the sun passes through  $360^\circ$ .

In 1 hr. the sun passes through  $15^\circ$ .

In 1 min. the sun passes through  $\frac{1}{60}$  of  $15^\circ = \frac{1}{4}^\circ = 15'$ .

In 1 sec. the sun passes through  $\frac{1}{60}$  of  $15' = \frac{1}{4}' = 15''$ .

*Hence, to reduce longitude expressed in time to longitude expressed in degrees, we multiply by 15, and to reduce longitude expressed in degrees to longitude expressed in time, we divide by 15.*

**195.** Make the multiplication table of 15 and memorize it, so as to be able to work questions in longitude and time by short multiplication and division.

#### Exercise 126

1. What is the difference in longitude between two places whose difference in time is 1 hr. ? 2 hr. ? 4 hr. ? 2 min. ? 3 min. ? 1 sec. ? 3 sec. ?

2. What is the difference in longitude between two places whose difference in time is 1 hr. 2 min. ? 1 hr. 3 min. 2 sec. ?

3. What is the difference in time between two places whose difference in longitude is  $30^\circ$  ?  $75^\circ$  ?  $120'$  ?  $90'$  ?  $135''$  ?  $105''$  ?

4. What is the difference in time between two places whose difference in longitude is  $15^\circ 45' 30''$  ?  $75^\circ 15' 45''$  ?

5. Find the difference in longitude between the following places, and illustrate your answers by diagrams:

Washington  $77^\circ$  west longitude and Helena  $112^\circ$  west longitude ?

Washington  $77^\circ$  west longitude and Hamburg  $10^\circ$  east longitude ?

Cairo  $32^\circ$  east longitude and Hamburg  $10^\circ$  east longitude ?

6. What is the difference in time between two places:

- (1) One  $64^\circ$  west longitude, the other  $34^\circ$  east longitude?
- (2) One  $64^\circ$  west longitude, the other  $26^\circ$  east longitude?
- (3) One  $64^\circ$  east longitude, the other  $34^\circ$  east longitude?

7. When it is 6 A.M. at San Francisco, what time is it at a place  $45^\circ$  east of San Francisco?  $30^\circ$  east?  $15^\circ 45'$  east?  $30^\circ 15' 45''$  east?

8. When it is 11 A.M. at Chicago, what time is it at a place  $60^\circ$  west of Chicago?  $30^\circ$  west?  $45'$  west?  $15^\circ 45'$ ?

**196.** (1) Find the difference in time between St. Louis  $90^\circ 19' 26''$  west longitude and Sacramento  $121^\circ 25' 41''$  west longitude.

$$\begin{array}{r}
 121^\circ \quad 25' \quad 41'' \\
 90 \quad 19 \quad 26 \\
 \hline
 15 \overline{) 31^\circ \quad 6' \quad 15''} \quad \text{difference in longitude} \\
 \underline{2 \text{ hr. } 4 \text{ min. } 25 \text{ sec.}} \quad \text{difference in time}
 \end{array}$$

A difference in longitude of  $31^\circ$  gives a difference of  $31 \div 15$ , or 2 hr. of time, with a remainder of  $1^\circ$ . A difference in longitude of  $1^\circ 6'$ , or  $66'$ , gives a difference of  $66 \div 15$ , or 4 min. of time, with remainder  $6'$ . A difference of  $6' 15''$ , or  $375''$ , gives a difference of  $375 \div 15$ , or 25 sec. of time.

(2) Berlin is  $13^\circ 23' 53''$  east longitude and Boston is  $71^\circ 4' 9''$  west longitude. When it is 1.15 P.M. at Boston, what time is it at Berlin?

$$\begin{array}{r}
 13^\circ \quad 23' \quad 53'' \\
 71 \quad 4 \quad 9 \\
 \hline
 15 \overline{) 84^\circ \quad 28' \quad 2''} \quad \text{difference in longitude} \\
 \underline{5 \text{ hr. } 37 \text{ min. } 52\frac{2}{3} \text{ sec.}} \quad \text{difference in time} \\
 \underline{1 \text{ hr. } 15 \text{ min.}} \quad \text{time in Boston} \\
 6 \text{ hr. } 52 \text{ min. } 52\frac{2}{3} \text{ sec.} \quad \text{time in Berlin}
 \end{array}$$

$\therefore$  it is 52 min.  $52\frac{2}{3}$  sec. after 6 P.M., or 7 min.  $7\frac{1}{3}$  sec. to 7 P.M.

### Exercise 127

Find the difference in time between the following cities:

1. Brooklyn  $73^\circ 58'$  W. and Omaha  $95^\circ 28'$  W.
2. St. Paul  $93^\circ 3' 45''$  W. and Cleveland  $81^\circ 39'$  W.

3. Indianapolis  $86^{\circ}6'57''$  W. and San Francisco  $122^{\circ}26'12''$  W.
4. Cincinnati  $84^{\circ}28'36''$  W. and Glasgow  $4^{\circ}17'6''$  W.
5. Detroit  $83^{\circ}5'7''$  W. and Vienna  $16^{\circ}22'22''$  E.
6. Pillsbury  $79^{\circ}55'43''$  W. and Amsterdam  $4^{\circ}52'13''$  E.
7. Newark  $74^{\circ}9'12''$  W. and Rome  $12^{\circ}27'58''$  E.
8. When it is 11 A.M. at Cleveland, what o'clock is it at St. Paul?
9. What time is it at Indianapolis at the opening of school at 9 A.M. in San Francisco?
10. When it is 7 A.M. at Cincinnati, what time is it at Glasgow?
11. When it is 8 A.M. at Omaha, what time is it at Brooklyn?
12. A man travels until his watch is 1 hr. 5 min. 16 sec. slow. Does he travel east or west, and how many degrees has he gone?
13. A vessel sailed from a port directly on a line of latitude a certain distance, and then due north to port, where the captain found that his chronometer was 40 min. slow. In which direction did he sail at first, and how many degrees?
14. What is the difference in longitude between two places whose difference in time is :
  - (a) 2 hr. 33 min. 18 sec.?
  - (b) 4 hr. 27 min. 46 sec.?
  - (c) 6 hr. 12 min. 29 sec.?
15. Buffalo is  $78^{\circ}57'48''$  W. and Constantinople is  $28^{\circ}59'3''$  E. What time is it in Constantinople when it is 20 min. after 6 A.M., July 6, in Buffalo?
16. What time is it in Buffalo when it is 20 min. after 6 A.M., July 6, in Constantinople?
17. Given the longitude of two places, state how to find the time in the place east at a given time in the place west.

## Exercise 128

1. Find values of the quantity measured by the number 6 according as the unit is £2 5s. or 6 oz. 10 pwt. 16 gr.
2. Find the value of a pile of cordwood 13'4" long by 3'9" high at \$4.50 a cd.
3. How many gr. are there in 9 oz. 17 pwt. 22 gr., and how many A., etc., in 167,412,715 sq. in.?
4. In 161,384 in. how many mi.?
5. If a sovereign weigh 123.274 gr., how many sovereigns will weigh 21 lb. 4 oz. 16 pwt. 10 gr.?
6. The working wheel of a locomotive is 226 in. in circumference. It turns 91 times in 1 min. Through how many rd., etc., does it draw the train in 1 min.?
7. How much must be paid for 360 ft. of boards at \$12.00 per M., 250 shingles at \$2.50 per M., and 760 ft. of timber at \$1.00 per C.?
8. How many min. are there in  $7\frac{5}{8}$  yr. +  $\frac{1}{8}$  wk. +  $\frac{5}{12}$  hr.?
9. A horse trotted 1 mi. in 2 min. 12 sec. Taking his stride at 16 ft., how many times per sec. did his feet touch the ground?
10. A bicyclist rode 39 mi. in 3 hr. 15 min. What was his rate in mi. per hr., in yd. per min., and in ft. per sec.?
11. Reduce  $\frac{1}{2}$  of £20 to the decimal of £100.
12. How many oz. of gold are worth £23 16s. when .53 oz. is worth 44s. 2d.?
13. If wire fencing cost 6¢ per yd., find what must be paid for enclosing a field 305 yd. long and 156 yd. wide, there being 4 rows of wire.
14. Convert £296 16s. sterling into dollars and cents, £1 being worth \$4.8665.

15. Make out the following account neatly, accurately, and in proper form. All fractions are to be retained.

John Wilson bought from you to-day :

$7\frac{1}{2}$  lb. cheese @  $12\frac{1}{2}$ ¢ per lb.;

$6\frac{1}{4}$  lb. butter @ 23¢ per lb.;

$2\frac{1}{2}$  lb. tea @ 55¢ per lb.;

27 lb. sugar @ \$1 per 18 lb.

16. If a yard measure is  $\frac{1}{8}$  of an inch too long, what is the actual distance between two points which is found by this measure to be 500 yd. 2 ft. 6 in.?

17. Express 1 lb. Troy as the fraction of 1 lb. Avoirdupois; and, conversely, express 1 lb. Avoirdupois as the fraction of 1 lb. Troy.

18. Express as the fraction of a ft. the remainder after .012 of a yd. has been subtracted as often as it is possible from 1.087 yd.

19. Which is the heavier, a pound of gold or a pound of feathers, and an ounce of gold or an ounce of feathers? By how much in each case?

20. How many silver spoons, each weighing 2 oz. 16 pwt., could be made out of a bar of silver, the weight of which is 50 oz. 8 pwt.?

21. If the unit of Troy weight, called the pennyweight, contained 14.25 gr. instead of 24, how many gr. would make 1 lb. Troy?

22. A man bought a quantity of tea supposed to be done up in packages of 1 lb. each, for which he was to pay \$64; on weighing them, however, it was found that each package was 1 oz. too light. How much should he pay for the tea?

23. Required the mean of the following observations of temperature:  $41^{\circ} 29'$ ;  $41^{\circ} 27\frac{1}{2}'$ ;  $39^{\circ} 13'$ ;  $41^{\circ} 33'$ ;  $37^{\circ} 47\frac{1}{2}'$ ;  $44^{\circ} 28'$ ;  $40^{\circ} 13'$ .

24. Sold 20,900 ft. of lumber for \$331.62 $\frac{1}{2}$ , gaining thereby \$78.37 $\frac{1}{2}$ . What had it cost per C.?

25. A lot 150 ft. long and 100 ft. wide is to be surrounded by a close board fence 6 ft. high. What will the boards cost at \$12.50 per thousand ft.?

26. If a room be 12 ft. square, what must its height be in order that the area of the walls may amount to 60 sq. yd.?

27. Find the value of a rectangular field 330 yd. by 156 yd. @ \$36.50 per A.

28. Find the surface area and the volume of a rectangular block  $3' 9'' \times 2' 4'' \times 1' 3''$ .

29. Express as a fraction of an A. the sum of the following:  $\frac{1}{2}$  of  $\frac{4}{5}$  of  $\frac{1}{8}$  of 1 A.;  $\frac{2}{3}$  of  $\frac{1}{2}$  of  $\frac{3}{8}$  of 100 sq. rd.; and  $\frac{1}{7}$  of  $2\frac{1}{8}$  times 605 sq. yd.

30. The Manufacturers and Liberal Arts Building of the Columbian Fair was in the form of a rectangle and covered an area of 30 A. 76 sq. rd. 19 sq. yd. 7 sq. ft. The building was 787 ft. wide. How many ft. in length was it?

31. A 200-acre farm is sown with grain as follows: Peas, 25 A. 126 sq. rd. 10 sq. yd.; oats, 46 A. 134 sq. rd. 15 sq. yd.; wheat, 75 A. 125 sq. rd. 25 sq. yd. The buildings, garden, and orchard occupy 12 A., and the rest is pasture. How many A. of pasture are there?

32. If a road is 4 rd. wide, how many mi. of it will make 10 A.?

33. A map is drawn to a scale of half an inch to a mile. How many acres are represented by a square inch on the map?

34. After drawing off 124 gal. of water from a cistern,  $\frac{3}{11}$  of the water still remained. How many gal. did the cistern at first contain? How many gal. were left in it?

35. Some Atlantic liners consume 200 T. of coal per day. They average 8 da. out and 8 back. For fear of accidents they carry a supply for 4 da. extra. How many cu. yd. of the hold of such a steamer will be occupied with coal for her round trip if each ton is 33 cu. ft.?



36. A load of wood 10 ft. long, 3 ft. 8 in. wide, and 3 ft. high was sold for \$ 3.

(a) What was the price per cd. ?

(b) At \$ 4 per cd., what would the load be worth ?

37. Find the value of a pile of tan bark 180 ft. long, 48 ft. wide, and 16 ft. high at \$ 2.25 per cd.

38. Find the amount of the following bill, retaining all fractions :

3 $\frac{3}{4}$ lb. tea	@ 80¢ ;
300 lb. sugar	@ 4 $\frac{3}{4}$ ¢ ;
45 yd. print	@ 11 $\frac{1}{2}$ ¢ ;
2 $\frac{1}{4}$ gal. syrup	@ 65¢ ;
12 $\frac{1}{2}$ yd. towelling	@ 12 $\frac{1}{2}$ ¢ ;
$\frac{3}{4}$ doz. knives and forks	@ \$ 2.50 ;
27 lb. cheese	@ 15¢ ;
1 lb. 10 oz. lemon peel	@ 32¢ per lb.

39. A train 80 yd. long crossed a bridge 140 yd. long in 22 $\frac{1}{2}$  sec. Find the average speed of the train while crossing.

40. Find the length of a bridge which a train 100 yd. long required 1 min. 15 sec. to cross, running at a speed of 15 mi. per hour.

41. Find the value of 2 $\frac{7}{8}$  cwt. + 37 $\frac{5}{8}$  lb. + 10 $\frac{3}{4}$  oz.

42. Find the weight of a bar 3 yd. 1 ft. 9 in. long, of which a yard weighs 15 lb.

43. Find the cost price of lead per cwt., if the sale of 48 cwt. for \$ 218.70 gives a profit of  $\frac{1}{8}$  of the original price.

44. Find the expense of fencing a railway (both sides) 73 mi. in length, at the rate of \$ 5.50 per rd.

45. If a wheel makes 260 revolutions in passing over 1 mi. 520 yd. 2 ft., what is its circumference ?

46. A block of stone is 4 ft. long, 2 ft. 6 in. broad, and 1 ft. 3 in. thick ; it weighs 27 cwt. Find the weight of 50 cu. in. of the stone.

47. A rectangular lot 45 ft. front by 99 ft. deep was sold for \$3150. What was the price per ft. frontage, and what the price per A. at the rate of the selling price of the lot?

48. If I buy 147 gal. of molasses at 19¢ a gal., and use 33 gal. of it, at how much must I sell the remainder per gal. so as to receive as much as the whole cost?

49. When 1 oz. of gold costs \$19.45, what is the cost of .04 lb.?

50. A bushel of wheat weighs 60 lb. and a barrel of flour weighs 196 lb. If 3 lb. of wheat make 2 lb. of flour, how many bbl. of flour can be made from 343 bu. of wheat?

51. A grocer receives \$9.60 for a bill of goods weighed on scales that gave only  $15\frac{1}{4}$  oz. to the pound. How many cents' worth did he cheat his customer?

52. If a cow gives 12 qt. 1 pt. of milk every day and 1 lb. 8 oz. of butter can be made from 25 qt. of milk, how many pounds of butter can be made in one week from the milk of 16 cows?

53. A man can run 100 yd. in 10 sec. How many mi. will a steamboat go in  $5\frac{1}{2}$  da. at the same rate?

54. How many mi. must be travelled by a team in ploughing lengthwise a piece of land 60 rd. long and 40 rd. wide, if each furrow is 10 in. wide?

55. A farmer exchanges  $3\frac{3}{5}$  T. of wheat at  $64\frac{1}{2}$ ¢ per bu. for coal at \$6.75 per T. How much coal does he get?

56. How much wheat is necessary to sow a field containing  $4\frac{2}{3}$  A., if  $\frac{3}{4}$  oz. is sown on every sq. yd.?

57. Two clocks point to 2 at the same instant; one loses  $3\frac{1}{2}$  sec. and the other gains 4 sec. in 12 hr. When will one be half an hour before the other, and what time will each clock then show?

58. A railway company pays \$24.75 per A. for a portion of road 100 mi. long and  $94\frac{1}{2}$  ft. wide. Find the whole amount paid.

59. A man mowing grass walks at the rate of .35 mi. an hour, and in 70 min. mows a grass plot of 1056 sq. yd. How broad does he mow?

60. Find the expense of sodding a plot of ground, which is 40 yd. long and 100 ft. wide, with sods each a yd. in length and a ft. in breadth; the sods when laid costing 75¢ per hundred.

61. A cd. of wood and 100 bu. of grain fill equal spaces. A cubic bin whose edge is 12 ft. contains 45,900 lb. of grain. Find the weight of 1 bu. of this grain.

62. Make out the following bill neatly and accurately. John Smith, a merchant of Chicago, sold to William Jones, on June 15, 1895:

5 lb. 8 oz. of butter	@ 16¢ per lb.;
2 lb. 10 oz. of tea	@ 3¢ an oz.;
4 doz. lemons	@ 4¢ for 3 lemons.;
8 lb. coffee	@ 37½¢ per lb.;
1 bu. 3 pk. chestnuts	@ 10¢ per qt.;
11 doz. penholders	@ 1½¢ each.

63. Find cost of digging a cellar 48 ft. long, 30 ft. wide, and 6 ft. deep, at 20¢ per cu. yd., and flooring it with Portland cement at 10¢ per sq. yd.

64. A piece of land is surrounded by a stone wall 8 ft. high and 2 ft. thick; the land inside the wall is 100 ft. long and 50 ft. wide. How many cu. ft. of stone does the wall contain?

65. How many bushels of potatoes can be sold out of a garden in which there are 160 rows of potatoes, in each row 240 hills, and on an average 10 potatoes in each hill, if 6 potatoes make 3 pt.?

66. Farmer B. sold to a merchant the following articles to apply on an overdue account of \$54.45:

1680 lb. of hay	@ \$ 15 per T. ;
3½ cd. of wood	@ \$ 4.80 per cd. ;
4 bbl. of apples	@ \$ 2.75 per bbl. ;
350 lb. of flour	@ \$ 2.50 per cwt. ;
30 lb. 10 oz. butter	@ 16¢ per lb.

Make out the account neatly, showing the balance and to whom due.

## CHAPTER XIII

### PERCENTAGE

**197.** The expressions  $\$ \frac{5}{100}$  and  $\$.05$  denote that the quantity  $\$1$  is conceived as made up of 100 equal parts or units, and that 5 of these parts or units have been taken to measure the quantity denoted by  $\frac{5}{100}$  or  $.05$ .

The phrase *per cent* means *hundredths*. Thus the fraction  $\frac{5}{100}$  and the decimal  $.05$  are also written 5 per cent or 5%.

Hence  $\frac{5}{100}$ ,  $.05$ , and 5% of any quantity are equal.

**198.** (1) Express  $\frac{1}{3}$  as hundredths and also as per cent.

$$\frac{1}{3} = .33\frac{1}{3} \text{ or } \frac{33\frac{1}{3}}{100} = 33\frac{1}{3} \%.$$

(2) A horse dealer who had 600 horses sold 480 of them. What per cent of his horses did he sell?

We are here given the *measured* quantity or 480 horses to compare with the quantity 600 horses, and we are required to find the per cent which is the *number*.

The number sold =  $\frac{480}{600}$  or  $\frac{4}{5}$  or 80% of the whole number.

**199.** The term *per cent* is used constantly in business. The merchant gains 20%, meaning that he gains \$20 on every \$100 he has invested in goods. The insurance company charges 2% for insuring furniture, meaning that \$2 is charged on every \$100 worth of furniture insured. A man borrows money at 5%, meaning that he is to pay \$5 interest on every \$100 borrowed. The commission merchant charges

2% of the buying or selling price. The broker charges  $\frac{1}{8}\%$  for buying stocks, and so on.

### Exercise 129

Express as hundredths and also as per cent :

1.  $\frac{1}{2}, \frac{1}{4}, \frac{3}{4}, \frac{1}{5}, \frac{2}{5}, \frac{4}{5}, \frac{5}{10}, \frac{3}{10}, \frac{7}{10}, \frac{1}{20}, \frac{3}{20}, \frac{9}{20}, \frac{17}{20}, \frac{1}{25}, \frac{11}{25}, \frac{23}{25}$ .

2.  $\frac{1}{3}, \frac{2}{3}, \frac{1}{6}, \frac{5}{6}, \frac{1}{8}, \frac{3}{8}, \frac{5}{8}, \frac{7}{8}, \frac{1}{12}, \frac{1}{15}, \frac{1}{16}, \frac{15}{16}, \frac{17}{18}, \frac{14}{17}, \frac{41}{45}$ .

3. Out of a class of 25 pupils 5 are absent. What part of the class is absent? How many hundredths? What per cent of the class?

4. A merchant paid \$3 for hats which he sold for \$4. What fraction of the cost price did he gain? How many hundredths? What per cent of the cost?

5. A person bought a house for \$5000 and afterward sold it for \$4000. The loss was what fraction of the cost? How many hundredths? What per cent?

6. A fruit dealer bought strawberries for \$1.75 a crate and sold them for \$2.25 a crate. What per cent did he gain?

7. A man bought a horse for \$234 and afterwards sold it for \$273. What per cent of the cost did he gain?

8. The population of a town of 32,000 inhabitants increases 1120 in one year. What is the per cent of increase?

9. How do you find the gain per cent when you are given the cost price and the selling price of an article?

200. Express 25% and  $37\frac{1}{2}\%$  as fractions in their lowest terms.

$$25\% = \frac{25}{100} = \frac{1}{4}.$$

$$37\frac{1}{2}\% = \frac{37\frac{1}{2}}{100} = \frac{75}{200} = \frac{3}{8}.$$

## Exercise 130

What fractions in their lowest terms are equivalent to the following:

1. 1%, 4%, 5%, 10%, 20%, 25%, 30%, 40%, 80%, 90%, 100%?
2. 60%, 75%, 35%, 24%, 70%, 16%, 85%?
3.  $12\frac{1}{2}\%$ ,  $37\frac{1}{2}\%$ ,  $62\frac{1}{2}\%$ ,  $87\frac{1}{2}\%$ ?
4.  $6\frac{1}{4}\%$ ,  $8\frac{1}{3}\%$ ,  $6\frac{2}{3}\%$ ,  $16\frac{2}{3}\%$ ,  $83\frac{1}{3}\%$ ?
5.  $11\frac{1}{3}\%$ ,  $14\frac{2}{7}\%$ ,  $9\frac{1}{11}\%$ ?
6.  $10\frac{5}{7}\%$ ,  $5\frac{5}{9}\%$ ,  $1\frac{2}{3}\%$ ?
7. 100%, 120%, 125%, 175%, 250%, 325%?
8. A horse which cost \$ 120 was sold at a gain of 25%. The gain is equal to what part of the cost? How much was gained? What was the selling price?
9. Cloth which cost 60¢ per yd. was sold at a loss of  $16\frac{2}{3}\%$ . The loss was what fraction of the cost? What was the loss on each yd.? What was the selling price per yd.?
10. An article costing \$ 4.20 was sold at a gain of  $8\frac{1}{3}\%$ . Find the gain. Find the selling price.
11. Tea is bought for 84¢ per lb. and sold at an advance of  $14\frac{2}{7}\%$ . What was the selling price of each lb.?
12. A drover sold 400 sheep at a gain of 10%. He gained the cost price of how many sheep?
13. How do you find the gain on an article when you are given the cost price and the gain per cent? How do you find the selling price?

**201.** The following results should be memorized so that the fractions or the per cent can be given rapidly in any order:

$20\% = \frac{1}{5}$	$33\frac{1}{3}\% = \frac{1}{3}$	$37\frac{1}{2}\% = \frac{3}{8}$	$100\% = 1$
$40\% = \frac{2}{5}$	$66\frac{2}{3}\% = \frac{2}{3}$	$50\% = \frac{1}{2}$	$6\frac{1}{4}\% = \frac{1}{16}$
$60\% = \frac{3}{5}$	$100\% = 1$	$62\frac{1}{2}\% = \frac{5}{8}$	$6\frac{3}{8}\% = \frac{1}{16}$
$80\% = \frac{4}{5}$	$12\frac{1}{2}\% = \frac{1}{8}$	$75\% = \frac{3}{4}$	$8\frac{1}{8}\% = \frac{1}{12}$
$100\% = 1$	$25\% = \frac{1}{4}$	$87\frac{1}{2}\% = \frac{7}{8}$	$16\frac{2}{3}\% = \frac{1}{6}$

NOTE. — The expression  $20\% = \frac{1}{5}$  signifies that 20 % of a quantity =  $\frac{1}{5}$  of it. Thus, 20 % of 80 A. =  $\frac{1}{5}$  of 80 A.

### Exercise 131

Read the following decimals as per cents:

1. .25, .16 $\frac{2}{3}$ , .40, .75, .03 $\frac{1}{3}$ , 1.20.
2. .15, .37 $\frac{1}{2}$ , .45 $\frac{1}{3}$ , .05 $\frac{1}{3}$ , .001 $\frac{1}{3}$ , 2.40.

**202.** What is the quantity which is measured by the unit \$840 and the number  $16\frac{2}{3}\%$ ?

The quantity =  $16\frac{2}{3}\%$  of \$840 =  $\frac{1}{3}$  of \$840 = \$140.

### Exercise 132

1. What is the quantity which is measured by the unit \$720 and the number  $8\frac{1}{3}\%$ ?
2. What is the quantity which is measured by the unit \$465 and the number 20%?
3. What is the gain that is measured by the cost \$893 and the number 6%?
4. What is the loss that is measured by the cost \$1268 and the ratio 32%?
5. What is the quantity that is measured by the sum of the unit \$468.25 and 8% of it?
6. What is the quantity that is measured by the difference between the unit \$4397.50 and 6% of it?



7. If the gain on selling is measured by the number 7% and the cost \$450, find the gain.

8. Find the selling price of an article which cost \$600, the loss on selling being measured by the number 5% and the cost price.

**203.** A speculator bought a house for \$2349 and sold it at a gain of 17%. Find the selling price.

In this question the selling price is the sum of the cost, which is known, and the gain, which is unknown. The gain is measured by the number 17% or .17, and the cost price \$2349.

$$\begin{array}{r}
 \$2349 \text{ cost} \\
 .17 \text{ number} \\
 \hline
 16443 \\
 2349 \\
 \hline
 \$399.33 \text{ gain}
 \end{array}$$

The gain = 17% of \$2349 = \$399.33.

$\therefore$  the selling price = \$2349 + \$399.33 = \$2748.33.

### Exercise 133

1. Write as decimals: 17%, 13%, 37%,  $23\frac{1}{2}\%$ , 146%, 346%, 6%, 8%,  $8\frac{1}{2}\%$ ,  $1\frac{1}{2}\%$ ,  $\frac{1}{2}\%$ ,  $\frac{1}{4}\%$ .

2. Find 34% of \$893.

3. Find 27% of 6594 bu. of wheat.

4. If 39% of a cargo of flour, consisting of 8492 bbl., was damaged, how many bbl. were damaged?

5. A farmer who sold his crop of wheat in 1892 for \$967.20, received 13% less the next year. How much less did he receive for his crop in 1893 than in 1892?

6. A grain dealer invested \$6459 in wheat, and 23% of that amount in oats. How much did he invest in oats?

7. What does a bill for \$1896 become after a reduction of 3%?

8. What is the selling price of an article costing \$18, and sold at a loss of 9%?

9. What is the selling price of an article costing \$7, and sold at a gain of 7%?

**204.** Express  $\frac{4}{5}\%$  as a decimal and also as a common fraction in its lowest terms.

$$\begin{aligned}\frac{4}{5}\% &= .00\frac{4}{5}. \\ \frac{4}{5}\% &= \frac{\frac{4}{5}}{100} = \frac{4}{500} = \frac{1}{125}. \\ \therefore \frac{4}{5}\% &= .00\frac{4}{5} = \frac{1}{125}.\end{aligned}$$

#### Exercise 134

Express as decimals and also as common fractions in their lowest terms:

- $\frac{1}{2}\%$ ,  $\frac{1}{5}\%$ ,  $\frac{3}{5}\%$ ,  $\frac{3}{4}\%$ ,  $\frac{2}{5}\%$ ,  $\frac{4}{5}\%$ ,  $\frac{5}{6}\%$ ,  $\frac{1}{7}\%$ .
- $\frac{3}{7}\%$ ,  $\frac{6}{7}\%$ ,  $\frac{1}{8}\%$ ,  $\frac{5}{8}\%$ ,  $\frac{3}{8}\%$ ,  $\frac{1}{9}\%$ ,  $\frac{8}{9}\%$ ,  $\frac{4}{11}\%$ .
- $\frac{1}{10}\%$ ,  $\frac{7}{10}\%$ ,  $\frac{2}{11}\%$ ,  $\frac{1}{25}\%$ ,  $\frac{9}{17}\%$ ,  $\frac{1}{3}\%$ ,  $\frac{4}{15}\%$ ,  $\frac{1}{5}\%$ .
- What part of 1% is  $\frac{1}{2}\%$ ?  $\frac{3}{5}\%$ ?  $\frac{3}{8}\%$ ?  $\frac{7}{10}\%$ ?
- If  $\frac{1}{4}\%$  is charged for sending money from Chicago to New York, what is charged for sending \$1200?
- If  $\frac{1}{8}\%$  is charged for sending money to St. Louis, find how much is charged when \$1632 is sent.
- If  $\frac{1}{8}\%$  is charged as commission for buying stock, what is the commission on buying \$2400 stock?
- If  $\frac{1}{8}\%$  is charged for selling stock, find the commission charged for selling \$1600 stock.
- 6% per annum is what per cent for 1 month?  $\frac{2}{3}\%$  a month is what per cent per annum?
- What is the cost of insuring 550 bbl. of flour, worth \$4 per bbl., the cost of insurance being  $\frac{1}{2}\%$  of the value of the flour?

**205.** (1) Express  $102\frac{1}{2}\%$  as a decimal.

$$102\frac{1}{2}\% = \frac{102\frac{1}{2}}{100} = 1.02\frac{1}{2} = 1.025.$$

(2) I send my agent \$5100, which is 102% of the money which he invested for me in cotton. What does my agent pay for cotton?

$$\begin{aligned} 102\% \text{ or } 1.02 \text{ of the cost of the cotton} &= \$5100. \\ \therefore \text{the cost of the cotton} &= \$5100 \div 1.02 = \$5000. \end{aligned}$$

### Exercise 135

1. Express as decimals: 103%, 104%, 101%, 102%.
2. Express as decimals: 97%, 96%, 99%, 98%.
3. Express as decimals:  $103\frac{1}{2}\%$ ,  $104\frac{1}{4}\%$ ,  $102\frac{3}{4}\%$ ,  $101\frac{1}{2}\%$ .
4. Express as decimals:  $97\frac{1}{2}\%$ ,  $96\frac{3}{4}\%$ ,  $99\frac{1}{4}\%$ ,  $98\frac{3}{8}\%$ .
5. A real estate broker in St. Louis received \$5047, which was 103% of the sum he was to invest in real estate. What sum did he invest in real estate?
6. My agent sent me \$3395, which was 97% of the selling price of some railway stock. What did the stock sell for?
7. My agent on selling a quantity of wheat sent me  $98\frac{1}{2}\%$  of the proceeds. If I received \$3546, what did the wheat sell for?
8. I sent my agent \$4545.30, which was  $104\frac{1}{4}\%$  of the sum he was to invest in buying silk. What did he pay for the silk?

**206.** (1) A house is sold for \$16,400, and 25% of the purchase money is paid down, the balance to remain on mortgage. How much remains on mortgage?

In this problem we are given the measured whole, *i.e.* the selling price, and the number or 25% of it. We are required to find the balance which is the difference between the selling price and the sum paid.

$$\begin{aligned} \text{The sum paid} &= 25\% \text{ or } \frac{1}{4} \text{ of } \$16,400 = \$4100. \\ \therefore \text{the balance} &= \$16,400 - \$4100 = \$12,300. \end{aligned}$$

(2) On Jan. 10 a merchant buys goods invoiced at \$876.40, on the following terms: 4 months, or less 6% if paid within 10 days. What sum will pay the debt on Jan. 15?

Since Jan. 15 is less than 10 days after Jan. 10, the sum due will be 6% less than \$876.40.

The discount = 6 % or .06 of \$876.40 = \$52.584.

$\therefore$  the sum due = \$876.40 - \$52.58 = \$823.82.

### Exercise 136

1. A maltster malts 7500 bu. of barley, which in the process increases  $12\frac{1}{2}\%$ . How many bu. of malt has he?

2. Certain books are bought at \$1.75 each. At what must they be sold to gain 12%?

3. A merchant asked 30% advance on goods which cost \$120, but finally took 25% less than the price asked. What did he sell them for?

4. A merchant bought apples at 60¢ per bu., and sold them at a gain of 25%. Find the selling price per bu. How many bu. did he sell if he received all together \$37.50?

5. Bought \$64 worth of apples at 80¢ per bu., part of which being damaged and rendered worthless, I sold the remainder at an advance of 50%, receiving \$76.80. How many bu. were damaged?

6. If 10% of an army of 23,400 men were slain in battle, and 5 per cent of the remainder were mortally wounded, find the sum of the killed and mortally wounded.

7. The population of a town of 64,000 inhabitants increases at the rate of  $2\frac{1}{2}\%$  in each year. Find its population 1, 2, and 3 years hence.

8. The population of a city is a million; it increases  $1\frac{1}{2}\%$  for 3 years successively. Find the population at the end of 3 years.

9. A lawyer collected \$287.50, and charged 5% for his services. How much did he retain and how much did he pay over? What per cent is the amount paid over of the amount collected?

10. The cost price of a book is \$1.60, the expense of sale 5% upon the cost price, and the profit 25% upon the whole outlay. Find the selling price of the book.

11. The cattle on a certain stock farm increase at the rate of  $18\frac{1}{2}\%$  per annum. If there are 4096 cattle in 1894, how many will there be in 1896?

12. A man bought a store and contents for \$4720; he sold the same for  $12\frac{1}{2}\%$  less than he gave, and then lost 15% of the selling price in bad debts. Find his entire loss.

13. A person having bought goods for \$40 sells half of them at a gain of 5%. For how much must he sell the remainder so as to gain 20% on the whole?

14. A grocer mixes two kinds of tea which cost him 38¢ and 44¢ per lb. respectively. What must be the selling price of the mixture in order that he may gain 15% on his outlay?

15. A grain dealer expended \$2150 in the purchase of wheat, one-half as much again in the purchase of barley, and twice as much in the purchase of corn; he sold the wheat at a profit of 6%, the barley at a loss of 5%, and the corn at a gain of 2%. Find his gain on the whole transaction.

16. A person gave \$150 for one horse and \$225 for another. He sold the first horse at a gain of 20%, and the second at a loss of 20%. Find the selling price of each horse and the gain or loss on the whole transaction.

17. A sells goods to B which cost him \$465, at a gain of 6%, B sells them to C at a loss of 3%, and C sells them to D, gaining 10%. What did D give for the goods?

18. A man having bought a lot of goods for \$450 sells  $\frac{1}{3}$  at a loss of 5%,  $\frac{1}{3}$  at a gain of 7%, and the remainder at a gain of 2%. Find the total gain.

19. A merchant began business with a capital of \$30,000. He gained  $16\frac{2}{3}\%$  the first year, which he added to his capital, and  $12\frac{1}{2}\%$  the second year, which he added to his capital. In the third year he lost 20%. Find his capital at the end of the third year.

20. Sugar being composed of 49.856% of oxygen, 43.265% of carbon, and the remainder hydrogen, find how many lb. of each of these materials there are in 1 T. of sugar.

21. A merchant buys a bill of dry goods, April 16, amounting to \$6377.84, on the following terms: 4 mo., or less 5% if paid within 30 da. How much would settle the account on May 16? The amount paid May 16 is what per cent of the full amount of the bill?

22. Water is composed of 88.9% of oxygen and 11.1% of hydrogen. How many lb. are there of each in 1 cu. yd. of water? (A cu. ft. of water weighs 1000 oz.)

207. If a gain of \$24 is measured by a certain number and the cost price, which is \$64, find the number as a rate per cent.

The required number  $\times$  \$64 = \$24.

$\therefore$  the required number =  $\frac{\$24}{\$64} = \frac{3}{8} = 37\frac{1}{2}\%$ .

### Exercise 137

1. If the quantity of land sold, which is 50 A., is measured by a certain number and a unit of 200 A., find the number as a rate per cent.

2. If a gain of \$45 is measured by a certain number and the cost \$225, find the number as a rate per cent.

3. If a loss \$36 is measured by a certain rate per cent and the cost, which is \$108, find the rate per cent.

4. If the sum of money paid for fruit, which is \$6, is measured by a certain number and the unit \$72, find the number as a rate per cent.

**208.** (1) A merchant sold 60 yd. of cloth from a web containing 150 yd. What per cent of the web did he sell?

We are here given the measured part and the measured whole, and we are required to find the number expressing the ratio of the part to the whole.

The quantity sold =  $\frac{60}{150}$  or  $\frac{2}{5}$  or 40 % of the web.

(2) An article which cost \$3.60 was sold for \$4.32. Find the gain per cent.

The gain = \$4.32 - \$3.60 = 72 cents.

$\therefore$  the gain =  $\frac{72}{360}$  or  $\frac{1}{5}$  or 20 % of the cost.

### Exercise 138

1. The cost price of an article is \$64, and the gain on selling \$16. Find the gain per cent.

2. Tea is bought at 84¢ per lb., and sold at 98¢. Find gain per cent.

3. Out of 48 eggs, 6 were broken. What per cent of the whole number was broken?

4. The cost price of an article was \$56, and the selling price \$49. Find the loss per cent.

5. What per cent is 1 in. of 1 ft.? 1 ft. of 1 yd.? 1 yd. of 1 rd.? 1 rd. of 1 mi.?

6. What per cent is 1 min. of 1 hr.? 1 hr. of 1 da.? 1 da. of 1 wk.? 1 wk. of 1 yr.?

7. What per cent is 1 pt. of 1 qt.? 1 qt. of 1 gal.?

8. What per cent of 1 pk. is 1 qt.? Of 1 bu. is 1 pk.?

9. A merchant by selling 1 lb. of butter gains the cost price of 1 oz. What is his gain per cent?

10. One lb. Troy is what per cent of 1 lb. Avoir.? One lb. Avoir. is what per cent of 1 lb. Troy?

11. The volume of 1 gal. is what per cent of 1 cu. ft.? A cu. ft. is what per cent of 1 gal.?

12. An area containing 1 sq. yd. is increased by 4 sq. ft. Find the per cent of increase.

13. If £1 is worth \$4.866, what per cent of £1 is \$1?

### Exercise 139

1. A paymaster receives \$150,000 from the treasury, but fails to account for \$2250. What is the percentage of loss to the government?

2. A city of 17,000 inhabitants increases in a given time to 20,000. Find the increase per cent.

3. \$640 increased by a certain per cent of itself equals \$720. Required the rate per cent.

4. A house worth \$3500 rents for \$400. For what per cent of its value does it rent?

5. If a tradesman gain \$1.32 on an article which he sells for \$5.28, what is his gain per cent?

6. An article which cost 84¢ is sold for 93¢. Find the gain per cent.

7. A city gained 2467 in population in 5 years. If its population was 14,802 five years ago, what was the gain per cent?

8. In a certain year the number of graduates of a school was 70. Ten years later it was 213. Find the rate per cent of increase.

9. A tea merchant mixes 40 lb. of tea at 45¢ per lb. with 50 lb. at 27¢ per lb., and sells the mixture at 42¢ per lb. What per cent profit does he make?



10. Paid \$664.25 for transportation on an invoice of goods amounting to \$8866. What per cent must be added to the invoice price to make a profit of 20% on the full cost?

11. A grocer uses for a 1 lb. weight one which only weighs 15.75 oz. What does he gain per cent by his dishonesty?

12. I bought 500 sheep at \$6 a head; their food cost me \$1.25 a head; I then sold them at \$10 a head. Find my whole gain, and also my gain per cent.

13. A man's income is derived from the proceeds of \$4550 at a certain rate per cent; and \$5420 at 1% more than the former rate. His whole income being \$453, find the rates.

14. If a commodity be bought for \$16.42 a cwt. and sold for 18¢ a lb., find the rate of profit per cent.

15. The police returns for a certain year give 1350 male offenders, and 1150 female; the next year's returns show a decrease of 6% in the number of male criminals, and an increase of 8% in number of female. Find increase or decrease per cent in whole number of criminals.

16. The area of North America is 8,000,000 sq. mi., and of the Mississippi and St. Lawrence basins respectively 1,250,000 and 350,000 sq. mi. Find what per cent of the area of the continent is drained by each of these rivers.

17. Turn to your geography and find the area of the basins of the principal rivers, and then find what per cent of the area of North America is drained by each river and by all of them together.

18. The length of the Missouri-Mississippi River is 4200 mi., and of the St. Lawrence 2000 mi. The length of the St. Lawrence is what per cent of that of the Missouri-Mississippi? The area of the St. Lawrence basin is what per cent of the area of the Mississippi basin?

**209.** If a gain of \$36 is measured by the number  $22\frac{2}{3}\%$  and the cost, find the cost.

The gain =  $22\frac{2}{3}\%$  or  $\frac{2}{3}$  of the cost.

$\frac{2}{3}$  of the cost = \$36.

$\frac{1}{3}$  of the cost = \$18.

$\frac{2}{3}$  of the cost = \$162.

$\therefore$  the cost = \$162.

#### Exercise 140

1. If the quantity \$18 is measured by the number 6% and a certain unit, find the unit.

2. If the gain \$32 is measured by the number 8% and the cost, find the cost.

3. If the loss \$60 is measured by the ratio 12% and the cost, find the cost.

4. If the selling price \$3810 is measured by the number 127% and the cost price, find the cost price.

5. If the selling price \$5640 is measured by the number 94% and the cost, find the cost.

**210.** A trader sold a horse at an advance of 12%, gaining \$18. Find the cost price of the horse.

12% or  $\frac{3}{25}$  of the cost = \$18.

$\frac{1}{25}$  of the cost = \$6.

$\frac{3}{25}$  of the cost = \$150.

$\therefore$  the cost = \$150.

#### Exercise 141

1. A quantity of sugar was sold at an advance of  $12\frac{1}{2}\%$ . If the gain was \$17, what was the cost?

2. A yd. of cloth was sold at a loss of  $37\frac{1}{2}\%$ . If the loss was 36¢ a yd., what was the cost?

3. Forty-five per cent of a piece of cloth was sold. If 135 yd. were sold, how many yd. were in the piece at first? How many remain unsold? What per cent remains unsold?

4. If 3% more be gained by selling a horse for \$333 than by selling him for \$324, find his original price.

5. A man bought a horse which he sold at a loss of 8%. If he had received \$24 more, he would have gained 7%. What did the horse cost him?

6. A clerk pays 16% of his salary each year for board. If his board costs him \$208 a year, what is his salary?

7. A man sold a field consisting of 15 A., which was  $6\frac{1}{4}\%$  of his farm. How many A. were in the farm at first?

8. If 25% of my money is invested in bank stock and the remainder in business, what per cent of my money is invested in business? My bank stock is worth what part of my business capital? If I have \$4800 invested in business, what is the value of my bank stock?

9. Twenty per cent of my money is invested in business, and the remainder, which is \$12,800, in real estate. How much have I invested in business?

10. I invested 25% of my money in business, and put  $6\frac{2}{3}\%$  of the remainder in the bank. If I put \$600 in the bank, how much money did I have at first?

11. Twenty-eight per cent of a sum of money was invested in business, and  $12\frac{1}{2}\%$  of the remainder in real estate. If the sum invested in business exceeds that invested in real estate by \$1900, find the amount of money I had at first.

**211.** (1) A man invests  $77\frac{1}{2}\%$  of his capital in bank stock, and has \$29,367 left. What is his capital?

The amount left =  $100\% - 77\frac{1}{2}\%$  or  $22\frac{1}{2}\%$  of his capital.

$22\frac{1}{2}\%$  or  $\frac{45}{200}$  of his capital = \$29,367.

$\frac{1}{40}$  of his capital = \$3263.

$\frac{4}{10}$  of his capital = \$130,520.

$\therefore$  his capital = \$130,520.

(2) If a profit of 17% is made by selling an article at an advance of \$24.50, what would have been the selling price if the loss had been 8%?

17% of the cost = \$24.50.

1% of the cost = \$1.4412.

100% of the cost = \$144.12.

The loss = 8% of the cost = \$11.53.

∴ the second selling price = \$144.12 - \$11.53 = \$132.59.

### Exercise 142

1. A man invests 42% of his capital in real estate, and has \$53,070 left. What is his capital?

2. A bankrupt's assets are \$23,625, and he pays 40% of his liabilities. What are his liabilities?

3. A merchant loses  $6\frac{1}{2}\%$  of the cost price by selling an article at a loss of \$27.30. Find the cost price, and also at what he must sell it to gain  $7\frac{1}{2}\%$ .

4. By selling a house at a loss of \$150, a real estate dealer loses  $6\frac{2}{3}\%$  of the cost. Find the cost and also the gain per cent if it had been sold for \$2625.

5. I sold a lot at a gain of  $8\frac{1}{3}\%$ , thereby gaining \$113. What should I have sold it for to gain 9%?

6. Coals are 20% cheaper this year than last. If the price were to rise \$1 per T., they would still be 50¢ per T. cheaper than last year. Find the price last year.

7. A person asked for a lot of land 40% more than it cost him, but finally reduced his price 15%, gaining on the whole \$1000. For how much did he sell the land?

8. A merchant sold  $\frac{3}{4}$  of a quantity of cloth at a gain of 20%, and the remainder at cost. His gain was what per cent of the cost? If he gained \$7.29, what was the cost of the goods?

9. A merchant sold  $\frac{2}{3}$  of a quantity of tea at a gain of 12%, and the remainder at a gain of 9%, gaining all together \$2.75. Find the cost of the tea.

10. A speculator gained 20% on  $\frac{3}{4}$  of his investment, and lost 24% on the remainder. All together he made \$ 270. Find the amount of his investment.

11. A business firm's resources consist of notes, merchandise, personal accounts, etc., to the amount of \$ 9117.61, and a balance, which is 44% of their entire capital on deposit in bank. How much is on deposit?

12. Ten per cent of an army were slain on the field of battle, and 5 per cent of the remainder were mortally wounded. The difference between the killed and mortally wounded was 1100. How many men went into battle?

**212.** (1) A horse was sold for \$ 117, which was  $8\frac{1}{3}\%$  more than it cost. Find the cost price.

$$\begin{aligned}\text{The gain} &= 8\frac{1}{3}\% \text{ or } \frac{1}{12} \text{ of the cost.} \\ \text{The selling price} &= \frac{13}{12} \text{ of the cost.} \\ \frac{13}{12} \text{ of the cost} &= \$ 117. \\ \frac{1}{12} \text{ of the cost} &= \$ 9. \\ \frac{13}{12} \text{ of the cost} &= \$ 108. \\ \therefore \text{the cost} &= \$ 108.\end{aligned}$$

(2) A horse was sold for \$ 154, which was  $8\frac{1}{3}\%$  less than it cost. Find the cost price.

$$\begin{aligned}\text{The loss} &= 8\frac{1}{3}\% \text{ or } \frac{1}{12} \text{ of the cost.} \\ \text{The selling price} &= \frac{11}{12} \text{ of the cost.} \\ \frac{11}{12} \text{ of the cost} &= \$ 154. \\ \frac{1}{12} \text{ of the cost} &= \$ 14. \\ \frac{13}{12} \text{ of the cost} &= \$ 168. \\ \therefore \text{the cost} &= \$ 168.\end{aligned}$$

### Exercise 143

1. A house and lot were sold for \$ 3600, which was 20% more than they cost. Find the cost price.

2. A house and lot were sold for \$ 4200, which was 25% less than they cost. Find the cost price.

3. A speculator gained 7% by selling wheat for \$2140. Find the cost price.

4. Eggs are sold at the rate of 15¢ per doz., a profit of 25% being made. What is the cost price per doz.?

5. I sold a book for 42¢, gaining  $16\frac{2}{3}\%$ . Find the cost price. How much would be gained by selling at a gain of 25%? What would then be the selling price?

6. By selling hats at 60¢ each, a merchant gains  $33\frac{1}{3}\%$ . Find the cost price. What would have been the actual loss, and what the loss per cent, if they had been sold at 36¢ each?

7. I sold a lot of land for \$600, thereby gaining 20%. Find the cost price.

8. I sold a lot of land for \$600, thereby losing 20%. Find the cost price.

9. What is the cost of both lots in questions 7 and 8? What is their selling price? How much is the cost of both greater than their selling price?

10. A dealer sold two bicycles for \$45 each, losing 25% on one and gaining 25% on the other. How much did he lose on the whole transaction?

**213.** (1) If a debt, after a reduction of 3%, becomes \$1008.80, what would it become after a reduction of 4%?

After a reduction of 3%, the amount owed = 97% of the original debt, and after a reduction of 4% it becomes 96% of the original debt.

$$97\% \text{ of the debt} = \$1008.80.$$

$$1\% \text{ of the debt} = \$10.40.$$

$$96\% \text{ of the debt} = \$998.40.$$

$$\therefore \text{after a reduction of } 4\% \text{ the debt} = \$998.40.$$

(2) The population of a city increases 2% yearly. It now has 132,651 inhabitants. How many had it 1, 2, and 3 years ago?

The population now = 102 % or 1.02 of that 1 year ago.

1.02 of the population 1 year ago = 132,651.

$\therefore$  the population 1 year ago =  $132,651 \div 1.02 = 130,050$ .

$\therefore$  the population 2 years ago =  $130,050 \div 1.02 = 127,500$ .

$\therefore$  the population 3 years ago =  $127,500 \div 1.02 = 125,000$ .

Prove these answers correct.

#### Exercise 144

1. A horse was sold for \$ 658, which was  $16\frac{2}{3}\%$  more than its cost. How much did it cost ?

2. A speculator gained 3% by selling wheat for \$ 6437.50. Find the cost price.

3. A merchant, after a business of 5 years, found his capital increased to \$ 28,000, showing a gain of 60% on his original capital. Find that capital.

4. Eggs are sold at the rate of 5 for 6¢, a profit of 20% being made. Find the price at which they are bought.

5. In 1896 a city has a population of 28,000 inhabitants. If its population increased  $17\frac{1}{4}\%$  in the two years previous, what was it in 1894 ? If its population decreased  $17\frac{1}{4}\%$  in the two years previous, what was it in 1894 ?

6. By selling an article for \$ 2.64 a merchant loses 12%. What was the cost, and for what must he sell it to gain  $16\frac{2}{3}\%$  ?

7. A merchant sells tea at 75¢ per lb., thereby losing 5%. What was the cost, and at what price per lb. must it be sold to gain  $4\frac{1}{2}\%$  ?

8. Flour is sold for \$ 6 per bbl., at a loss of 17%. Find what selling price would give 16%.

9. If, by selling an article for \$ 25.30, 8% be lost, what per cent is gained or lost if it be sold for \$ 38 ?

10. I sold goods at \$ 21.60 per cwt., thereby gaining  $14\frac{2}{3}\%$ . Find the cost per lb.

11. A farmer sold his crop of wheat in 1871 for 8% more than he obtained for his crop of the preceding year; he received for both crops \$2600; how much did he get for his crop of 1870?

12. I sold two houses, receiving \$2400 for each. On the first I gained 25%, and on the second lost 25%. Find the actual loss and also the loss per cent.

13. By selling a lot of land for \$600, thereby gaining 20%; a second for \$600, losing 20%; and a third at an advance of 20% on cost; I find I have made \$75 on the whole transaction. Find the cost of each lot.

14. A ship depreciates in value each year at the rate of 10% of its value at the beginning of the year, and its value at the end of three years is \$14,580. What was its original value?

15. The cattle on a stock farm increase at the rate of  $18\frac{3}{4}\%$  per annum. In 1889 there were 6859 head of cattle on the farm; how many were there in 1888? In 1887? In 1886?

16. An importer pays for freight and duty 10% on cost price, and sells to the retailer at a profit of 20%; the retailer sells to the consumer at a profit of 25%. Find the amount paid by the consumer for goods which cost the importer \$7500.

#### PROFIT AND LOSS

**214.** The **Profit** is the amount by which the selling price exceeds the buying price.

The *rate of profit* is usually expressed as a certain per cent of the cost price.

The **Loss** is the amount by which the selling price falls short of the cost price.

The *rate of loss* is usually expressed as a certain per cent of the cost price.



**215.** (1) At a forced sale a bankrupt's house was sold for \$8000, which was 20% less than its real value. If the house had been sold for \$12,000, what per cent of its real value would it have brought?

$$80\% \text{ of the value of the house} = \$ 8000.$$

$$1\% \text{ of the value of the house} = 100.$$

$$100\% \text{ of the value of the house} = 10,000.$$

$$\therefore \text{ the second selling price} = \frac{12000}{10000} \text{ of the value} \\ = 120\% \text{ of the value.}$$

(2) The manufacturer of an article makes a profit of 25%, the wholesale dealer makes a profit of 20%, and the retail dealer makes a profit of 30%. What is the cost to the manufacturer of an article that retails at \$15.60?

$$\text{Let the cost to the manufacturer} = 100 \text{ units of money.}$$

$$\text{The selling price of the manufacturer} = 125 \text{ units of money.}$$

$$\text{The gain of the wholesale dealer} = 25 \text{ units of money.}$$

$$\text{The selling price of the wholesale dealer} = 150 \text{ units of money.}$$

$$\text{The gain of the retail dealer} = 45 \text{ units of money.}$$

$$\text{The selling price of the retail dealer} = 195 \text{ units of money.}$$

$$195 \text{ units of money} = \$15.60.$$

$$1 \text{ unit of money} = .08.$$

$$100 \text{ units of money} = 8.00.$$

$$\therefore \text{ the prime cost} = 8.00.$$

#### GENERAL STATEMENT OF SOLUTION

(3) Represent the cost to the manufacturer by 100 units of money, and then find the number of units representing respectively the selling prices of the manufacturer, the wholesale dealer, and the retail dealer. Put the number of units of money which represent the retail price equal to \$15.60 and find the value of 100 units of money, which is the cost of manufacturing.

## QUESTION IN PROOF

(4) The manufacturer of an article makes a profit of 25%, the wholesale dealer a profit of 20%, and the retail dealer a profit of 30%. What is the retail price of an article which cost the manufacturer \$8?

## PROOF

The manufacturer's gain = 25% of \$8 = \$2.  
 The manufacturer's selling price = \$10.  
 The wholesale dealer's gain = 20% of \$10 = \$2.  
 The wholesale dealer's selling price = \$12.  
 The retail dealer's gain = 30% of \$12 = \$3.60.  
 The retail dealer's selling price = \$15.60.  
 $\therefore$  \$8 is the correct answer to the previous question.

## MAKING QUESTIONS

(5) Make a question in which you are given the selling price and the gain per cent, and are required to find the cost price.

## MAKING

Let the cost of a house = \$6250.  
 Let the gain per cent on selling =  $37\frac{1}{2}\%$ .  
 The gain =  $37\frac{1}{2}\%$  of \$6250 = \$2343.75.  
 The selling price = \$6250 + \$2343.75 = \$8593.75.

## Problem

A house was sold at  $37\frac{1}{2}\%$  above cost. If the selling price was \$8593.75, find the cost price.

Other questions may also be written down from the same making, thus :

A house which cost \$6250 was sold at a gain of  $37\frac{1}{2}\%$ . Find the selling price.

A house which cost \$6250 was sold for \$8593.75. Find the gain per cent.

A house which cost \$6250 was sold at a gain of \$2343.75. Find the gain per cent.

In the following exercise state in general terms how to solve each question. Prove some of your answers correct, framing the question in proof. Make questions similar to problems in the exercise.

**Exercise 145**

1. A lot of dry goods was sold at an advance of 18%. If the gain was \$436.50, what was the cost?
2. I made a mixture of wine consisting of 1 gal. at 50¢, 3 at 90¢, 4 at \$1.20, and 12 at 40¢. I sell the mixture at \$1.60 a gal. Find my gain per cent.
3. A merchant's price is 25% above cost. If he allow a customer a discount of 12% on his bill, what per cent profit does he make?
4. If cloth when sold at a loss of 25% brings \$5 a yd., what would be the gain or loss per cent if sold at \$6.40 a yd.?
5. Eggs are bought at 27¢ a doz., and sold at the rate of 8 for 25¢. Find the rate of profit.
6. A merchant sells goods to a customer at a profit of 60%, but the buyer becomes bankrupt and pays only 70 cents on the dollar; what per cent does the merchant gain or lose on the sale?
7. A man bought a horse which he sold again at a loss of 10%. If he had received \$45 more for him he would have gained  $12\frac{1}{2}\%$ . Find the cost of the horse.
8. A bookseller sold a book at 17% below cost, but had he charged 50 cents more for it, he would have gained 7%. Find the cost of the book to the bookseller, and the price at which he sold it.
9. A tradesman bought goods for \$1200 and sold  $\frac{1}{3}$  of them at a loss of 10%. For how much must he sell the remainder to gain 20% on the whole?

10. A man bought a house and lot for \$4750. After spending \$1143 on repairs and improvements, and paying \$128 for taxes and other expenses, he sold the property for \$6800. What rate per cent of profit did his investment yield him?

11. By selling cloth at \$1.20 per yd., a tradesman lost  $6\frac{1}{4}\%$  on his outlay. At what price must he sell it to gain  $12\frac{1}{2}\%$ ?

12. If a manufacturer sells an article of which the first cost is \$400, to a wholesale dealer at 10% profit, the wholesale dealer to the retailer at 15% profit, and the retailer to the consumer at 30% profit, what sum is paid by the consumer as profits in addition to the first cost of the article?

13. A grocer sold, at 51¢ per lb., a portion of a stock of tea, incurring a loss of 15% and a total loss of \$18 on the quantity sold. How many lb. did he sell?

14. A merchant marks his goods so that he may allow a discount of 5%, and still make a profit of 15%. Find the marked price of broadcloth that cost him \$3.80 a yd.

15. A drover bought 400 sheep at a certain price per head. He sold  $\frac{3}{8}$  of them at a gain of 20%,  $\frac{3}{10}$  of them at a gain of 15%, and the remainder at a loss of 10%, gaining on the whole \$217. How much did he pay for the 400 sheep?

16. A grain dealer expended a certain sum of money in the purchase of wheat, half as much again in the purchase of barley, and twice as much in the purchase of oats; he sold the wheat at a profit of 5%, the barley at a profit of 8%, and the oats at a profit of 10%, receiving all together \$9740. Find the sum laid out in each grain.

17. A person sold two horses at \$160 each, losing 20% on one and gaining 20% on the other. Did he gain or lose on the whole transaction, and how much?

18. A man bought 360 bu. of wheat at a certain price per bu. and sold  $\frac{1}{2}$  of it at a gain of 10%,  $\frac{1}{4}$  at a loss of 25%, and

the remainder at a gain of 45%, and by so doing realized \$594 for the whole lot. What was the cost price per bu.?

19. A speculator paid \$1400 for two lots, the price of one of them being 40% that of the other. He sold the cheaper lot at a gain of 50%, and the dearer one at a loss of 30%. Find his gain or loss per cent on the whole transaction.

20. A merchant buys 3150 yd. of cloth. He sells  $\frac{1}{3}$  of it at a gain of 6%,  $\frac{1}{5}$  at a gain of 8%,  $\frac{1}{7}$  at a gain of 12%, and the remainder at a loss of 3%. Had he sold the whole at a gain of 5% he would have received \$28.98 more than he did. Find the prime cost of 1 yd.

21. The manufacturer of an article charged 20% profit, the wholesale dealer charged 25% of an advance on the manufacturer's price, and the retail dealer charged 30% of an advance on the wholesale price. Find the cost to the manufacturer of an article for which the retail dealer charged \$23.40.

22. I buy two cows for \$55; if I sell the first at a loss of 5%, and the second at a gain of 5%, I should gain  $\frac{5}{11}$ %. What was the price of each cow?

#### COMMERCIAL OR TRADE DISCOUNT

**216.** Commercial discount is an allowance made by merchants upon their catalogue prices.

The commercial discount is reckoned at a certain rate per cent.

Sometimes several discounts are allowed to a purchaser.

In such a case, the first discount is to be deducted, and then the second discount is to be reckoned upon the remainder and then deducted, and so on for each successive discount.

**217.** What is the net amount of a bill for \$720 subject to discounts of 20% and 6%? Find a single discount equivalent to these successive discounts.

The first discount = 20 % of \$720 = \$144.

The first remainder = \$720 - \$144 = \$576.

The second discount = 6 % of \$576 = \$34.56.

∴ the net amount = \$576 - \$34.56 = \$541.44.

Again, the single equivalent discount = \$720 - \$541.44 = \$178.56.

∴ the rate of a single discount = \$178.56 ÷ \$720 = .248 = 24.8%.

### Exercise 146

1. An invoice was \$650, trade discounts 20% and 8% off. Find the cost of the goods.

2. What is the net amount of a bill of goods, the list price of which is \$245, trade discounts 18% and 5% off for cash?

3. What is the difference on an invoice of \$540, between 40% direct discount, and discounts of 25% and 15%?

4. A dealer buys a book, list price 80¢, at a discount of 25%; he sells the book for 80¢. What per cent is the profit?

5. What is the net amount of a bill of \$480, discounts being  $12\frac{1}{2}\%$  and 8%? Find a single discount equivalent to these successive discounts.

6. A man paid \$380 for goods, at discounts of 20% and 5%. Find the list price of the goods.

7. A dealer paid \$299.20 for goods at 15% and 12% off. Find the list price of the goods.

8. Find the net cash amount of a bill for \$1266, subject to discounts of  $33\frac{1}{3}\%$ , 10%, and 5%, for cash.

9. Find the difference between a single discount of 40%, and successive discounts of 25% and 15%.

10. Find the net amount of a bill of \$340, discounts being 30, 15, and 6. Find a single discount equivalent to these three discounts.

11. Find the net cash amount of a bill of \$ 254, discounts being 25%, 12½%, 5%. Find a single discount equivalent to these three successive discounts.

12. A merchant who receives successive discounts of 20%, 15%, and 10%, on a bill of \$ 750, sells at an advance of 33⅓%. What does he sell his goods for? His selling price is what per cent less than the list price?

13. What is the difference between discounting a bill of \$ 3000 at 40%, and then taking a discount off the remainder of 5% for cash, and discounting the whole at 45%?

14. A merchant buys goods at 40 and 20 off the list price and sells them at 30 and 10 off the list price. What is his gain per cent?

15. An invoice of crockery, amounting to \$ 1473.20, was sold Jan. 3, at 90 days, subject to 40% and 10% discount, with an additional discount of 6% if paid within 20 days. How much will be required to pay the bill on Jan. 21?

#### COMMISSION AND BROKERAGE

**218.** A **Commission Merchant** is one who buys or sells goods for other persons by their authority. Commission merchants are usually placed in possession of the goods bought.

A **Broker** is a person who, in the name of his principal, effects contracts to buy or sell.

The broker is not in general placed in possession of the goods bought or sold.

The title Broker is also applied to persons who deal in stocks, bonds, bills of exchange, promissory notes, etc., and to mercantile agents, who transact the business for a ship in port.

**Commission** is the charge made by an agent for transacting business.

In buying, the commission is reckoned on the *cost* price ;  
in selling, the commission is reckoned on the *selling* price.

**219.** (1) A commission merchant sold 270 bbl. of flour at \$6 a bbl., and received 5% commission. What was his commission? How much did he remit to his employer?

$$\text{The selling price} = 270 \times \$6 = \$1620.$$

$$\therefore \text{the commission} = 5\% \text{ of } \$1620 = \$81.$$

$$\therefore \text{the amount remitted} = \$1620 - \$81 = \$1539.$$

(2) A commission of \$242.58 was charged for selling \$3772 worth of goods. What was the rate of commission?

$$\text{The commission} = \frac{242.58}{3772} \text{ of the selling price}$$

$$= .0643 \text{ of the selling price.}$$

$$\therefore \text{the rate of commission} = 6.43\%.$$

(3) A grain dealer charged  $3\frac{1}{2}\%$  for selling a quantity of wheat, and received for his commission \$218.40. For how much did he sell the wheat?

$$\text{The commission} = 3\frac{1}{2}\% \text{ or } .035 \text{ of the selling price.}$$

$$.035 \text{ of the selling price} = \$218.40.$$

$$\therefore \text{the selling price} = 218.40 \div .035 = \$6240.$$

(4) If \$512.50 include the price paid for certain goods, and  $2\frac{1}{2}\%$  commission to the agent, how much money does the agent expend in purchasing the goods?

$$\text{Let the cost price of the goods} = 100 \text{ units of money.}$$

$$\text{Then the commission} = 2\frac{1}{2}\% \text{ units of money.}$$

$$\text{The amount sent to the commission merchant} = 102\frac{1}{2}\% \text{ units of money.}$$

$$102\frac{1}{2}\% \text{ units} = \$512.50.$$

$$1 \text{ unit} = 512.50 \div 102.5 = \$5.$$

$$100 \text{ units} = \$500.$$

$$\therefore \text{the cost of the goods} = \$500.$$

As in Exercise 145, give the general statements of solutions, prove answers, and make questions. Do this also in each of the following exercises.



## Exercise 147

1. A commission merchant sold 480 bbl. of flour at \$3.50 a bbl. on a commission of 2%. What was his commission? How much did he remit to his employer? The amount remitted was what per cent of the selling price?

2. My agent sold coffee to the amount of \$850 on a commission of 3%. Find his commission and also the amount remitted to his employer. The amount remitted is what per cent of the selling price?

3. An agent sold 210 bu. of oats at 60¢ a bu., and charged \$3.78 for doing so. Find his rate of commission.

4. On a debt of \$1725 a creditor receives a dividend of 60%, on which he allows his attorney 5%. He receives a further dividend of 25%, on which he allows his attorney 6%. What is the net amount that he receives?

5. If a commission of \$212.94 is paid for buying \$6552 worth of goods, find the rate per cent of commission.

6. An agent received \$40.62½ for selling a house worth \$1625. Find his rate per cent of commission.

7. An agent, who is paid a commission on what he invests, received \$4896, and invests \$4800. Find his rate per cent of commission.

8. An agent received \$56 for selling grain on a commission of 4%. Find value of grain sold.

9. A commission merchant charged 2½% for selling a quantity of pork, and received for his commission \$64.82. Find the selling price of the pork.

10. The owner of a house offered an agent \$500 commission, if the agent could sell the house for \$10,500. What rate per cent commission was the owner offering? Had the owner offered 5% commission, what would have been the commission on \$10,500?

11. I bought a bicycle for \$70, which was  $\frac{1}{6}$  of my commission at  $3\frac{1}{2}\%$  for selling a quantity of land. For how much was the land sold?

12. A real estate dealer sold land for 100 units of money, on a commission of 4%. How many units of money did he keep for his commission? How many units of money did he send his employer? If his employer received \$2880, what did the land sell for? What was the agent's commission?

13. An agent remits \$4850 to his employer after taking out his commission of 3%. Find the selling price.

14. My agent sent me as my share of the selling price of flour \$2038.40. If the flour sold for \$3.25 a bbl., and the agent's commission was 2%, how many bbl. did he sell?

15. My agent bought a quantity of goods for me on a commission of 2%. If the cost of the goods was 100 units of money, how many units of money did his commission equal? How many units did I have to send him to cover the cost of the goods and his commission?

16. A merchant in Buffalo sends a commission merchant in New York \$3120, instructing him to purchase goods, reserving his commission at 4%. Find his commission.

17. A merchant sent \$3238.30 to New Orleans to be expended in cotton. The broker in New Orleans charged 6% commission. What sum was paid for the cotton?

18. Sent to a commission merchant in Chicago \$2080.80 to invest in flour, his commission being 2% on the amount expended. How many bbl. of flour could be purchased at \$4.25 a bbl.?

19. A real estate broker sold a house on  $3\frac{1}{2}\%$  commission, and sent to the owner \$6176. What was the broker's commission, and what sum did he receive for the house?

20. I send \$5250 to a commission merchant in St. Louis, who charges 5% for investing, with instructions to purchase certain

goods, deducting his commission from the amount of money sent him. Find his commission.

21. (a) Received \$4100 from my agent, who had deducted his commission at 5% as proceeds of sale of goods. What were the goods sold at? (b) Remitted \$4100, including commission, to my agent to invest for me on commission of 5%. What was his commission?

22. An agent sold a quantity of flour for \$5100 on commission of 3%, and invested the remainder in tea, first taking out his commission of 2% on the price paid for the tea. Find the amount paid for the tea, and also the total commission.

## INSURANCE

220. Insurance is a contract by which a person whose property is insured receives security against loss by fire or accident in consideration of a sum of money paid to the insurance company.

The **Premium** is the sum paid for insurance. It is always a certain per cent of the *sum insured*.

The **Policy** is the written contract of insurance.

221. (1) A factory valued at \$35,000 was insured for  $\frac{3}{5}$  of its value, the rate of insurance being  $\frac{5}{8}\%$  for one year. What was the amount of the premium?

The premium =  $\frac{5}{8}\%$  of  $\frac{3}{5}$  of \$35,000.

$$= \frac{5}{800} \times \frac{3}{5} \times \frac{\$35000}{1} = \$131.25.$$

(2) The sum of \$285 was paid for the insurance at  $\frac{3}{4}$  of its value of a ship worth \$50,000. What was the rate per cent of premium if \$3.75 was charged for the policy and the preliminary survey?

The premium = \$285 - \$3.75 = \$281.25.

The amount insured =  $\frac{3}{4}$  of \$50,000 = \$37,500.

The rate of insurance =  $\$281.25 \div \$37,500 = .0075$ .

$\therefore$  the rate per cent =  $\frac{75}{100}\%$  or  $\frac{3}{4}\%$ .

(3) For what sum must a cargo worth \$33,950 be insured at 3% so that in case of loss the owner may recover both the value of the cargo and the premium?

Let the amount of insurance = 100 units of money.

Then the premium = 3 units of money.

The value of the goods = 97 units of money.

97 units of money = \$33,950.

1 unit of money = \$350.

100 units of money = \$35,000.

$\therefore$  the cargo must be insured for \$35,000.

#### Exercise 148.

1. A warehouse valued at \$62,500 was insured for  $\frac{3}{8}$  of its value. The rate of insurance was  $1\frac{1}{4}\%$  for three years, and the cost of the policy and the agent's expenses were \$2.50. What was the amount paid for the insurance?

2. An insurance company took a risk at  $2\frac{1}{4}\%$ , and reinsured  $\frac{3}{8}$  of the risk at 2%. The premium received exceeded the premium paid by \$42. Find the amount of the risk.

3. What will be the cost of insuring a cargo of 24,000 bu. of wheat valued at \$1.05 per bu., the insurance covering  $\frac{4}{5}$  of the value of the cargo, the premium rate being  $1\frac{1}{8}\%$ , and the other expenses of the insurance being  $2\frac{1}{2}\%$  of the premium?

4. A merchant's stock was insured for \$42,000,  $\frac{1}{2}$  of this amount being at  $\frac{7}{8}\%$ ,  $\frac{2}{3}$  of the remainder at  $\frac{3}{4}\%$ , and the remainder at  $\frac{5}{8}\%$ . Find the total amount of premium paid.

5. A merchant insured his stock for \$33,000 for one year at  $\frac{7}{8}\%$ . Six months thereafter the policy was cancelled at the request of the insured. Find the amount of premium returned, the short rate for six months being  $\frac{5}{8}\%$ .

6. A factory and the machinery therein is insured for \$65,000;  $\frac{2}{3}$  of this sum is at  $\frac{3}{4}\%$  premium, and the remainder is at  $\frac{1}{8}\%$ . What is the average rate per cent of premium paid on the whole?

7. A fire insurance company charged \$196.88 for insuring a house for \$17,500. What was the rate per cent of insurance?

8. A merchant's stock was worth \$120,000. He insured it at  $\frac{2}{3}$  its value, paying \$700 premium. What was the rate per cent of insurance? What was the rate in cents per \$100?

9. A shipment of goods is insured for \$7500, and \$18.75 is paid as premium. At that rate, what would be the amount of the premium on \$18,750?

10. For what sum was a house insured if the premium paid was \$17.50 and the rate of insurance  $\frac{1}{2}\%$ ?

11. For what sum was a shop insured if the rate of insurance was 65¢ per \$100 and the premium paid was \$81.25?

12. A fire insurance company received \$350 for insuring a factory at  $1\frac{1}{4}\%$  premium, and charged  $\frac{1}{2}\%$  for insuring a less hazardous property of the same valuation as the factory. What was the amount of the premium paid on the second property?

13. A building and contents are insured as follows: \$12,000 in the first, \$8000 in the second, and \$5000 in the third insurance company. Were a loss to the extent of \$3500 to occur through fire, what portion of the loss should each company bear?

14. Merchandise valued at \$63,000 was insured in the first insurance company for \$15,000, in the second for \$12,000, and in the third for \$8000. If the merchandise is damaged by fire to the extent of \$10,500, how much of the damage should each company pay?

15. A fire insurance company insured a building for \$60,000 at  $\frac{1}{2}\%$  premium, and reinsured  $\frac{1}{2}$  of the risk in another company at  $\frac{3}{8}\%$ , and  $\frac{1}{3}$  of the risk in a third company at  $\frac{1}{4}\%$ . What

amount and what rate of premium did the company net on the remainder of their risk ?

16. A steamboat worth \$ 60,000 is insured in three companies; in two to the amount of \$ 15,000 each, and in the third to the amount of \$ 20,000. For what sum would each company be liable if the vessel were to sustain damage to the extent of \$ 6600 ?

17. For what amount must property worth \$ 7600 be insured, at 5%, so that in case of loss both the premium and the value of the goods may be recovered ?

#### TAXES AND DUTIES

**222.** A **Tax** is a sum of money assessed on persons or property for public purposes.

The tax on property is reckoned at a certain *rate per cent* of the assessed value of the property.

Direct taxes are levied by the state, county, township, city, or the school district.

Some states levy a tax upon each voter, independent of the property he owns. Such a tax is called a **Poll-tax**, and as a rule does not exceed \$2 a year.

Indirect taxes, called **Duties**, are levied by the general government on imported goods.

An **Ad Valorem Duty** is reckoned at a certain *rate per cent* of the cost of the goods in the country from which they have been imported.

A **Specific Duty** is a fixed charge on the quantity of goods without reference to their cost, as a specific tax of one cent a pound.

**223.** The people of a school section wish to build a new school-house, which will cost \$2850. The taxable property

of the section is valued at \$190,000; what will be the rate of taxation, and what will be a man's tax whose property is valued at \$7500?

The tax on \$190,000 = \$2850.

∴ the rate of taxation =  $\$2850 \div \$190,000 = .015$  or  $1\frac{1}{2}\%$ .

∴ the tax on \$7500 =  $1\frac{1}{2}\%$  of \$7500 = \$112.50.

#### Exercise 149

1. State expenses which the government meets by taxation; the county; the township; the village or city; the school district.

2. What is the tax on property assessed at \$6400, the rate of taxation being  $1\frac{1}{2}\%$ ?

3. In a school section a tax of \$4000 is to be raised. If the assessed valuation of the property is \$250,000, what will be the tax on the dollar, and what is A's tax, whose property is valued at \$1800?

4. What is the assessed value of property taxed \$37.50, at the rate of 15 mills on the dollar?

5. What is the assessed value of property taxed \$37.80, at the rate of  $4\frac{1}{2}$  mills on the dollar?

6. A person, after paying an income tax of 16 mills on the dollar, has \$8265.60 left. What is his income?

7. What amount must a town be taxed so that after allowing the collector 3% the net amount realized may be \$24,250?

8. What sum must be assessed to raise \$12,250, the collector's commission being 2%?

9. If it costs 2% to collect, and 5% of the tax assessed is non-collectible, what amount must be levied in order to raise \$27,930?

10. The municipal rates being reduced from  $19\frac{1}{2}$  mills to  $17\frac{1}{2}$  mills on the dollar, my taxes are lowered by \$4.05. For how much am I assessed?

11. In a certain section a school-house is to be built at an expense of \$8400, to be defrayed by a tax upon property valued at \$700,000. What is the rate of taxation to cover both the cost of the school-house and the collector's commission at 4%?

12. The assessed valuation of a town is \$972,250, and the town has 320 polls paying \$1.50 each; what is the rate of taxation when the tax levy is \$19,925? What tax must a person pay whose property is assessed for \$7500, and who pays for one poll?

13. If the assessed value of a town is \$1,260,000, and the town has 420 polls paying \$1.25 each, what is the rate of taxation on property when the tax levy is \$29,925? What does A pay, whose property is assessed at \$8500 and who pays one poll?

14. A house assessed at \$2200 was rented for \$23 a month, the tenant to pay taxes and water-rates. The taxes were  $17\frac{3}{4}$  mills on the dollar, and the water-rates were \$5 per quarter year. How much all together did the tenant pay per year for the house? If the property had cost the landlord \$2500, what rate per cent per year was he receiving on his investment?

15. Paid 30% duty on a watch, and sold it at a loss of 5%; but had it been sold for \$21.06 more, there would have been a gain of  $8\frac{1}{2}\%$ . Find the cost price.

16. Distinguish specific and ad valorem duties. A quantity of raisins invoiced at \$877 cost \$990.25 in store, after paying duty and \$16.12 for freight. Find rate of duty.

17. An importer purchased goods, paying freight 10% and duty 20% on the original outlay; he was obliged to sell the goods at a loss of 20%, but had he received \$585 more than he actually sold them for, he would have made a profit of 4%. Find the original cost of the goods.

18. What is the duty on 600 yd. of cloth invoiced at 6 francs per yd., the duty being 30%? (1 franc = 19.3¢.)



**Miscellaneous Exercise 150**

1. If I buy an article for \$ 3.60 and sell it for \$ 4.20, what is my gain per cent ?
2. If I sell goods for \$ 3360 and gain 12%, what was the cost price ?
3. If 425 yd. of silk be sold for \$ 1657.50, and 20% profit be made, what did it cost per yd. ?
4. If, by selling goods for \$ 1088, I lose 16%, how much per cent should I have lost or gained if I had sold them for \$ 1344 ?
5. A tradesman's prices are 25% above cost price. If he allow a customer 8% on his bill, what profit does he make ?
6. If 8% be gained by selling a piece of ground for \$ 8251.20, what would be gained per cent by selling it for \$ 8404 ?
7. Find the brokerage on \$ 1324 at  $\frac{1}{4}\%$ .
8. Find the brokerage on \$ 375 at 5%.
9. What amount of money was invested, when the broker's charges at  $1\frac{1}{4}\%$  amounted to \$ 150 ?
10. My agent has purchased real estate, on my account, to the amount of \$ 19,384. What is his commission at  $1\frac{1}{2}\%$  ?
11. I send my agent \$ 3654, with instructions to deduct his commission at  $1\frac{1}{2}\%$ , and invest the balance in tea. How much did he invest ?
12. Gave \$ 15,037.50 to a broker to invest, with instructions, after deducting brokerage at  $\frac{1}{4}\%$ , to invest the balance in government bonds. What will be the sum invested, and how much will be the brokerage ?
13. What will be the premium of insurance on the furniture of a house valued at \$ 1500, at  $2\frac{1}{2}\%$  ?
14. What is the premium for insuring a cargo, valued at \$ 16,450, at  $3\frac{1}{4}\%$  ?

15. A person at the age of 40 insures his life in each of two offices for \$4500, the premiums being at the rate of  $3\frac{1}{3}\%$  and  $3\frac{1}{2}\%$  respectively. Find his annual payment.

16. A trader gets 600 bbl. of flour insured for 80% of its cost, at  $2\frac{1}{4}\%$ , paying \$37.80 premium. At what price per bbl. did he purchase the flour?

17. A shipment of dry goods was insured at  $1\frac{3}{5}\%$  to cover  $\frac{1}{5}$  of its value. The premium was \$28. What were the goods worth?

18. A man who owns \$12,750 worth of property pays a tax of \$216.75. Find the rate on the dollar.

19. A certain town has property assessed at \$520,000, and levies a tax of \$7800. What should B pay, whose property is assessed at \$2500?

20. A town has levied a tax of \$7690, which sum includes the amount voted for building a town hall and the collector's fees, at 3%. What was expended on the town hall?

21. What is the rate per cent of commission when I receive \$5 for selling goods to the value of \$180?

22. I sold a quantity of goods for \$273.68, on a commission of  $2\frac{3}{8}\%$ . Find my commission.

23. A and B insure their houses against fire, and A has to pay \$7.50 more than B, who pays \$28.75. Find the value of their houses, the rate of insurance being  $\frac{5}{8}\%$ .

24. A. B. bought goods amounting to \$7460 subject to 25 and 5 off, \$3730 subject to 30 off, and \$1492 subject to 20 and 10 off. Find the net cost of the goods. Were the invoice-clerk to bill A. B. with goods amounting to \$12,682 subject to 30 off, what would be the amount of the error in the net cost of the goods?

25. A mixture of coffee and chiccory in the proportion of 8 parts of coffee to 1 part of chiccory is sold at 35¢ per lb., being an advance of 40% on the cost. The chiccory cost 9¢ per lb. Find the cost of the coffee per lb.

26. A man bought a house and lot for \$4750. After spending \$1143 on repairs and improvements, and paying \$128 for taxes and other expenses, he sold the property for \$6800. What rate per cent of profit did his investment yield him?

27. In an examination A obtained 78% of the full number of marks, beating B by 16% of the full number. If A received 975 marks, how many did B receive? What percentage of B's number was A's?

28. By selling a certain book for \$3.96, I would lose 12% of the cost. What advance on this proposed selling price would give a profit of 12% of the cost? What rate per cent on the proposed selling price would this advance be?

29. Goods are sold at a loss of 15% on the cost. By what percentage of itself should the selling price be advanced to yield a profit of 15% on the cost?

30. A man having bought a certain quantity of goods for \$150, sells  $\frac{1}{3}$  of them at a loss of 4%. By what increase per cent must he raise that selling price that by selling the whole at that increased rate he may gain 4% on his entire outlay?

31. The cost of freight and insurance on a certain quantity of goods was 15%, and that of duty 10% on the original outlay. The goods were sold at a loss of 5%, but had they brought \$3 more there would have been a gain of 1%. How much did they cost?

32. A man began business with a certain capital; he gained 20% the first year, which he added to his capital, and  $37\frac{1}{2}\%$  the second year, which he added to his capital; in the third year he lost 40%; had he received \$600 more for the goods sold the last year, he would have cleared in the three years 2 per cent of his original capital. Find the capital with which he commenced business.

33. A merchant bought 400 lb. of tea and 1600 lb. of coffee, the cost of the latter per lb. being  $16\frac{2}{3}\%$  that of the former;

he sold the tea at a profit of  $33\frac{1}{3}\%$ , and the coffee at a loss of 20%, gaining, however, on the whole \$60. Find his buying prices and his selling prices.

34. A sells a quantity of wheat at \$1 per bu. and gains 20%. Afterwards he sold some of the same wheat to the amount of \$37.50 and gained 50%. How many bu. were there in the last lot and at what rate per bu. did he sell it?

35. A person marks his goods so that he may allow a discount of 4%, and still make a profit of 15%. What must be the marked price of an article that cost him \$4.80?

36. A manufacturer who employed men at \$1.60 a day found that he could save 15% by employing women. What wages were paid the latter, supposing a man could do  $\frac{1}{3}$  more than a woman in the same time?

37. A merchant buys goods; the cost of freight is 8%, and that of insurance 12% on the original outlay; he is obliged to sell them at a loss of 7%; but if he had received \$5.10 more for them he would have gained  $11\frac{1}{2}\%$ . Find the original outlay.

38. A merchant sells 50 yd. of broadcloth at a gain of 15%, and 75 yd., which cost the same per yd., at a gain of 10%, and finds that if he had sold the whole at a uniform gain of  $12\frac{1}{2}\%$ , he would have received \$2.25 more than he actually did receive. What was the cost price per yd.?

39. A man buys goods for a certain sum, and marks  $\frac{1}{3}$  of them at a profit of 24%, and  $\frac{2}{3}$  of them at a profit of 36%; but had he marked  $\frac{2}{3}$  of them at 24% gain, and  $\frac{1}{3}$  at 36% gain, he would have realized \$240 less than before. Find the cost of the goods.

40. A wheat buyer sold  $\frac{1}{4}$  of his wheat at a certain gain per cent,  $\frac{1}{3}$  of it at a gain of twice the former rate per cent, and the remainder at a gain per cent of 3 times the first gain. If the gain on the entire stock was 26%, what did he gain on each part?

If he gained 5% on the first part, what was the entire gain per cent?

41. A merchant wishes to mark some goods which cost \$1.20 per yd., so that after making a reduction of 20% off the marked prices, he may yet gain 10%. At what price per yd. must he mark the goods?

42. Sold goods to a certain amount on a commission of 5%, and having remitted the net proceeds to the owner, received for prompt payment  $\frac{1}{2}\%$ , which amounted to \$24.22 $\frac{1}{2}$ . What was the selling price of the goods?

43. My agent sold a quantity of flour for \$2550 on a commission of 3%, and invested the proceeds (after taking out his commission) in tea on a commission of 2% on the price paid for the tea. Find how much he paid for the tea and also his total commission.

44. At 2 $\frac{1}{2}\%$ , for what must property worth \$3600 be insured, so that in the event of loss the worth of the goods and the premium of insurance may be recovered?

45. What sum must be insured on a house worth \$665, so that in case of loss the owner may receive  $\frac{4}{7}$  of this sum, and also  $\frac{5}{8}$  of the premium, which was at 6%?

46. What single discount is equivalent to successive discounts of 20% and 10%?

47. Show that successive discounts of specified rates may be taken off a list price in any order without affecting the net price. Thus 20 and 10 off is equivalent to 10 and 20 off, so also 30 and 10 and 5 off, 10 and 30 and 5 off, and 5 and 30 and 10 off are all equivalent.

48. An agent sold 6 mowing-machines at \$120 each, and 12 at \$140 each. He paid for transportation \$72, and, after deducting his commission, remitted \$2208 to his employer. What was the rate of commission?

49. A man allows his agent 5% of his gross rentals, and receives a net rental of \$ 3488.40. If the gross rental is 6% of the value of the property, what is the value of the property?

50. A shipment of goods is insured for \$ 6000, which sum covers the value of the goods, the premium at  $1\frac{1}{8}\%$  and \$ 2.50 for expenses. What was the value of the goods?

## CHAPTER XIV

### INTEREST

**224.** Interest is money paid for the *use* of money.

The **Principal** is the sum loaned.

The **Amount** is the sum of the principal and interest.

The **Rate of Interest** is always expressed as a rate per cent of the principal.

The unit of time is 1 yr.

**225.** (1) What is the interest on \$638 for 1 yr. at 6%?

$$\begin{array}{r} \$638 \text{ principal} \\ .06 \text{ rate per unit} \\ \hline \$38.28 \text{ interest for 1 yr.} \end{array}$$

$\therefore$  the interest on \$638 for 1 yr. at 6% = 6% of \$638 = \$38.28.

(2) Find the interest on and the amount of \$473.28 for 81 da. at 7%.

$$\begin{array}{r} \$473.28 \text{ principal} \\ .07 \text{ rate} \\ \hline \$33.1296 \text{ interest for 1 yr.} \\ \$33.13 \text{ interest for 1 yr.} \\ 9 \\ \hline 40) \$298.17 \\ 7.45 \text{ interest for 81 da.} \\ \hline 473.28 \\ \$480.73 \text{ amount} \end{array}$$

The interest for 1 yr. = 7% of \$473.28 = \$33.13.

The interest for 81 da. =  $\frac{81}{360}$  or  $\frac{9}{40}$  of \$33.13 = \$7.45.

The amount = \$473.28 + 7.45 = \$480.73.

(3) Find the amount of \$385.35, from July 7, 1895, to Oct. 13, 1895, at  $7\frac{1}{2}\%$ .

	mo.	da.
Oct. 13 =	10	13
July 7 =	7	7
The time =	3	$6 = 3\frac{1}{2}$ mo. = $\frac{4}{15}$ yr.

$$\text{The rate} = \frac{4}{15} \times \frac{1}{2} \% = 2 \%$$

$$\text{The interest} = 2 \% \text{ of } \$385.35 = \$6.71.$$

$$\text{The amount} = \$385.35 + \$6.71 = \$392.06.$$

### 226. Six Per Cent Method.

The interest at 6% for 1 yr. = .06 of the principal.

The interest at 6% for 1 mo. =  $\frac{1}{12}$  of .06 or .005 of the principal.

The interest at 6% for 1 da. =  $\frac{1}{360}$  of .005 or .000 $\frac{1}{8}$  of the principal.

Find the interest on \$435 for 9 mo. 24 da. at 6%.

$$\text{The interest for 9 mo.} = 9 \times .005 = .045$$

$$\text{The interest for 24 da.} = 24 \times .000\frac{1}{8} = .004$$

$$\text{The interest for 9 mo. 24 da.} = .049$$

$$\therefore \text{the interest} = .049 \times \$435 = \$21.315.$$

To find the interest at any other rate per cent, divide the interest at 6% by 6 and multiply by the given rate per cent.

The interest at  $7\frac{1}{2}\%$  may be found by increasing the interest at 6% by  $\frac{1\frac{1}{2}}{6}$  or  $\frac{1}{4}$ ; that at  $5\frac{1}{2}\%$  by diminishing the interest at 6% by  $\frac{1}{6}$  or  $\frac{1}{12}$ .

By what fraction must the interest at 6% be increased in order to give the interest at each of the following rates: 7%, 8%, 9%,  $6\frac{1}{4}\%$ ,  $6.2\%$ ?

By what fraction must the interest at 6% be diminished in order to give the interest at each of the following rates: 5%, 4%, 3%,  $4\frac{1}{2}\%$ ,  $4\frac{1}{4}\%$ ,  $5\frac{3}{4}\%$ ?

### Exercise 151

Find the interest on:

- |                              |                                              |
|------------------------------|----------------------------------------------|
| 1. \$449 for 1 yr. at 5%.    | 5. \$587.50 for 5 mo. at 6%.                 |
| 2. \$757 for 1 yr. at 4%.    | 6. \$628.90 for 9 mo. at $4\frac{1}{2}\%$ .  |
| 3. \$643.17 for 1 yr. at 7%. | 7. \$323.75 for 60 da. at 8%.                |
| 4. \$725 for 4 mo. at 8%.    | 8. \$958.50 for 90 da. at $4\frac{1}{4}\%$ . |



9. \$2865 for 33 da. at 6%.      12. \$225.90 for 63 da. at 7%.  
10. \$312.80 for 93 da. at 6%.      13. \$390.50 for 93 da. at 6%.  
11. \$612.94 for 33 da. at  $7\frac{1}{2}\%$ .      14. \$8396.40 for 123 da. at 8%.  
15. \$4087.50 for 1 mo. 3 da. at 9%.  
16. \$1465.53 for 3 mo. 3 da. at 5%.  
17. \$1350 for 3 mo. 21 da. at 7%.  
18. \$295.36 for 57 da. at 6.2%.  
19. \$1200 from May 7 to June 6 at 7%.  
20. \$975.65 from Sept. 16 to Dec. 8 at  $6\frac{1}{4}\%$ .  
21. \$450 from Sept. 4 to Oct. 27 at 7%.  
22. \$79.50 from Dec. 23 to Feb. 20 at  $7\frac{1}{2}\%$ .  
23. \$586.67 from Jan. 15, 1892, to May 1, 1892, at 8%.  
24. State how to find the simple interest when the principal, rate per cent, and time are given.  
25. Name the terms in problems in Profit and Loss, Commission and Insurance, which correspond to principal and rate per cent of interest.  
26. Find the relation between the interest, principal, and amount, when the time is 3 mo., and rate 8%; time 120 da., rate 9%.  
27. Find the amount of \$473.28 for 3 mo. at  $\frac{1}{2}\%$  per month.  
28. Find the amount of \$885.85 for 1 mo. 15 da. at 5%.  
29. Find the amount of \$628.25 for 185 da. at  $4\frac{1}{2}\%$ .  
30. Find the amount of \$935.68 for 66 da. at  $6\frac{1}{2}\%$ .  
31. Find the amount of \$147.50 for 93 da. at 7%.  
32. Find the amount of \$250 from July 9 to Aug. 18 at 8%.  
33. Find the amount of \$2394 from May 8 to Sept. 21 at 4%.  
34. Find the amount of \$5246 from March 1 to Aug. 3 at 5%.  
35. Find the amount of \$230.80 from Jan. 4, 1896, to June 23, 1896, at 6%.  
36. Find the amount of \$657.60 from Aug. 9 to Dec. 5 at 8%.

37. A person loaned \$ 480 for 2 mo. and 13 da. at 9%. What interest did he receive ?

38. On March 20, a merchant sold goods to the value of \$ 1168, and received a note, due June 8, next, for that sum with interest at 7% per annum. For what amount was the note drawn ?

39. A debt of \$ 175 became due on June 13, after which date interest was charged at the rate of 8%. What must be paid to settle the debt Sept. 14 ?

40. A owes \$ 15,000 bearing interest at 5% per annum; he pays at the end of each year for interest and part payment of principal \$ 2500. Find the amount of his debt at the end of the third year.

41. A man engaged in business with a capital of \$ 10,920 is making  $12\frac{1}{2}\%$  per annum on his capital, but on account of ill health he quits the business and loans his money at  $7\frac{3}{4}\%$ . How much does he lose by the change in 2 yr.  $5\frac{1}{2}$  mo. ?

42. \$ 420.

CHICAGO, June 4, 1895.

Sixty days from date I promise to pay Samuel Jones, or order, four hundred and twenty dollars, with interest at six per cent, value received.

RICHARD WALSH.

What is the amount of this note on maturity, 63 da. after June 4 ?

#### EXACT INTEREST

227. In order to find the *exact interest* we must reckon 365 da. to a year. Exact interest is used by the United States Government and sometimes in business transactions.

228. The exact interest at 5% for 1 da. is  $\frac{5}{365}$ , or  $\frac{1}{73}$  of the principal.

The common interest is  $\frac{5}{360}$  or  $\frac{1}{72}$  of the principal. Therefore the exact interest is  $\frac{1}{73} \div \frac{1}{72}$  or  $\frac{72}{73}$  of the common interest. Hence the exact interest is equal to the common interest diminished by  $\frac{1}{73}$  of itself.

229. Find the exact interest on \$ 4250 from May 12 to Oct. 3 at 7%.

The number of days from May 12 to Oct. 3 =  $19 + 30 + 31 + 31 + 30 + 3 = 144$ .

The interest on \$4250 at 7% for 1 yr. = \$297.50.

The interest on \$4250 at 7% for 144 da. =  $\frac{144}{365}$  of \$297.50 = \$117.37.

**Exercise 152**

Find the exact interest on :

1. \$2450 for 146 da. at 6%.
2. \$3475 for 292 da. at 7%.
3. \$1560 for 60 da. at 5%.
4. \$629 for 113 da. at 6%.
5. \$1400 from July 6 to Dec. 4 at  $5\frac{1}{2}\%$ .
6. \$1850 from March 1 to Aug. 6 at  $6\frac{1}{2}\%$ .

**230.** In Exercise 151 we were given the principal, rate per cent, and time, and were required to find the interest or the amount. In the following exercise we shall have given the principal, interest or amount, and the time, and will be required to find the rate per cent.

**231.** (1) At what rate per cent will \$480 yield \$18.20 interest in 7 mo.?

The interest on \$480 for 7 mo. = \$18.20.

The interest on \$480 for 1 mo. = \$2.60.

The interest on \$480 for 1 yr. = \$31.20.

$\therefore$  the rate per cent =  $\$31.20 \div \$480 = .06\frac{1}{2}$  or  $6\frac{1}{2}\%$ .

(2) At what rate per cent must I loan \$2840 for 145 da. to amount to \$2922.36?

The interest =  $\$2922.36 - \$2840 = \$82.36$ .

The interest for 145 da. or  $\frac{145}{365}$  yr. = \$82.36.

The interest for 1 yr. =  $\$82.36 \times \frac{365}{145} = \$204.48$ .

$\therefore$  the rate per cent =  $\$204.48 \div \$2840 = .072$  or  $7\frac{1}{4}\%$ .

**Exercise 153**

1. A gentleman invests \$2500 for his son in order to give him a yearly income of \$175. Find the rate per cent.

Find the rate per cent :

2. When the interest on \$1200 for 8 mo. is \$48.

3. When the interest on \$640 for 9 mo. is \$35.

4. When the interest on \$585 for 1 mo. 18 da. is \$4.94.

5. When a loan of \$600 amounts to \$626.75 in 7 mo. 4 da., what rate per cent is charged ?

6. A man pays \$14.91 for the use of \$568 for 4 mo. 15 da. Find the rate per cent.

7. A man lent \$4800 for 6 mo. 6 da., and at the expiration of the time received in payment of interest and principal \$4955. Find the rate per cent.

8. At what rate will \$438 borrowed on April 17 amount at simple interest to \$445.519 on July 29 next following, if exact interest is reckoned ?

**232.** To find the time when we are given the principal, interest, or amount, and rate per cent.

In what time will the interest on \$845 be \$32.95½ at 6½% ?

The interest on \$845 at 6½% for 1 yr. = \$54.92½.

Therefore the fraction of a year =  $\$32.955 \div \$54.925 = \frac{3}{5}$ .

∴ the time =  $\frac{3}{5}$  yr. or 7 mo. 6 da.

**Exercise 154**

1. In what time will the interest on \$750 at 7% equal \$26.25 ?

2. In what time will \$400 amount to \$415 at 5% per annum ?

3. A man invests \$4760 at 8%, in order that his son may have an income of \$95.20 at certain equal intervals of time. How often is the interest payable a year ?

4. A gentleman gives his note for \$ 350, together with interest at  $4\frac{1}{2}\%$ . If he pays \$ 362.60 to settle the note, when is it paid?

5. A person leaves unpaid a sum of money, on which he pays  $7\frac{1}{2}\%$  interest, until the interest equals  $\frac{1}{20}$  of the principal. Find the time.

6. A principal of \$ 1200 was loaned May 12, 1892, at  $8\%$ . At what date did it amount to \$ 1216.80?

7. In what time will \$ 273.85 yield \$ 8.86 simple interest at  $6\%$ ?

8. In how many da. will \$ 733.65 amount to \$ 743.70 at  $5\%$  simple interest?

9. A debt of \$ 175 became due on June 13, after which date interest was charged at  $7\%$  per annum; when the debt was paid the interest accrued on it was \$ 4.10. When was the debt paid?

10. In what time will \$ 143 amount to \$ 150 at  $7\%$  interest?

**233.** To find the principal when the interest, time, and rate are given.

Find the principal that will produce \$ 40.77 in 9 mo. at  $8\%$ .

The interest for 9 mo. or  $\frac{3}{4}$  yr. = \$ 40.77.

The interest for  $\frac{1}{4}$  yr. = \$ 13.59.

The interest for 1 yr. = \$ 54.36.

$8\%$  or .08 of the principal = \$ 54.36.

$\therefore$  the principal = \$ 54.36  $\div$  .08 = \$ 679.50.

**Exercise 155**

1. What principal will produce \$ 60 in 216 da., at  $7\frac{1}{2}\%$  per annum?

2. A man borrowed money at  $6\%$ , and paid \$ 323.70 interest a yr. Find what sum he borrowed.

3. A man loans money at 8% per annum, interest payable semiannually. If his semiannual interest is \$38.40, find the sum loaned.

4. A man left to his wife a yearly income of \$1750, to the oldest son \$1540 yearly, and to the youngest \$1260 yearly. Find what sum must be invested at  $6\frac{1}{2}\%$  to produce these amounts.

5. What principal will yield \$43.25 interest in  $\frac{1}{4}$  yr. at  $5\frac{1}{2}\%$ ?

**234.** To find the principal when the amount, time, and rate per cent are given.

Find the principal that will amount to \$1312.50 in 8 mo. at  $7\frac{1}{2}\%$ .

The interest on \$1 for 8 mo. at  $7\frac{1}{2}\% = $.05.$

The amount of \$1 for 8 mo. at  $7\frac{1}{2}\% = $1.05 = 1.05$  of \$1.

Hence \$1312.50 is the amount of  $\$1312.50 \div 1.05 = \$1250.$

$\therefore$  the principal = \$1250.

### Exercise 156

1. Find what sum would pay now a debt of \$450 due in 6 mo. at 6% per annum.

2. What must be paid now to cancel a debt of \$1368.25 9 mo. before it is due, money being worth 7%?

3. Which is cheaper, lumber bought at \$35 a thousand on 9 mo. credit, or at \$34.30 on 6 mo. credit, money being worth 6%?

4. I bought a lot, paying \$400 cash, and the balance \$800 in 9 mo. What is the cash value of the lot, money bringing 6%?

5. What principal will amount to \$1000 in 4 mo. at  $4\frac{1}{2}\%$ ?

6. What principal will amount to \$73.56 in 63 da. at 8%?

7. A debt due on March 3 was not paid, and interest at  $6\frac{1}{4}\%$  was charged on it from that date. On June 9 following, the debt amounted to \$100. What was the sum due on March 3?

8. A merchant bought 500 bbl. of flour at \$6.25 a bbl. on a credit of 8 mo. He sold it at \$6.50 a bbl. on a credit of 4 mo. What was his net cash gain, money being worth 6%?

9. A merchant borrows \$1600 for 1 yr. at 7%. Find what he owes at the end of the yr. In case he pays only \$12 interest, how much will he owe at the beginning of the next yr.? What will he owe at the end of the yr.?

10. If I borrowed \$1200 Jan. 1, 1894, at 6%, what would I owe Jan. 1, 1895? If I kept the money until Jan. 1, 1896, what would I then owe?

## BANK DISCOUNT

**235.** A merchant, who desires to obtain a loan of \$800 for 90 da., makes a note and takes it to the bank, which deducts the interest on \$800 for 93 da. at a certain rate per cent, which varies from time to time. This bank gives him the *proceeds*, and collects the \$800 at the end of 93 da.

The 3 da. added to the specified time are called days of grace, which must elapse before payment is *due*.

California, Idaho, New Jersey, New York, Oregon, Utah, Vermont, and Wisconsin have abolished *days of grace*.

**236.** **Bank Discount** is, therefore, simple interest collected *in advance* upon the sum due on a note at its maturity.

Nearly all notes specify the place of payment. In case the place of payment is not specified in the note, it is to be paid at the *business office* of the maker of the note.

**237.** \$450.75.

CHICAGO, July 3, 1896.

Sixty days after date I promise to pay to the order of James Smith, four hundred fifty and  $\frac{75}{100}$  dollars at the First National Bank. Value received.

HORACE WARD.

Discounted July 3, at 6%. Find proceeds.

The day of maturity = 63 da. after July 3 = Sept. 4.

The discount = the interest on \$450.75 at 6% for 63 da. = \$4.73.

The proceeds = \$450.75 - \$4.73 = \$446.02.

**238.** The **Day of Maturity** is the day on which the note becomes legally due.

The **Proceeds of a Note** is the sum of money received for it when discounted.

It is found by subtracting the discount from the value of the note at maturity.

The **Time to run** is the time between the day on which the note is discounted and the day of maturity.

### Exercise 157

1. \$600.

CHICAGO, July 6, 1896.

Thirty days after date I promise to pay to George Boies, or order, six hundred dollars, value received.

ROBERT BROWN.

Discounted at 7%, July 6, 1896. Find proceeds.

- | Face of Note                                                      | Date of Note   | Time    | Rate of Discount |                |
|-------------------------------------------------------------------|----------------|---------|------------------|----------------|
| 2. \$312.80;                                                      | May 13, 1895;  | 90 da.; | 6%.              | Find proceeds. |
| 3. \$225.90;                                                      | June 14, 1896; | 2 mo.;  | 7%.              | Find proceeds. |
| 4. \$100.00;                                                      | Feb. 12, 1896; | 30 da.; | 5%.              | Find proceeds. |
| 5. State how to find the proceeds of any note discounted at once. |                |         |                  |                |

- | Face of Note                                                                                                           | Date of Note  | Time    | Rate of Discount |                |
|------------------------------------------------------------------------------------------------------------------------|---------------|---------|------------------|----------------|
| 6. \$1.00;                                                                                                             | Jan. 7, 1894; | 57 da.; | 6%.              | Find proceeds. |
| 7. In question 6 what face value would give \$99 proceeds? \$198? \$495? Prove your face value correct by discounting. |               |         |                  |                |
| 8. Write the notes corresponding to questions 2 and 3.                                                                 |               |         |                  |                |

9. \$390 $\frac{50}{100}$ .

SPRINGFIELD, ILL., May 1, 1890.

Three months after date I promise to pay to the order of Thomas A. Stuart, three hundred ninety and  $\frac{50}{100}$  dollars. Value received.

JAMES HENDERSON.

Discounted May 1, 1890, at 6%. Find proceeds.



**239.** (1) \$712.65.

CHICAGO, July 6, 1895.

Sixty days from date I promise to pay George Wilson, or order, seven hundred twelve and  $\frac{65}{100}$  dollars, for value received.

SAMUEL JONES.

Discounted at 7%, Aug. 6, 1895.

In the above note, find the *day of maturity*, the *time to run*, the *discount*, and the *proceeds*.

The day of maturity = 63 da. after July 6 = Sept. 7, 1895.

The time to run = the number of days between Aug. 6 and Sept. 7.  
= 32 da. =  $\frac{32}{360}$  yr.

The discount = the interest on \$712.65 for 32 da. at 7% = \$4.43.

The proceeds = \$712.65 - \$4.43 = \$708.22.

(2) \$450.76.

ST. LOUIS, May 5, 1895.

Three months after date, for value received, I promise to pay Thomas King, or order, four hundred fifty and  $\frac{76}{100}$  dollars, at the First National Bank, with interest at 6%.

ARTHUR HILL.

Discounted July 1, 1895, at 8%.

The day of maturity = 3 mo. 3 da. after May 5 = Aug. 8, 1895.

The amount of the note, Aug. 8, 1895 = the amount of \$450.76 for 3 mo.  
3 da. at 6% = \$457.75.

The time to run = the number of days between July 1 and Aug. 8 = 38 da.

The discount = the interest on \$457.75 for 38 da. at 8% = \$3.87.

The proceeds = \$457.75 - \$3.87 = \$453.88.

In the following exercise find the *day of maturity*, the *time to run*, the *discount*, and the *proceeds*.

### Exercise 158

1. \$2400.

CLEVELAND, O., March 3, 1894.

Three months after date I promise to pay Ralph Barker, or order, twenty-four hundred dollars, value received.

ROBERT PETERSON.

Discounted at 7%, May 7.

2. A note for \$572.80 drawn on June 13, and payable 4 mo. after date, was discounted at 7% on June 27. Find the proceeds.

3. \$2400.

CLEVELAND, O., March 3, 1894.

Three months after date I promise to pay Ralph Barker, or order, twenty-four hundred dollars, for value received, with interest at 6%.

ROBERT PETERSON.

Discounted at 7%, May 7.

4. State business transactions which may have preceded the giving of the notes in questions 1, 2, and 3.

5. On July 7, James Monroe bought a farm from John Harris, paying \$2000 cash, and giving his note, without interest, for \$1200, payable in 60 da. Write the note.

	Face of Note	Date of Note	Time	Date of Disc.	Rate of Disc.
6.	\$312.80	May 13, 1890	90 da.	May 13	6½%
7.	\$975.65	Sept. 5, 1892	3 mo.	Sept. 16	7%
8.	\$450.00	Aug. 28, 1891	60 da.	Sept. 4	7%
9.	\$79.50	Dec. 17, 1889	2 mo.	Dec. 23	7½%
10.	\$586.67	Dec. 28, 1891	4 mo.	Jan. 15, 1892	8%

11. \$2480.

BUFFALO, N. Y., Nov. 19, 1892.

Six months after date I promise to pay Alfred Jameson, or order, two thousand four hundred and eighty dollars, value received, with interest at 5%.

WILLIAM O'CONNOR.

Discounted at 6%, Jan. 4, 1893.

12. \$2065.76.

NEW ORLEANS, June 4, 1895.

Ninety days after date I promise to pay to the order of Edgar Johnston, two thousand sixty-five and  $\frac{76}{100}$  dollars, for value received, with interest at 6%.

ALEXANDER GRANT.

Discounted at 8%, July 4, 1895.

13. State how to find the proceeds of a note, not bearing interest, when discounted. What change is to be made in the solution when the note bears interest?

14. Find the proceeds of a note payable in 87 da., whose face value is \$1, discounted immediately at 8%.

15. In question 14, what would have been the face value of the note if the proceeds had been \$98? \$980? \$196? \$392?

**240.** (1) Find the face of a note payable in 60 da., that will realize \$840 when discounted at  $6\frac{1}{2}\%$ .

Let us consider a similar note whose face is \$1.

The discount on \$1 for 63 da. at  $6\frac{1}{2}\%$  = \$.011375.

The proceeds of a note whose face is \$1 = \$1 - .011375 = \$.988625.

Hence .988625 of the face = \$840.

$\therefore$  the face = \$840  $\div$  .988625 = \$849.66.

#### PROOF

The discount on \$849.66 for 63 da. at  $6\frac{1}{2}\%$  = \$9.66.

The proceeds = \$849.66 - \$9.66 = \$840.

$\therefore$  \$849.66 is the correct answer.

(2) In solving the preceding question it is unnecessary to carry out the discount on \$1 beyond the *fourth* figure in the decimal.

Thus the discount on \$1 = \$.0114.

The proceeds of \$1 = \$.9886.

The face of the note = \$840  $\div$  .9886 = \$849.68.

This is correct within 2 ¢.

#### Exercise 159

1. Find the face value of a note for 27 da. that will realize \$1990 when discounted at 6%.

2. Write the note corresponding to question 1, and prove that the face as written in the note is correct.

3. For how much must a 2 months' note be drawn so that when discounted at 7% it may yield \$500? Prove your answer correct and write the note.

4. A gentleman wishes to borrow \$ 800 at a bank. For what sum must he give a 60 days' note which is discounted at  $7\frac{1}{2}\%$ ?

5. What must be the face of a 4 months' note discounted at 8%, to realize \$ 89.50?

6. For what sum must a note be drawn in order that if discounted 89 da. before maturity, the proceeds may be \$ 425; the rate of discount being 7%?

7. What must be the face of a note so that when discounted at a bank for 4 mo. and 9 da. at 9%, it will give \$ 240?

8. State how to find the face value of a note, when the term of discount, the rate per cent, and the proceeds are given.

9. Make a question in which it is required to find the face of a note—given the proceeds, rate of discount, and term of discount.

10. I owe a man \$ 575, and gave him a note at 60 da. What must be the face of the note to pay him the exact debt, when discounted at (bank discount)  $1\frac{1}{2}\%$  a month?

11. A sold B a bill of goods amounting to \$ 7600, but B, not having the money, gave A a note for 3 mo., which, when discounted at the bank at 8%, paid the debt. Required the face value of the note.

#### Miscellaneous Exercise 160

1. Find the interest on \$ 794.35 for 188 da. at 5%.

2. To what sum would \$ 87.68 amount in 97 da. at  $6\frac{1}{2}\%$  interest?

3. A certain sum amounts to \$ 1488 in 8 mo., and \$ 1530 in 15 mo., simple interest. What is the rate per cent?

4. A merchant borrowed \$ 1680 June 16, and \$ 1728 Sept. 28; the merchant repays the whole sum, with interest, Jan. 2 next. Find the amount repaid, interest being  $7\frac{1}{2}\%$  per annum.

5. Bought 9000 bu. of wheat at 75¢ a bu., payable in 6 mo.; I sold it immediately for 72¢ a bu., cash, and put the money at interest at 6%. At the end of the 6 mo. I paid for the wheat. Did I gain or lose by the transaction, and how much?

6. What is the bank discount and proceeds of a note of \$1168, drawn Jan. 18 at 11 mo., discounted at the bank May 20 at 6%?

7. I owe a bill amounting to \$219.75, and I give my note for 60 da. How must I draw it to cover the discount at  $6\frac{1}{2}\%$ ?

8. A. B. has a note of \$800 to pay at the Merchants' National Bank. At the time of its maturity he pays \$200, and gives a note for 3 mo., days of grace included, for the balance. The rate of discount being 8% per annum, what was the face of the note?

9. A note for \$1750 was drawn July 10 for 4 mo. It was discounted Sept. 2 at the Granite Bank at  $7\frac{1}{2}\%$  per annum. What sum was received for it?

10. A note is drawn for 3 mo., days of grace included, and when discounted at 7% per annum at the Farmers' Bank it realizes \$4.55 less than its face value. What is the face value of the note?

11. Find the proceeds of the following joint note discounted in New York Dec. 18, 1896, at  $7\frac{1}{2}\%$ .

\$347 $\frac{50}{100}$ .

NEW YORK, Dec. 18, 1896.

Ninety days after date we jointly and severally promise to pay to the order of Jno. Locke & Co., three hundred and forty-seven  $\frac{50}{100}$  dollars, at the Standard Bank. Value received.

ISAAC HARPER.

A. C. EARLY.

12. For how much must a 90-day note be drawn to realize \$190 when discounted at 6%?

13. Find the proceeds of a note whose face value is \$950, payable in 3 mo. from Feb. 1, 1896, and discounted Feb. 6, 1896, at 7%.

14. A note for \$360 was discounted 40 da. before maturity and the proceeds were \$356.80. What was the rate of discount, there being no exchange?

15. The proceeds of a note for \$137.50, discounted 40 da. before maturity, were \$136.40. What was the rate of discount charged on the face of the note?

16. June 18, 1895, a merchant purchased goods amounting per catalogue prices to \$647.80, subject to 25 and 5 off. He was allowed 3 months' credit, after which he was charged interest at 8%. Find the amount of the account Feb. 21, 1896.

#### \* PARTIAL PAYMENTS

**241.** A **Partial Payment** is a payment of only a part of a debt.

An **Indorsement** is an acknowledgment of the receipt of a partial payment, written on the back of a note, stating the amount and the date of payment.

**242.** The following is the method of solving questions in partial payments when the note is paid in full in a year or less:

What amount is due Dec. 20, 1895, on a note for \$1600, dated Jan. 14, 1895, with interest at 6%, on which the following payments are indorsed:

Feb. 20, 1895, \$400; May 5, 1895, \$200; Aug. 2, 1895, \$600.

The time between Jan. 14 and Dec. 20 = 11 mo. 6 da. (\$1600).

The time between Feb. 20 and Dec. 20 = 10 mo. (\$400).

The time between May 5 and Dec. 20 = 7 mo. 15 da. (\$200).

The time between Aug. 2 and Dec. 20 = 4 mo. 18 da. (\$600).

The amount of \$1600 for 11 mo. 6 da. @ 6% = \$1689.60.

The amount of \$400 for 10 mo. @ 6% = \$420.

The amount of \$200 for 7 mo. 15 da. @ 6% = \$207.50.

The amount of \$600 for 4 mo. 18 da. @ 6% = \$613.80.

The total amount of the payments Dec. 20 = \$420 + \$207.50 + \$613.80 = \$1241.30.

$\therefore$  the amount due Dec. 20 = \$1689.60 - \$1241.30 = \$448.30.

**243.** This solution is in accordance with the **Merchants' Rule**, which is as follows: *When the note is paid in full in a year or less, find the amount of the note and of each payment at the date of settlement. From the amount of the note subtract the sum of the amounts of the payments. The difference thus found is the sum due on the date of settlement.*

#### Exercise 161

1. What is due Nov. 1, 1895, on a note for \$2000, dated Feb. 1, 1895, with interest at 6%, on which the following payments are indorsed: July 1, 1895, \$600; Sept. 1, 1895, \$800?

2. On a note for \$2400, dated Sept. 8, 1892, and drawing 6%, the following payments were indorsed: Oct. 8, 1892, \$600; Jan. 8, 1893, \$250; July 15, 1893, \$850. What was due on the note Sept. 8, 1893?

3. A note for \$1500, dated April 14, 1894, with interest at 5%, bears the following indorsements: June 8, 1894, \$450; Oct. 29, 1894, \$750. What is due April 14, 1895?

4. On a note for \$1650, dated Oct. 9, 1893, and bearing interest at  $5\frac{1}{2}\%$ , the following payments were made: Dec. 5, 1893, \$240; Feb. 25, 1894, \$320; June 30, 1894, \$300. What is due Aug. 15, 1894?

5. How much was due on the following note, on Dec. 31, 1889?

\$950.

NEW YORK, Jan. 2, 1889.

For value received, I promise to pay James Brown or order, on demand, nine hundred and fifty dollars, with interest from date at 6% per annum.

GEORGE THOMPSON.

On this note the following payments were indorsed: Feb. 22, 1889, \$ 225; May 22, 1889, \$ 85; July 21, 1889, \$ 125; Sept. 21, 1889, \$ 325.

**244.** The following is the method of solving questions in partial payments when the note runs longer than a year:

A note for \$ 2400, dated March 15, 1893, and drawing interest at 6%, had the following payments indorsed upon it: June 30, 1893, \$ 250; Sept. 12, 1893, \$ 25; April 9, 1894, \$ 450; Sept. 14, 1894, \$ 84.50. How much was due on the note March 4, 1895?

yr.	mo.	da.	
1893	6	30	(June 30)
1893	3	15	(March 15)
	3	15	(\$ 250)

1893	9	12	(Sept. 12)
1893	6	30	(June 30)
	2	12	(\$ 25)

yr.	mo.	da.	
1894	4	9	(April 9)
1893	9	12	(Sept. 12)
	6	27	(\$ 450)

1894	9	14	(Sept. 14)
1894	4	9	(April 9)
	5	5	(\$ 84.50)

yr.	mo.	da.	
1895	3	29	(March 29)
1894	9	14	(Sept. 14)
	6	15	

The interest on \$ 2400 from March 15, 1893, to June 30, 1893 = \$ 42.

The amount due June 30, 1893 = \$ 2400 + \$ 42 - \$ 250 = \$ 2192.

The interest on \$ 2192 from June 30, 1893, to Sept. 12, 1893 = \$ 26.30.

The interest on \$ 2192 from Sept. 12, 1893, to April 9, 1894 = \$ 75.62.

The amount due April 9, 1894 = \$ 2192 + \$ 26.30 + \$ 75.62 - \$ 25 - \$ 450 = \$ 1818.92.

The interest on \$ 1818.92 from April 9, 1894, to Sept. 14, 1894 = \$ 46.99.

The amount due Sept. 14, 1894 = \$ 1818.92 + \$ 46.99 - \$ 84.50 = \$ 1781.41.

The interest on \$ 1781.41 from Sept. 14, 1894, to March 29, 1895 = \$ 57.90.

∴ the amount due March 3, 1895 = \$ 1781.41 + \$ 57.90 = \$ 1839.31.



**NOTE.**— In the solution of the above example, the interest on \$2192 from June 30, 1893, to Sept. 12, 1893, is found to be \$26.30, which is greater than the payment made on Sept. 12; consequently, the interest is again computed on \$2192 from Sept. 12, 1893, to April 9, 1894, and the interest is found to be \$75.02. The sum of the two payments \$26 and \$450 is now greater than the sum of the two interests \$26.30 and \$75.02, and the solution proceeds as given above.

**245.** The following is the rule for finding the amount due on a note at a given date, when the sum is not paid within a year. It is known as the **United States Rule** because it is adopted by the Supreme Court of the United States. *Find the amount of the principal until the time of a payment. Subtract the payment made from this amount, and use the remainder for a new principal. Continue this process until the time of settlement, when the last amount is the sum due. If, however, any payment is less than the accrued interest, compute the interest for the next period on the same principal as before, and do this until the sum of the payments equals or is greater than the sum of the interests.*

#### Exercise 162

1. On a note for \$2000, dated Jan. 24, 1890, and drawing 6% interest, are indorsed the following payments: May 24, 1890, \$440; Aug. 24, 1890, \$324; Feb. 24, 1891, \$139. How much is due July 24, 1891?

2. A note for \$1200, dated Jan. 18, 1889, and drawing interest at 5%, had payments indorsed upon it as follows: March 18, 1889, \$210; Sept. 18, 1889, \$15; Feb. 12, 1890, \$260. Find the balance due May 9, 1890.

3. On a mortgage for \$3750, dated May 16, 1887, and bearing interest at 6%, there were paid May 16, 1888, \$350; Sept. 18, 1888, \$280; Jan. 22, 1889, \$750; May 16, 1889, \$925. What sum was due on the mortgage Oct. 31, 1889?

4. How much was due on the following note, Oct. 30, 1889?

\$ 850.

NEW YORK, Oct. 30, 1887.

For value received, I promise to pay Alex. Thompson or order, on demand, eight hundred and fifty dollars, with interest from date at 6%.

JOHN STUART.

On this note the following payments were indorsed: April 20, 1888, \$ 125; Nov. 20, 1888, \$ 125; Jan. 20, 1889, \$ 75; July 20, 1889, \$ 425.

### COMPOUND INTEREST

**246. Compound Interest** is interest which is found for stated periods and added at the end of each period to the principal, the sum of the principal and interest becoming the new principal.

The unit of time is 1 yr., although the interest may be compounded annually, semiannually, quarterly, and so on.

Thus 6% compounded semiannually means that each new principal is increased each 6 mo. by 3% of itself.

**247.** If \$5000 deposited at a savings bank draws interest at 4%, semiannually, the interest due at the end of the first half-year will be 2% of \$5000 or \$100.

If this \$100 is not drawn, it is placed to the credit of the depositor, who has now \$5100 on deposit.

The interest for the second half-year is 2% of \$5100 or \$102.

If this is not drawn, it is placed to the credit of the depositor, making his deposit \$5202.

The interest for the third half-year is 2% of \$5202 or \$104.04.

If this is not drawn, it is placed to the credit of the

depositor, making his deposit \$5306.04 at the end of 1 yr. 6 mo.

Thus \$5000 at 4% interest, compounded semiannually, will in 1 yr. 6 mo. amount to \$5306.04; and the compound interest for that time will be \$5306.04 - \$5000 = \$306.04.

$$\begin{array}{r}
 \$5000 \text{ original principal} \\
 \underline{1.02} \\
 10000 \\
 50000 \\
 \$5100.00 \text{ amount at the end of the first period} \\
 \underline{1.02} \\
 10200 \\
 51000 \\
 \$5202.00 \text{ amount at the end of the second period} \\
 \underline{1.02} \\
 10404 \\
 52020 \\
 \$5306.04 \text{ amount at the end of the third period}
 \end{array}$$

**248.** Find the compound interest on \$5000 for 1 yr. 10 mo. 15 da. at 4%, payable semiannually.

As in the last paragraph, find the amount of \$5000 for 1 yr. 6 mo., and then complete the work thus:

The rate per cent for 4 mo. 15 da. or  $\frac{2}{3}$  yr. =  $\frac{2}{3} \times 4\% = 1\frac{1}{3}\%$ .

$$\begin{array}{r}
 \$5306.04 \text{ amount at the end of the third period} \\
 \underline{1.01\frac{1}{3}} \\
 265302 \\
 530604 \\
 5306040 \\
 \$5385.6306 \text{ amount at the end of the fourth period}
 \end{array}$$

$\therefore$  the compound interest = \$5385.63 - \$5000 = \$385.63.

Instead of finding the rate for 4 mo. 15 da., we might have found the interest on \$5306.04 for 1 yr. at 4%, and then have taken  $\frac{2}{3}$  of it to find the interest for the last period, since 4 mo. 15 da. is  $\frac{2}{3}$  of 1 yr.

**249.** Find what principal will in 1 yr. 10 mo. and 15 da. amount to \$5385.63.

From a study of the preceding paragraph it will appear that to find the principal at the beginning of the fourth period, or the amount at the end of the third period, we must divide \$5385.63 by  $1.01\frac{1}{2}$  or 1.015, since the fourth amount is found by multiplying the fourth principal by  $1.01\frac{1}{2}$ .

$$\begin{aligned}\text{The principal at the beginning of the fourth period} &= \$5385.63 \div 1.015 \\ &= \$5306.04.\end{aligned}$$

$$\begin{aligned}\text{The principal at the beginning of the third period} &= \$5306.04 \div 1.02 \\ &= \$5202.\end{aligned}$$

$$\begin{aligned}\text{The principal at the beginning of the second period} &= \$5202 \div 1.02 \\ &= \$5100.\end{aligned}$$

$$\begin{aligned}\text{The principal at the beginning of the first period} &= \$5100 \div 1.02 \\ &= \$5000.\end{aligned}$$

$$\therefore \text{the principal} = \$5000.$$

In practice it is often more convenient to divide by the 1.02 three times in succession and then by 1.015 for the last division.

### Exercise 163

Find the amount and the compound interest of:

1. \$800 for 3 yr. at 5%, compounded annually.
2. \$425 for 4 yr. at 4%, compounded annually.
3. \$250 for 2 yr. at 6%, compounded semiannually.
4. Find the amount and also the compound interest on \$1000 for 3 yr. at 5%.
5. In question 4 what would the amount have been at simple interest? How much has to be paid as interest on interest? What fraction is it of the first year's interest? What per cent?
6. Find the amount of \$360 for 2 yr. at 6%, interest payable semiannually.

7. Find the amount of \$ 650 for 1 yr. 3 mo., interest payable quarterly at 4% per annum.
8. Find the compound interest on \$ 8240 for 2 yr. at 5%, payable semiannually.
9. State how to find the amount of a sum of money at compound interest, for a given time and rate.
10. Find the amount and also the compound interest on \$ 2500 for 1 yr. 10 mo. and 15 da. at 6%, payable semiannually.
11. What principal will amount to \$ 2247.20 in 2 yr. at 6%?
12. What sum of money put out at compound interest for 2 yr. at 7% will amount to \$ 100?
13. What sum of money put out for 2 yr. at 5%, payable half-yearly, will amount to \$ 600?
14. State how to find the principal that amounts to a given sum of money at compound interest for a given time and rate.
15. Find the difference between the simple and compound interest of \$ 1050 for 3 yr. at 4%.
16. A sum of money put out at simple interest for 2 yr. at 6% amounted to \$ 896. To what sum would it have amounted had it been lent at compound interest?
17. The simple interest on a sum of money for 3 yr. at 7% is \$ 420. What is the compound interest of the same sum for the same time?
18. A man deposits in the savings bank \$ 1500, on which the interest at 3% per annum is to be added to the principal every 6 mo. How much money has the man in the bank at the end of 2 yr.?
19. What will be the amount, compound interest, of \$ 2400 for  $1\frac{1}{2}$  yr. at 10% per annum, paid half-yearly, and at what rate, simple interest, will it amount to the same sum in the same time?

## \* ANNUAL INTEREST

**250. Annual Interest** is the sum of the simple interest on the principal and on each year's interest, if unpaid, from the time it is due until the date of settlement.

**Annual Interest** is charged when the words "interest payable annually" are in the note.

**251.** Find the amount due Oct. 9, 1895, on a note for \$1200, dated July 5, 1891, with interest payable annually at 6%.

	yr.	mo.	da.
(1)	1895	10	9
	1891	7	5
	4	3	4

The interest on \$1200 for 1 yr. at 6% = \$72.

The interest on \$1200 for 4 yr. 3 mo. 4 da. at 6% = \$306.80.

(2) The interest due July 5, 1892, bears interest for 3 yr. 3 mo. 4 da.

The interest due July 5, 1893, bears interest for 2 yr. 3 mo. 4 da.

The interest due July 5, 1894, bears interest for 1 yr. 3 mo. 4 da.

The interest due July 5, 1895, bears interest for 3 mo. 4 da.

∴ the interest on \$1200 for 1 yr., i.e. \$72, bears interest for 7 yr. 16 da.

The interest on \$72 for 7 yr. 16 da. @ 6% = \$30.43.

∴ the amount due = \$1200 + \$306.80 + \$30.43 = \$1537.23.

**Exercise 164**

1. Find the amount due July 16, 1896, on a note for \$900, dated July 16, 1892, with interest payable annually at 6%.

2. Find the amount due March 9, 1896, on a note dated Jan. 3, 1892, for \$1500 at 5%, interest payable annually.

3. Find the amount due Sept. 26, 1896, on a note for \$280, dated June 5, 1893, with interest at  $4\frac{1}{2}\%$ , payable annually.

4. Find the amount due June 8, 1896, on a note dated Aug. 12, 1892, for \$712.50, with interest at 6%, payable annually.

5. Find the amount due Nov. 4, 1896, on a note dated Dec. 24, 1892, for \$842, with interest at  $4\frac{1}{2}\%$ , payable annually.

6. Find the simple, annual, and compound interest on a note for \$1000, dated Aug. 12, 1892, and due Aug. 12, 1896, interest at  $6\%$ .

7. Compare the methods of finding the simple, annual, and compound interest on a sum of money, for a given time, at a given rate per cent.

#### \* STOCKS AND BONDS

**252.** The capital of a bank or other public company is called **Stock**.

It is usually divided into a definite number of equal parts or **Shares**.

The original value of a share, generally \$100, \$50, or \$25, is called its **Par Value**.

**253.** The **Market Value** of a share is the sum for which it can be sold.

Stock is said to be *above par*, or at a premium, when the market value is greater than its par value; it is said to be *below par*, or at a discount, when the market value of the share is less than its par value.

Thus if \$100 stock sells for \$112 money, the stock is at  $12\%$  premium, and it is said to sell at 112.

If \$100 stock sells for \$96 money, the stock is at  $4\%$  discount, and is quoted at 96.

**254.** A **Stock Broker** is a person who buys or sells stocks, bonds, or similar securities. His commission, called **Brokerage**, is reckoned at a certain *rate per cent*, which varies, the most common rate being  $\frac{1}{8}$  of  $1\%$  or  $\frac{1}{8}\%$ .

**255.** A **Bond** is a note bearing interest issued by a government or corporation. There are two kinds of bonds, — *registered* and *coupon* bonds.

A **Registered Stock** or **Bond** is one which is registered on the books of the company or government issuing it, and which cannot be sold or transferred except in writing at the office of the treasurer.

An **Interest Coupon** is an interest certificate payable to the bearer, which is attached to the bond, and which is detached when the interest becomes due.

One coupon is attached to the bond for each instalment of interest to be paid on it.

**256.** The following is the quotation of U. S. bonds in the market of July 8, 1896:

		Bid	Asked
Registered	2's	95	. . .
Registered	4's	108	108 $\frac{1}{2}$
Coupon	4's	109	109 $\frac{1}{4}$
New Coupon	4's	116 $\frac{1}{2}$	116 $\frac{3}{4}$
Registered	5's	112 $\frac{3}{4}$	113

**257.** The following is the quotation of stock in the market of July 8, 1896:

Stocks	Opening	Highest	Lowest	Closing	
				July 8	July 7
Am. Sugar	109 $\frac{1}{2}$	111 $\frac{1}{8}$	109 $\frac{3}{8}$	110	110 $\frac{1}{4}$
Am. Sugar pfd.	101 $\frac{3}{4}$	101 $\frac{3}{4}$	101 $\frac{3}{4}$	101 $\frac{3}{4}$	101 $\frac{1}{4}$
C. B. & Q.	71 $\frac{7}{8}$	72 $\frac{3}{8}$	71 $\frac{5}{8}$	72 $\frac{7}{8}$	72 $\frac{1}{8}$
C. R. I. & P.	63	63 $\frac{3}{8}$	62 $\frac{3}{8}$	62 $\frac{5}{8}$	63 $\frac{1}{4}$
Michigan Central	96	96	96	96	. . .
Manhattan	97 $\frac{1}{4}$	97 $\frac{1}{4}$	96 $\frac{1}{2}$	96 $\frac{5}{8}$	97
Del. & Hud.	124 $\frac{1}{8}$	124 $\frac{1}{8}$	124 $\frac{1}{8}$	124 $\frac{1}{8}$	124 $\frac{1}{2}$

#### Exercise 165

1. At what different prices is Am. Sugar stock quoted at, July 8, 1896?



2. What will a seller receive from his broker for 1 share of C. B. & Q. stock, July 8, 1896, at each of the quoted prices, brokerage being  $\frac{1}{8}\%$ ? What from 1 share of Am. Sugar pfd.?

3. What will a buyer have to pay for 1 share of Manhattan stock at each quotation, July 8, 1896, brokerage  $\frac{1}{8}\%$ ? What for C. R. I. & P.?

4. At what per cent premium are the different quotations for Am. Sugar, Am. Sugar pfd., and Del. & Hud. stock, July 8, 1896?

5. At what per cent discount are the different quotations for C. B. & Q., C. R. I. & P., Michigan Central, and Manhattan stock, July 8, 1896?

6. What would I receive for 1 share of Del. & Hud., July 8, 1896, sold at the highest price, brokerage  $\frac{1}{8}\%$ ? What for 10 shares? What for 100 shares?

7. What would I have to pay for 1 share of C. B. & Q. stock, July 8, 1896, bought at the opening price, brokerage  $\frac{1}{8}\%$ ? What for 10 shares? What for 100 shares?

8. What would I have to pay for 1 share of Am. Sugar stock, July 8, 1896, at the lowest quoted price, brokerage  $\frac{1}{8}\%$ ? What for 10 shares? What for 100 shares?

9. What would I receive for 1 share of C. B. & Q. R. R. stock, July 8, 1896, sold at the lowest quotation, brokerage  $\frac{1}{8}\%$ ? What for 10 shares? What for 100 shares?

10. What will 1 share of C. B. & Q. stock cost, July 8, 1896, at the opening price, brokerage  $\frac{1}{8}\%$ ? How many shares can I buy for \$144? For \$216? For \$360?

11. What will 1 share of C. R. I. & P. stock cost, July 8, 1896, at the lowest quotation, brokerage  $\frac{1}{8}\%$ ? How many shares can be bought for \$125? For \$625?

12. What is the difference between the highest and lowest quotations of Manhattan stock, July 8, 1896?

13. What is the difference between the closing prices of Del. & Hud. stock, July 7 and July 8, 1896?

14. What is the difference between the opening and closing prices of C. B. & Q. stock, July 8, 1896?

258. (1) How much will be realized by selling out 66 shares of N. Y. Central R. R. stock at  $95\frac{1}{2}$ , brokerage  $\frac{1}{8}\%$ ?

1 share of stock sells for  $\$95\frac{1}{2} - \$\frac{1}{8}$  or  $\$95\frac{3}{8}$  money.

$\therefore$  66 shares of stock sell for  $66 \times \$95\frac{3}{8}$  or  $\$6294.75$  money.

NOTE. — The brokerage =  $66 \times \$\frac{1}{8} = \$8.25$ .

(2) How many shares of Del. & Hud. stock at  $124\frac{1}{8}$ , brokerage  $\frac{1}{8}\%$ , can I buy for  $\$5591.25$ ?

1 share costs  $\$124\frac{1}{8} + \$\frac{1}{8}$  or  $\$124.25$ .

$\therefore$  the number of shares =  $\$5591.25 \div \$124.25 = 45$ .

NOTE. — The brokerage =  $45 \times \$\frac{1}{8} = \$5.62\frac{1}{2}$ .

(3) A broker realizes  $\$7.25$  from a sale of stock, brokerage  $\frac{1}{8}\%$ . What was the par value of the stock sold?

$\frac{1}{8}\%$  of the par value =  $\$7.25$ .

1% of the par value =  $\$58$ .

100% of the par value =  $\$5800$ .

$\therefore$  the par value =  $\$5800$ .

(4) I sold through my broker 95 shares of Chicago and Northwestern R. R. stock, receiving for it  $\$9476.25$ , brokerage  $\frac{1}{8}\%$ . Find at what price the stock was quoted.

95 shares sell for  $\$9476.25$ .

1 share sells for  $\$9476.25 \div 95 = \$99.75$ ;

*i.e.* excluding brokerage, the selling price =  $99\frac{3}{4}$ .

$\therefore$  stock is quoted at  $99\frac{3}{4} + \frac{1}{8}$  or  $99\frac{5}{8}$ .

(5) What annual income will be realized from  $\$3828.12\frac{1}{2}$ , invested in the U. S. 4's at  $109\frac{1}{4}$ , brokerage  $\frac{1}{8}\%$ ?

1 share costs  $\$109\frac{1}{4} + \$\frac{1}{8} = \$109\frac{3}{8} = \$109.375$ .

The number of shares =  $\$3828.125 \div \$109.375 = 35$ .

$\therefore$  the income =  $35 \times \$4 = \$140$ .

(6) What amount of money must be invested in 6% stock, at  $119\frac{3}{4}$ , brokerage  $\frac{1}{8}\%$ , to realize an income of \$978?

1 share yields an income of \$6.

The number of shares =  $\$978 \div \$6 = 163$ .

1 share costs  $\$119\frac{3}{4} + \$\frac{1}{8} = \$119\frac{7}{8}$ .

163 shares cost  $163 \times \$119\frac{7}{8} = \$19,539.62\frac{1}{2}$ .

$\therefore \$19,539.62\frac{1}{2}$  must be invested.

(7) If 6% stock is bought at  $109\frac{7}{8}$ , what per cent does it pay on the investment, brokerage  $\frac{1}{8}\%$ ?

1 share costs  $\$109\frac{7}{8} + \$\frac{1}{8} = \$110$ .

\$110 yields an income of \$6.

$\therefore$  the rate per cent =  $\frac{6}{110}$  or  $5\frac{5}{11}\%$  of the investment.

(8) What must I pay for 8% stock to realize an income of 6% on the investment, brokerage  $\frac{1}{8}\%$ ?

8% of the cost of 1 share = \$6.

1% of the cost of 1 share =  $\$ \frac{3}{4}$ .

100% of the cost of 1 share = \$75;

i.e. excluding brokerage, the cost price is \$75.

$\therefore$  stock is quoted at  $74\frac{1}{4}$ .

### Exercise 166

1. What will 25 shares of Adams Express stock cost at 148, brokerage  $\frac{1}{8}\%$ ?

2. What is realized from the sale of 208 shares of C. B. & Q. R. R. stock at  $71\frac{1}{2}$ , brokerage  $\frac{1}{8}\%$ ?

3. What did I pay for 39 shares Chicago and Northwestern, July 8, 1896, stock selling at  $99\frac{1}{4}$  and brokerage being  $\frac{1}{8}\%$ ?

4. Find what I received from the sale of 84 shares of Western Union stock at  $82\frac{1}{2}$ , brokerage  $\frac{1}{8}\%$ .

5. What is the cost of \$20,000 U. S. 4's at  $112\frac{3}{4}$ , brokerage  $\frac{1}{8}\%$ ?

6. Find the cost of \$24,000 U. S. 4's at  $116\frac{3}{4}$ , brokerage  $\frac{1}{8}\%$ .

7. July 8, 1896, 40 shares of Chicago City Railway, reg. at 220, were sold, brokerage  $\frac{1}{8}\%$ . Find what was received by the owner of the stock.

8. How many shares of Manhattan R. R. stock at  $97\frac{1}{4}$  can I buy for \$3505.50, brokerage  $\frac{1}{8}\%$ ?

9. July 8, 1896, Am. Sugar pfd. stock was quoted at  $101\frac{1}{4}$ . How many shares were bought for \$4257.75, brokerage  $\frac{1}{8}\%$ ?

10. A stockholder sold D. L. and W. R. R. stock at  $157\frac{1}{4}$ , receiving all together \$3771. How many shares did he sell, brokerage being  $\frac{1}{8}\%$ ?

11. How many shares of Wells Fargo Express stock must I sell at 95, brokerage  $\frac{1}{8}\%$ , to receive \$9677.25?

12. If from my sales of New York Central R. R. stock at  $95\frac{1}{4}$ , I receive \$6278.25, how much stock did I sell, brokerage being  $\frac{1}{8}\%$ ?

13. How many shares of D. L. and W. R. R. stock at  $158\frac{1}{2}$  can be bought for \$2855.25, brokerage  $\frac{1}{8}\%$ ?

14. A broker sells 24 shares of stock on a commission of  $\frac{1}{8}\%$ . How much does he realize?

15. How many shares of stock does a broker sell to realize a commission of \$16.25, brokerage  $\frac{1}{8}\%$ ?

16. A broker realizes \$12.50 from the sale of stock, brokerage  $\frac{1}{8}\%$ . What was the par value of the stock sold and what did it sell for at  $70\frac{3}{8}$ ?

17. A broker received \$46.50 for buying stock on a commission of  $\frac{5}{8}\%$ . How much stock did he buy?

18. I sold through my broker 40 shares of stock, receiving for it \$4860, brokerage  $\frac{1}{8}\%$ . At what price was the stock quoted?

19. A person received \$6053.12 $\frac{1}{2}$  for \$6500 stock after paying his broker  $\frac{1}{8}\%$ . Find at what per cent discount the stock was sold.

20. July 7, 1896, \$1654.25 was paid for 26 shares of Rock Island R. R. stock, brokerage  $\frac{1}{8}\%$ . At what was Rock Island stock quoted, July 7?

21. What annual income will be obtained from \$ 6071, invested in U. S. 4's coup. of 1925 at 116 $\frac{3}{4}$ , brokerage  $\frac{1}{8}\%$ ?

22. A person paid \$ 8578.50 for U. S. 4's at 112 $\frac{3}{4}$ , brokerage  $\frac{1}{8}\%$ . What was his income from the bonds?

23. If I invest \$ 8583.75 in stock at 95 $\frac{1}{4}$ , paying 5% dividend, what will be my income, brokerage  $\frac{1}{8}\%$ ?

24. What income will be realized from \$ 9229.50 invested in stock at 109 $\frac{3}{4}$ , brokerage  $\frac{1}{8}\%$ , paying a dividend of 5 $\frac{1}{2}\%$ ?

25. What amount of money must be invested in 8% stock at 158 $\frac{1}{2}$ , brokerage  $\frac{1}{8}\%$ , to realize an income of \$ 1096?

26. What sum must I invest in 4 $\frac{1}{2}\%$  stock at 99 $\frac{3}{4}$ , to produce an annual income of \$ 1638, brokerage  $\frac{1}{8}\%$ ?

27. How much must I invest in U. S. 5's at 112 $\frac{3}{4}$  to realize an annual income of \$ 450, brokerage  $\frac{1}{8}\%$ ?

28. If street railway stock bought at 232 yields a half-yearly dividend of 6 $\frac{1}{2}\%$ , how much must I invest to obtain a semiannual income of \$ 325, brokerage  $\frac{1}{8}\%$ ?

29. If I buy stock through a broker who charges  $\frac{1}{8}\%$ , how much must I invest in stock at 153, paying 9% dividends, to secure an income of \$ 1350?

30. If 4 $\frac{1}{2}\%$  stock is bought at 74 $\frac{1}{8}$ , brokerage  $\frac{1}{8}\%$ , what per cent does it pay on the investment?

31. If 8% stock is bought at 159 $\frac{1}{8}$ , what per cent does it pay on the investment, brokerage  $\frac{1}{8}\%$ ?

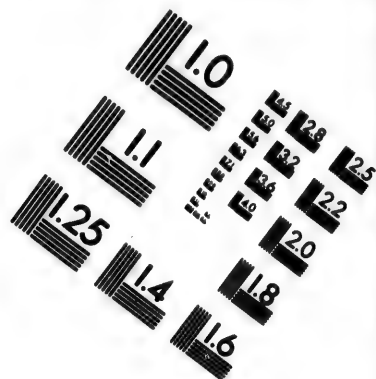
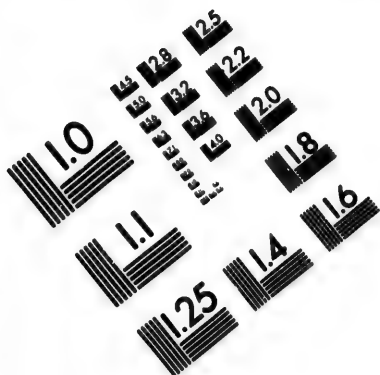
32. What must I pay for 4% stock to pay 5% on the investment, brokerage  $\frac{1}{8}\%$ ?

33. What rate of interest do I realize on an investment in 6% stock at 107 $\frac{1}{8}$ , brokerage  $\frac{1}{8}\%$ ?

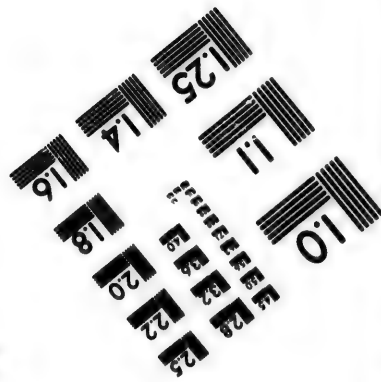
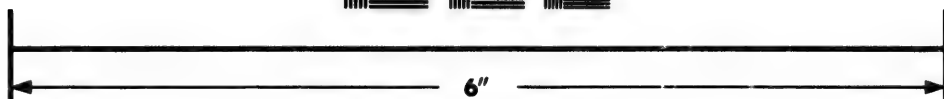
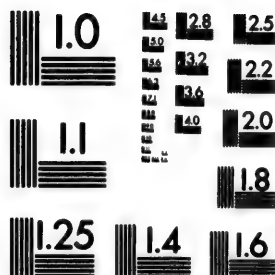
34. What must I pay for 5% stock to yield an income of 6% on my investment?

35. A person receives \$ 600 from an 8% bank dividend. How much stock does he own?





# IMAGE EVALUATION TEST TARGET (MT-3)



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10



36. A person having \$5000 bank stock sells out when it is at 40% premium. What amount of money does he receive, brokerage being  $\frac{1}{8}\%$ ?

37. Bought through a broker 1600 shares (\$100) R. R. stock at  $69\frac{1}{4}$ , brokerage  $\frac{1}{8}\%$ . What was the gross cost of the stock?

38. A speculator bought 36,500 shares (\$100) R. R. stock at  $39\frac{3}{4}$ , and sold them at  $40\frac{3}{8}$ . What was his gain, brokerage  $\frac{1}{8}\%$  on both transactions?

39. A bank declared a dividend of  $3\frac{1}{2}\%$ . How much should a stockholder owning 120 shares (\$50) receive?

40. One company guarantees to pay 6% on shares of \$100 each; another guarantees at the rate of  $5\frac{4}{5}\%$  on shares of \$30 each; the price of the former is \$124.50, and of the latter \$34. Find the rates of interest which they return to the purchaser.

41. A broker receives \$42,100 to invest in U. S. 5-20 bonds, after reserving  $\frac{1}{4}\%$  on the par value of the amount purchased. What was his commission, the bonds being at a premium of 5%?

42. A man bought through a broker 1900 shares (\$100) R. R. stock at  $54\frac{3}{4}$  and sold them at  $55\frac{3}{8}$ . What was his net profit on the transaction, brokerage each way  $\frac{1}{8}\%$ ?

43. An insurance company declared a dividend of 9%. What rate is that on the market value of the shares which are at 185?

44. Compare the rates on the cash values of 6% on stock at 216 and  $3\frac{1}{2}\%$  on stock at 125.

45. Sold 37 shares (\$25) B. and L. Association stock, receiving therefor \$1019.81. At what rate was the stock sold?

46. Bought through a broker 750 shares (\$50) in the Farmers' Loan and Savings Society, paying therefor \$43,968.75. At what quotation were they bought, brokerage  $\frac{1}{8}\%$ ?

47. Bought stock at  $197\frac{1}{8}$  and sold it at  $194\frac{1}{8}$ , having meanwhile received a dividend of 6% on it. My net gain on the transaction after paying  $\frac{1}{8}\%$  brokerage each way is \$336. How many shares (\$40) did I buy?

48. How many railway shares (\$100) at 40% discount must be sold in order that the proceeds invested in bank stock, which is 4% below par, and pays a dividend of 7%, may yield an income of \$1680, brokerage included?

49. Explain the terms: Stocks, Shares, Dividends. When is stock at par? At a premium? At a discount?

50. When the  $3\frac{1}{2}$  per cents are at 98, what must be the price of another stock yielding  $4\frac{1}{2}\%$ , so that the latter may be as profitable as the former, brokerage included?

## EXCHANGE

259. If A of Chicago owes B of St. Paul a sum of money, he can discharge the debt in any one of several ways. He can buy a *post-office order* at the Chicago post-office payable to B at the post-office at St. Paul; he can buy an *express order* at the office of an express company, payable to B at any office of the same company; or he can buy a *draft* at a bank payable to B at a bank in St. Paul.

Give some reasons why it is better to discharge a debt by means of a post-office order, express order, or draft than by sending the money in a registered letter or by express or check.

260. The following are the rates charged for express orders to any part of the United States or Canada:

Rates for orders not over

\$5.00,	5¢.	\$40.00,	18¢.
10.00,	8¢.	60.00,	20¢.
20.00,	10¢.	75.00,	25¢.
30.00,	15¢.	100.00,	30¢.

Over \$100 at above rate.

**261.** The rates charged for post-office orders to any part of the United States are the same as for express orders up to \$100, but orders not exceeding \$2.50 are sold at a charge of 3¢. Single post-office orders are not issued for more than \$100, and for larger amounts additional orders are issued.

**Exercise 167**

1. What is the cost of an express order for \$25? \$44? \$73? \$78?

2. What is the cost of a post-office order for \$80? \$32? \$95? \$1.50?

3. What is the cost of an express order for \$75? \$100?

4. What is the cost of a post-office order for \$75? \$100?

5. What is the cost of a draft for \$75? \$100? \$150? \$240? \$325? \$180? The charge in each case is  $\frac{1}{4}\%$  and the least charge 25¢.

6. By which of the three methods given in questions 3, 4, and 5 is it cheaper to send money in sums greater than \$75? In sums less than \$75, if 25¢ is the smallest charge for a draft?

**262.** Exchange is generally conducted through bankers, who issue *drafts* directing a second bank to pay a specified sum of money to the order of the person named in the draft.

A **Time Draft** is one payable at a specified time after sight or date.

If A in Chicago owe B in St. Paul a sum of money, B may send a draft to A for the amount. If A accepts the draft, he writes the word "accepted" with the date across the face and signs his name.

**Exchange** is the system of paying debts to persons in distant places without actually sending the money, by means of money orders and drafts.

**263.** (1) Find the cost of a draft on New York for \$600, when exchange is  $\frac{1}{4}\%$  premium.

The premium =  $\frac{1}{4}\%$  of \$600 = \$1.50.

$\therefore$  the cost = \$600 + \$1.50 = \$601.50.

(2) Find the cost of a draft on New Orleans for \$1200, payable 60 da. after date, exchange being  $\frac{1}{4}\%$  discount, and interest 6%.

The discount =  $\frac{1}{4}\%$  of \$1200 = \$3.00.

The discount for 63 da. = 6% of \$1200 for 63 da. = \$12.60.

$\therefore$  the cost = \$1200 - \$3.00 - \$12.60 = \$1184.40.

Show that if exchange had been  $\frac{1}{4}\%$  premium, the cost would have been \$1190.40.

### Exercise 168

1. Find the cost of a draft for \$900 at  $\frac{1}{4}\%$  premium.
2. Find the cost of a draft for \$1600 at  $\frac{1}{8}\%$  discount.
3. Find the cost of a draft for \$4500 at  $\frac{5}{8}\%$  discount.
4. Find the cost of a draft for \$2800 at  $\frac{3}{8}$  of 1% premium.
5. Find the cost of a draft for \$1000, payable in 60 da., exchange being  $\frac{1}{4}\%$  premium, and interest 6%.
6. Find the cost of a draft for \$360, payable in 30 da., exchange being  $\frac{1}{4}\%$  discount, and interest 5%.
7. Find the cost of a draft for \$1250, payable in 60 da., exchange being  $\frac{1}{4}\%$  premium, and interest  $4\frac{1}{2}\%$ .
8. Find the cost of a draft for \$1800, payable in 30 da., when exchange is at par, and interest 4%.
9. Find the cost of a bill of exchange on London for £600, when exchange is quoted at \$4.88.
10. Find the cost of a 60-da. draft on Liverpool for £750, exchange at 60 da. being \$4.86.
11. What will be the cost of a bill of exchange in Paris for 2400 francs at 5.16 $\frac{1}{4}$  francs per \$1?
12. What will be the cost of a bill of exchange on Berlin, for 2400 marks, the rate of exchange being 95 $\frac{1}{2}$  ct. for 4 marks?

## CHAPTER XV

### RATIO AND PROPORTION

**264.** If two quantities be expressed in terms of the same unit, their **Ratio** is the quotient obtained by dividing the number measuring the first quantity by the number measuring the second quantity.

Thus the ratio of \$3 to \$5 =  $\frac{3}{5}$ , or, as it is frequently written, 3 : 5.

The first term of a ratio is called the **Antecedent**, and the second the **Consequent**.

Since a ratio may be expressed as a fraction, both terms of a ratio may be multiplied or divided by the same number without changing its value.

**265.** (1) If 15 bbl. of flour cost \$111, what will 35 bbl. cost?

Multiply \$111 by  $\frac{35}{15}$ .  $\therefore$  35 bbl. will cost  $\frac{35}{15}$  of what 15 bbl. cost.

$$x = \$111 \times \frac{35}{15} = \$259.$$

Or the question may be solved thus :

15 bbl. cost \$111.

1 bbl. costs  $\$ \frac{111}{15}$ .

$$\therefore 35 \text{ bbl. cost } \frac{35 \times \$111}{15} = \$259.$$

(2) If 56 men do a piece of work in 21 da., how long will 24 men require to do it?

Multiply 21 da. by  $\frac{56}{24}$ .  $\therefore$  24 men will take  $\frac{56}{24}$  as long as 56 men.

$$x = \overset{\text{da.}}{21} \times \frac{56}{24} = 49 \text{ da.}$$

Or thus :

56 men do the work in 21 da.

1 man can do the work in  $56 \times 21$  da.

$\therefore$  24 men can do the work in  $\frac{56 \times 21}{24}$  or 49 da.

### Exercise 169

1. If 6 articles cost \$14.30, how much will 13 cost at the same rate?

2. If 25 lb. of tea cost \$16, how many lb. can be bought for \$56?

3. If the 4-lb. loaf costs 11¢ when flour is \$6 a bbl., find its cost when flour is \$7½ a bbl.

4. A bankrupt owes \$3000; his assets are \$1740. What sum will a creditor receive whose claim is \$350?

5. The expense of carpeting a room was \$100; if the breadth of the room had been 4 ft. greater, the expense would have been \$120. Find the breadth.

6. If a man working  $9\frac{3}{4}$  hr. per da. finishes a piece of work in 6 da., in what time would he have finished it if he had worked  $8\frac{1}{2}$  hr. per da.?

7. If a garrison of 1500 men have provisions for 13 mo., how long will their provisions last if it be increased to 2200?

8. If 4 men or 6 women can do a piece of work in 20 da., how long will it take 3 men and 15 women to do the same work?

9. A creditor receives \$1.50 for every \$4 of what was due to him, and thereby loses \$301.05. What was the sum due?

10. In a certain business one partner, whose share is  $\frac{3}{11}$  of the whole, receives from it a profit of \$859.20. What share is owned by another, whose profit is \$1969?

11. A person contracts to do a piece of work in 30 da., and employs 15 men upon it; the work is half finished in 24 da. How

many additional workmen must be then introduced in order to perform the contract?

12. The profits of a garden for 2 yr. were \$1456; the profits of the second yr. being  $\frac{2}{5}$  of those of the first. Find the profits of each yr.

13. If 10 men can do a piece of work in 12 da., how soon after beginning must they be joined by 3 more so as to finish the work in 10 da.?

14. If \$120 gain \$5.81 in 126 da., find the gain in 360 da.

15. A bankrupt who is paying  $37\frac{1}{2}\%$  on the dollar divides among his creditors \$6300. What do his debts amount to?

16. If 3 men or 5 boys can do a piece of work in 17 da., in how many da. will 5 men and 3 boys do a piece of work 3 times as great?

17. If 3 men can do as much work in a da. as 4 boys, how long will it take 64 boys to finish a piece of work of which 12 men have done  $\frac{1}{4}$  in 16 da.?

18. If a debt after a deduction of 3% becomes \$1008.80, what would it have become after a deduction of 4% had been made?

19. Six sheets of paper measuring 8 in. by 10 in. weigh an ounce. Find the weight of 120 sheets of the same kind of paper, each sheet measuring 11 in. by 17 in.

20. A person walks from his house to his office at the rate of 4 mi. per hr.; but finding he has forgotten something, returns at the rate of 5 mi. per hr. Compare the time spent in going with that spent in returning.

21. One train travels  $8\frac{1}{2}$  mi. in 20 min., and a second train 9 mi. in 15 min. Compare their rates per hr.

22. A man can row 6 mi. an hr. in still water. Compare his rate of rowing down a stream which flows at the rate of  $2\frac{1}{2}$  mi. an hr. with his rate of rowing up.

23. One water pipe discharges 141 gal. per hr., another discharges 235 gal. per hr. Compare their rates of discharge (a) per hr.; (b) per min.; (c) per sec.; (d) per da.; (e) per seventh of a da. Also compare the times in which the pipes would each discharge (a) 705 gal.; (b) 705 qt.; (c) 705 pt.

24. Two taps when both open discharge water at the rate of 481 gal. per hr.; the discharge of the smaller of the two being at the rate of 148 gal. per hr. Compare the volume discharged by the larger tap in any given time with the volume discharged by the smaller tap in the same time. Compare also the time in which the larger tap will discharge a given number of gal. with the time required by the smaller to discharge the same number of gal.

25. A greyhound pursuing a hare takes 3 leaps to every 4 the hare takes; but 2 leaps of the hound are equal in length to 3 leaps of the hare. Compare the speed of the hound with that of the hare.

26. Milk is worth 20¢ a gal., but by watering it the value is reduced to 15¢ a gal. Find the proportion of water to milk in the mixture.

27. Two men receive \$15 for doing a certain piece of work. Now one man had worked but 3 da. while the other had worked 5 da. on the job. If the money is to be divided in proportion to the lengths of time the men worked, how much should each receive?

#### \* COMPOUND PROPORTION

266. (1) If 20 men can dig 60 yd. of earth in 4 da., how many yards can 30 men dig in 9 da.?

Men	Yd.	Da.
20	60	4
30	$x$	9

Multiply 60 yd. by  $\frac{30}{20}$ .  $\therefore$  30 men can dig  $\frac{30}{20}$  as many yd. as 20 men.

Multiply the result by  $\frac{9}{4}$ .  $\therefore$  in 9 da. 30 men can dig  $\frac{9}{4}$  as much as in 4 da.

$$\therefore x = \frac{60}{1} \times \frac{30}{20} \times \frac{9}{4} = 202\frac{1}{2} \text{ yd.}$$



(2) If 120 bu. of oats last 14 horses 56 da., in how many da. will 6 horses consume 90 bu.?

Bu.	Horses	Da.
120	14	56
90	6	$x$

Multiply 56 da. by  $\frac{90}{120}$ .  $\therefore$  90 bu. will last  $\frac{90}{120}$  as many da. as 120 bu.

Multiply the result by  $\frac{14}{6}$ .  $\therefore$  90 bu. will last 6 horses  $\frac{14}{6}$  as long as 14 horses.

$$\therefore x = \frac{56}{1} \times \frac{90}{120} \times \frac{14}{6} = 98 \text{ da.}$$

**267.** In each of the above two solutions, in order that the ratio may easily be seen, the items in the question have been written in horizontal lines. In number (1) we are required to find the number of yards, and the problem is to determine the ratio resulting from each comparison, and how it affects the number of *yards*.

In number (2) we are required to find the number of days, and the problem is to determine the ratios, and how they affect the number of *days*.

**268.** Both problems may, if preferred, be solved by the unitary method, thus:

$$\begin{aligned}
 (1) \quad & 20 \text{ men in 4 da. dig } 60 \text{ yd.} \\
 & 1 \text{ man in 4 da. digs } \frac{60}{20} \text{ yd.} \\
 & 1 \text{ man in 1 da. digs } \frac{60}{20 \times 4} \text{ yd.} \\
 & 30 \text{ men in 1 da. dig } \frac{60 \times 30}{20 \times 4} \text{ yd.} \\
 \therefore & 30 \text{ men in 9 da. dig } \frac{60 \times 30 \times 9}{20 \times 4} = 202\frac{1}{2} \text{ yd.}
 \end{aligned}$$

$$\begin{aligned}
 (2) \quad & 14 \text{ horses eat 120 bu. in 56 da.} \\
 & 1 \text{ horse eats 1 bu. in } \frac{56 \times 14}{120} \text{ da.} \\
 \therefore & 6 \text{ horses eat 90 bu. in } \frac{56 \times 14 \times 90}{120 \times 6} = 98 \text{ da.}
 \end{aligned}$$

To prove the answer correct, substitute the answer in place of  $x$  in the horizontal line and omit one of the quantities, frame the question and then solve.

Bu.	Horses	Da.
120	14	56
90	$x$	98

If 120 bu. of oats last 14 horses for 56 da., how many horses will 90 bu. last 98 da.?

On solving,  $x$  will be found equal to 6, which proves the former solution correct. How many questions can be made from the numbers in the two lines, including the original one?

Solve the following questions. State one or more questions in proof for each problem, and prove your answers correct.

#### Exercise 170

1. If 7 horses are kept 20 da. for \$14, how many will be kept 7 da. for \$28?
2. If 3 men earn \$75 in 20 da., how many men will earn \$78.75 in 9 da. at the same rate?
3. If 16 horses eat 96 bu. of corn in 42 da., in how many days will 7 horses eat 66 bu.?
4. If 16 horses can plough 1280 A. in 8 da., how many A. will 12 horses plough in 5 da.?
5. If 20 men can perform a piece of work in 12 da., find the number of men who could perform another piece of work 3 times as great in  $\frac{1}{5}$  of the time.
6. If 252 men can dig a trench 210 yd. long, 3 wide, and 2 deep, in 5 da. of 11 hr. each, in how many days of 9 hr. each will 22 men dig a trench of 420 yd. long, 5 wide, and 3 deep?
7. If 10 men can reap a field of  $7\frac{1}{2}$  A. in 3 da. of 12 hr. each, how long will it take 8 men to reap 9 A., working 16 hr. a day?

8. If 25 men can do a piece of work in 24 da., working 8 hr. a day, how many hours a day would 30 men have to work in order to do the same piece of work in 16 da.?

9. A town which is defended by 1200 men, with provisions enough to sustain them 42 da., supposing each man to receive 18 oz. a day, obtains an increase of 200 men to its garrison. What must now be the allowance to each man, in order that the provisions may serve the whole garrison for 54 da.?

10. If 560 flagstones, each  $1\frac{1}{2}$  ft. square, will pave a courtyard, how many will be required for a yard twice the size, each flagstone being 14 in. by 9 in.?

11. If 20 men in 3 wk. earn \$900, in what time will 12 men earn \$1500?

12. If  $\frac{3}{4}$  of a meadow be mown by 12 men in 6 da., find in what time the remainder could be mown by 10 men.

13. If 36 men, working 16 da., can dig a trench 72 yd. long, 18 yd. wide, and 12 yd. deep, how many men can dig a trench 64 yd. long, 27 yd. wide, and 18 yd. deep in 24 da.?

14. If 25 men build a wall 15 ft. high, 2 ft. thick, and 50 ft. long, in 12 da. of 9 hr. each, how many hours per day must 40 men work to build a wall 60 ft. long, 3 ft. thick, and 20 ft. high in 25 da.?

15. Twenty men can do a piece of work in 12 da. Find how many men will do half as much again in one-fifth part of the time, supposing that they work the same number of hours in the day, and that 2 of the second set can do as much work in an hour as 3 of the first set.

16. If 12 men do a piece of work in 21 da., in what time will 10 men do a piece of work  $1\frac{3}{4}$  as great, if 3 of the first set do as much in an hour as 4 of the second set?

17. A miller has a bin 8 ft. long,  $4\frac{1}{5}$  ft. wide, and  $2\frac{1}{2}$  ft. deep, holding 75 bu. How deep must he make another bin which is to be 18 ft. long and  $3\frac{5}{8}$  ft. wide, so that its capacity may be 450 bu.?

18. What is the weight of a block of stone 12 ft. 6 in. long, 6 ft. 6 in. broad, and 8 ft. 3 in. deep, when a block of the same stone 5 ft. long, 3 ft. 9 in. broad, and 2 ft. 6 in. deep, weighs 7500 lb.?

## PROPORTIONAL PARTS

269. (1) Divide \$720 into parts proportional to 4, 5, and 6.

The total number of parts =  $4 + 5 + 6 = 15$ .

$\therefore$  the first part =  $\frac{4}{15}$  of \$720 = \$192,

the second part =  $\frac{5}{15}$  of \$720 = \$240,

the third part =  $\frac{6}{15}$  of \$720 = \$288.

(2) Divide 316 lb. into parts proportional to  $\frac{1}{3}$ ,  $\frac{1}{5}$ , and  $\frac{1}{8}$ .

Multiplying  $\frac{1}{3}$ ,  $\frac{1}{5}$ , and  $\frac{1}{8}$  by their L. C. M. 120, we have the parts proportional to 40, 24, and 15.

The total number of parts =  $40 + 24 + 15 = 79$ .

$\therefore$  the parts are respectively  $\frac{40}{79}$ ,  $\frac{24}{79}$ , and  $\frac{15}{79}$  of 316 lb. = 160, 96, and 60 lb.

PROOF. — Dividing 160, 96, and 60 by 480, the denominator which reduces 160 to  $\frac{1}{3}$ , we have  $\frac{1}{3}$ ,  $\frac{1}{5}$ ,  $\frac{1}{8}$ , which proves the results found to be correct.

## Exercise 171

1. Divide 1331 into parts proportional to 2, 4, 5.
2. Divide \$73.50 into parts proportional to  $\frac{1}{4}$ ,  $\frac{2}{5}$ ,  $\frac{1}{6}$ .
3. Divide 19 T. 1104 lb. into parts proportional to  $\frac{1}{2}$ ,  $\frac{1}{3}$ ,  $\frac{1}{4}$ .
4. Divide \$1064 into parts proportional to 2,  $2\frac{1}{4}$ ,  $2\frac{2}{5}$ .
5. Divide 180 lb. into parts proportional to 3.3, .7, .5.
6. Divide \$4500 between two persons in proportion to their ages, which are 21 and 24 yr.
7. Two men receive \$15 for doing a certain piece of work. Now one man had worked only 3 da., while the other had worked 5 da. on the job. If the money is to be divided in proportion to the lengths of time the men worked, how much should each receive?

8. Divide 4472 into parts which shall be to each other in the ratio of 3, 5, 7, 11.

9. Divide \$84.42 into two parts which shall be to each other as 5:16.

10. A company of militia consisting of 72 men is to be raised from 3 towns which contain respectively 1500, 7000, and 9500 men. How many must each town provide?

11. Sugar is composed of 49.856 parts oxygen, 43.265 carbon, and 6.879 hydrogen. How many lb. of each are there in 1300 lb. of sugar?

12. Gunpowder is composed of nitre, charcoal, and sulphur in the proportion of 33, 7, and 5.

(1) How many lb. of sulphur are there in 180 lb. of powder?

(2) How many lb. of powder can be made with 30 lb. of sulphur?

(3) How much nitre and sulphur must be mixed with 112 lb. of charcoal to form gunpowder?

13. A man divides \$3300 amongst his three sons, whose ages are 16, 19, and 25 yr., in sums proportional to their ages; 2 yr. afterwards he similarly divides an equal sum, and again after 3 yr. more. How much does each receive in all?

14. Two persons travelling together agree to pay expenses in the ratio of 7 to 5. The first (who contributes the greater sum) pays on the whole \$103.40, the second \$63.40. What must one pay the other to settle their expenses according to agreement?

15. Divide \$480 among A, B, C, and D, so that B may receive as much as A; C as much as A and B together; and D as much as A, B, and C together.

## PARTNERSHIP

**270.** In **Simple Partnership** the capital of each partner is supposed to be invested for the same time.

In **Compound Partnership** the time is taken into account as well as the capital in determining the gain or loss of each partner.

**271.** A, B, and C engage in business. A furnishes \$7500, B \$5000, and C \$4500. If they gain \$2380, what is each one's share?

Dividing their capitals by \$500, we have their capitals, and therefore their gains proportional to 15, 10, and 9.

The total number of parts =  $15 + 10 + 9 = 34$ .

$\therefore$  their respective gains are  $\frac{15}{34}$ ,  $\frac{10}{34}$ , and  $\frac{9}{34}$  of \$2380 = \$1050, \$700, and \$630.

**Exercise 172**

1. Two merchants, A and B, form a joint capital. A puts in \$1200 and B \$1800. They gain \$400. How ought the gain to be divided between them?

2. A bankrupt owes three creditors, A, B, and C, \$175, \$210, and \$265 respectively; his property is worth \$422.50. What ought each to receive?

3. A, B, and C entered into partnership. A puts in \$6000, B \$4000, and C \$2000. They gained \$2250. What is each one's share of the gain?

4. Two men purchase a house for \$3600, the first contributing \$1600 and the second \$2000. If it rents so as to pay 12% on its value, what share of the rent should each receive?

5. Two persons have gained in trade \$3456; the one put in \$10,560 and the other \$8640. What is each person's share of the profits?

6. R. Stuart and G. Armstrong enter into partnership. Stuart contributes \$4500 to the partnership and Armstrong contributes \$7500. Their net gain at the end of the year is \$1750. How much of the sum should each partner receive?

7. Three partners invest respectively \$7800, \$5750, and \$9450 in business. At the end of the first year they find their net gain to be \$3156. What is the amount of each partner's share of this gain?

8. A, B, and C form a partnership with a capital of \$20,000. A contributes \$5000, B \$7000, and C the remainder. They gain 20% of the total capital. Find each man's share of the profits.

9. T. Allan and E. Jamieson engage in business with a joint capital of \$19,200, and agree to share gains and losses in proportion to their investments. At the end of a year Allan receives a dividend of \$1100, and Jamieson a dividend of \$1300. What was the amount of the investment of each?

10. D. Rowan, F. Galbraith, and J. Munro enter into partnership. They gain \$7500, of which Rowan receives \$2100, Galbraith \$3100, and Munro the balance. How much did Rowan and Galbraith respectively invest if the amount of Munro's investment was \$18,000?

11. A, B, and C pay \$37.80 as rent for a pasture. A puts in 5 horses, B 12 cows, and C 60 sheep. If 1 horse eats as much as 2 cows, and 1 cow as much as 3 sheep, what rent should each pay?

**272.** (1) A, B, and C enter into partnership. A puts in \$700 for 12 mo., B \$500 for 9 mo., and C \$600 for 8 mo. Divide a profit of \$2065 equitably among them.

The gain on \$700 for 12 mo. = the gain on \$8400 for 1 mo.

The gain on \$500 for 9 mo. = the gain on \$4500 for 1 mo.

The gain on \$600 for 8 mo. = the gain on \$4800 for 1 mo.

∴ the proportional parts representing the gains are 84, 45, and 48, or 28, 15, and 16.

$$28 + 15 + 16 = 59.$$

∴ the respective gains are  $\frac{28}{59}$ ,  $\frac{15}{59}$ , and  $\frac{16}{59}$  of \$2065 = \$980, \$525, and \$560.

(2) A commenced business with \$4000 stock; 3 mo. after, he took in B with a capital of \$2000; and 4 mo. after B became a partner, he took in C with a capital of \$600; at the end of the year the firm had gained \$3450. Find the share of each.

A's capital = \$4000 for 12 mo. = \$48,000 for 1 mo.

B's capital = \$2000 for 9 mo. = \$18,000 for 1 mo.

C's capital = \$600 for 5 mo. = \$3000 for 1 mo.

∴ the respective gains are proportional to 48, 18, and 3; i.e. 16, 6, and 1.

$$16 + 6 + 1 = 23.$$

∴ the respective shares are  $\frac{16}{23}$ ,  $\frac{6}{23}$ , and  $\frac{1}{23}$  of \$3450 = \$2400, \$900, and \$150.

### Exercise 173

1. D and E enter into partnership; D puts in \$480 for 3 mo., and E \$900 for 4 mo. They gain \$840. What is each man's share in the gain?

2. A, B, and C entered into partnership; A put in \$1200 for 8 mo., B \$800 for 10 mo., and C \$400 for 12 mo. They gained \$3920. What was each man's share of the gain?

3. A, B, and C are partners; A puts in \$5000 for 6 mo., B \$6000 for 8 mo., and C \$900 for 11 mo. The profit is \$5575.50. What is the share of each?

4. Three graziers hire a pasture for their common use, for which they pay \$318. One puts in 20 oxen for 6 mo., another 24 oxen for 8 mo., and the third 28 oxen for 4 mo. How much of the rent should each pay?

5. A and B enter into partnership; A contributes \$15,000 for 9 mo., and B \$12,000 for 6 mo. They gain \$5750. Find each man's share of the gain.

6. A, B, and C rent a field for \$56.50; A puts in 70 cattle for 6 mo., B 40 for 9 mo., and C 50 for 7 mo. What ought each to pay?



7. Three merchants enter into partnership; the first invests \$1855 for 7 mo., the second invests \$887.50 for 10 mo., and the third invests \$770 for 11 mo., and they gain \$434. What should be each partner's share of the gain?

8. L, M, and N entered into partnership and invested respectively \$19,200, \$22,500, and \$28,300. At the end of 5 mo. L invested \$3800 additional, M \$2500, and N \$3700. At the end of a year the net gain of the firm was found to be \$7850. What was each partner's share of this?

9. A and B enter into partnership; A puts in \$400 at first, and \$500 at the end of 2 mo.; B puts in \$300 at first, and \$600 at the end of 3 mo. The profit at the end of the year is \$470. How should this be divided?

10. A and B engage in trade; A invests \$6000, and at the end of 5 mo. withdraws \$2000; B puts into the business \$4000, and at the end of 7 mo. \$6000 more. Divide a gain of \$6800 at the end of the year.

11. A, B, and C form a partnership with a joint stock of \$15,600; A's stock continues in trade 6 mo., B's 8 mo., and C's 12 mo. A's gain is \$1200, B's \$2400, and C's \$1800. What stock did each put in?

12. Two men complete in a fortnight a piece of work for which they are paid \$46.75; one of them works alternately 9 hr. and 8 hr. a day, the other works  $8\frac{1}{2}$  hr. for 5 da. in the week, and does nothing on the remaining day. What part of the sum should each receive?

13. A and B are partners; A's capital is to B's as 4 to 9. At the end of 4 mo. A withdraws  $\frac{1}{2}$  of his capital, and B  $\frac{2}{3}$  of his. At the end of the year their whole gain is \$4600. How much belongs to each?

14. A, B, and C rent a pasture for \$92; A puts in 6 horses for 8 wk., B 12 oxen for 10 wk., C 50 cows for 12 wk. If 5 cows are

reckoned equivalent to 3 oxen, and 4 oxen to 3 horses, what should each pay?

15. Three men, working respectively 8, 9, and 10 hr. a day, receive the same daily wages. After working thus for 3 da., each works 1 hr. a day longer, and the work is finished in 3 da. more. If \$114 is paid for the work, how much should each man receive?

16. Three workmen, A, B, and C, did a certain piece of work and were paid daily wages according to their several degrees of skill. A's efficiency was to B's as 4 to 3, and C's to B's as 5 to 6; A worked 5 da., B 6 da., and C 8 da. The whole amount paid for the work was \$36.25. Find each man's daily wages.

# CHAPTER XVI

## POWERS AND ROOTS

### SQUARE ROOT \*

**273.** (1) Find the square root of 17.3056.

	17.3056	4.16
	16	
81	130	
	81	
826	4956	
	4956	

To prove 4.16 the right answer, square 4.16 and the result will be found to be 17.3056.

(2) Extract the square root of 35 to three decimal places.

	35	5.916
	25	
109	1000	
	981	
1181	1900	
	1181	
11826	71900	
	70956	

(3) Extract the square roots of  $\frac{25}{49}$ ,  $\frac{35}{49}$ ,  $\frac{5}{8}$ .

$$\sqrt{\frac{25}{49}} = \frac{\sqrt{25}}{\sqrt{49}} = \frac{5}{7}$$

\* See Chapter VIII.

$$\sqrt{\frac{35}{49}} = \frac{\sqrt{35}}{\sqrt{49}} = \frac{5.916}{7} = .845.$$

$$\sqrt{\frac{3}{8}} = \sqrt{.625} = .7905.$$

Which denominator is not a perfect square? Why reduce  $\frac{3}{8}$  to a decimal before extracting the square root?

### Exercise 174

Find the square root of:

1. 40.96; 65.61; 2.1025.
2. 167.9616; 28.8369; 57648.01.
3. .042849; .00139876; .00203401.
4. 5774409; 5.774409.
5. 10.3041; 2321.3124; .0050367409.
6.  $\sqrt{2}$ ;  $\sqrt{20}$ ;  $\sqrt{.4}$ ;  $\sqrt{1000}$  to four decimal places.
7.  $\frac{144}{289}$ ;  $\frac{324}{881}$ ;  $6\frac{1}{4}$ .
8.  $20\frac{1}{4}$ ;  $1\frac{56}{169}$ ;  $\frac{1}{8}$ ;  $\frac{2209}{881}$ .
9.  $\frac{3}{8}$ ;  $\frac{5}{9}$ ;  $\frac{7}{11}$ .

### \* CUBE ROOT

**274.** The product of  $3 \times 3 \times 3$  is 27; of  $5 \times 5 \times 5$  is 125. The cubes whose sides measure 3 and 5 units of length contain 27 and 125 units of volume. We say that 27 is the cube of 3 and that 125 is the cube of 5; that 3 is the cube root of 27, and that 5 is the cube root of 125. The cube of 5 is written  $5^3$ , and the cube root of 5 is indicated thus:  $\sqrt[3]{5}$ .

$5^3$  is also called the third power of 5.

**275.** The cubes of

	1, 2, 3, 4, 5, 6, 7, 8, 9, 10,
are	1, 8, 27, 64, 125, 216, 343, 512, 729, 1000.

**276.** The cube roots of  
1, 8, 27, 64, 125, 216, 343, 512, 729, 1000,  
are 1, 2, 3, 4, 5, 6, 7, 8, 9, 10.

**277.** These two paragraphs should be mastered by the pupils as the corresponding paragraphs in square root.

**278.** The product  $4 \times 4 \times 4$  is written  $4^3$ .

The product  $4.6 \times 4.6 \times 4.6$  is written  $(4.6)^3$ .

The product  $\frac{3}{4} \times \frac{3}{4} \times \frac{3}{4}$  is written  $(\frac{3}{4})^3$ .

The cube root of 4 is written  $\sqrt[3]{4}$ .

The cube root of .4 is written  $\sqrt[3]{.4}$ .

The cube root of  $\frac{3}{4}$  is written  $\sqrt[3]{\frac{3}{4}}$ .

#### Exercise 175

Write the following products as powers:

1.  $2 \times 2 \times 2$ .

7.  $\frac{4}{5} \times \frac{4}{5} \times \frac{4}{5}$ .

2.  $3 \times 3$ .

8.  $\frac{7}{8} \times \frac{7}{8} \times \frac{7}{8}$ .

3.  $5 \times 5 \times 5$ .

9.  $2.3 \times 2.3 \times 2.3$ .

4.  $5 \times 5 \times 5 \times 5$ .

10.  $3.12 \times 3.12 \times 3.12$ .

5.  $5 \times 5 \times 5 \times 5 \times 5$ .

11.  $.12 \times .12 \times .12$ .

6.  $5 \times 5 \times 5 \times 5 \times 5 \times 5$ .

12.  $.1 \times .1 \times .1$ .

13.  $.02 \times .02 \times .02$ .

Write the following powers as products and find their values:

14.  $4^3$ .

15.  $12^3$ .

16.  $2.5^3$ .

17.  $(\frac{2}{3})^3$ .

18.  $(\frac{1}{5})^3$ .

19.  $.02^3$ .

20.  $.1^3$ .

Prove the following statements:

21.  $\sqrt[3]{216} = 6.$

26.  $\sqrt[3]{.064} = .4.$

22.  $\sqrt[3]{15625} = 25.$

27.  $\sqrt[3]{\frac{8}{27}} = \frac{2}{3}.$

23.  $\sqrt[3]{15.625} = 2.5.$

28.  $\sqrt[3]{\frac{125}{848}} = \frac{5}{8}.$

24.  $\sqrt[3]{1.728} = 1.2.$

29.  $\sqrt[3]{\frac{1728}{8375}} = \frac{4}{5}.$

25.  $\sqrt[3]{.008} = .2.$

30.  $\sqrt[3]{\frac{512}{1728}} = \frac{2}{3}.$

### Exercise 176

1. Find the length of one edge of a cube containing 512 cu. in. Find the length of all its edges. Find the area of one of its faces. Of all its faces.

2. Find the area of one face of a cube containing 729 cu. in.

3. Find the number of units of length in a cube containing 343 units of volume. Find the number of units of area in one face.

4. Find the edge of a cube one of whose faces contains 144 sq. in. Find its volume.

5. Find the volume of a cube one of whose faces contains 225 sq. in.

6. What is the edge of a cube whose volume is 8 units of volume? 27 units of volume?

7. The ratio of the volumes of two cubes is  $\frac{8}{27}$ . What is the ratio of their edges?

8. The ratio of the volumes of two cubes is 64:125. What is the ratio of their edges?

9. The ratio of the edges of two cubes is  $\frac{5}{7}$ . What is the ratio of their volumes?

10. The edges of two cubes are as 7:9. What is the ratio of their volumes?

## Exercise 177

1. Find the cubes of 14, 25, 36, 54, 75, and 99.
2. From the results in § 275, state how many digits there are in the cube of a number of 1 digit.
3. From the results in question 1, state how many digits there are in the cube of a number of 2 digits.
4. In long division how many figures form a group? In square root? In cube root?
5. How many digits are there in the cube root of 512? 64? 8? What are they?
6. Divide 389,017 into groups of figures; 29,791; 3375. How many figures are there in the cube root of each of these numbers?
7. Cube 73, 31, and 15.
8. Judging from the results given in § 276, state the number of digits in the cube root of a number containing 1, 2, or 3 digits.
9. Write the cubes of 10, 20, 30, 40, 50, 60, 70, 80, and 90.
10. State how many digits there are in the cube root of a number containing 4, 5, or 6 digits.
11. What is the first digit in the cube root of 2744? 39,304?

**279.** To find the cube root of a number, we shall first see how the cube of a number is found.

Since  $54 = 50 + 4$ , we can cube 54 thus:

$$\begin{array}{r}
 50 + 4 \\
 \hline
 50 + 4 \\
 \hline
 (4 \times 50) + 4^2 \\
 \hline
 50^2 + (4 \times 50) \\
 \hline
 50^2 + 2(4 \times 50) + 4^2 \\
 \hline
 50 + 4 \\
 \hline
 (4 \times 50^2) + 2(4^2 \times 50) + 4^3 \\
 \hline
 50^3 + 2(4 \times 50^2) + (4^2 \times 50) \\
 \hline
 50^3 + 3(4 \times 50^2) + 3(4^2 \times 50) + 4^3
 \end{array}$$

Since 4 divides each of the last three terms, we can put this result  $= 50^3 + 4\{3 \times 50^2 + 3 \times 4 \times 50 + 4^2\}$ .

We now wish to recover from such a number as 157,464 its cube root. Plainly the tens' digit of the root is 5, *i.e.* the first part of the root is 50.

	50	157'464   50 + 4
		125000
$3 \times 50^2 = 7500$		32464
$3 \times 4 \times 50 = 600$		
$4^2 = 16$		
	8116	32464

To find the second term, note that in the expression

$$4\{3 \times 50^2 + 3 \times 4 \times 50 + 4^2\}$$

the number 4 is the second digit in the number 54 that was cubed; hence in the work of taking the cube root, the other factor  $\{3 \times 50^2 + 3 \times 4 \times 50 + 4^2\}$  will be the real divisor and  $3 \times 50^2$  the trial divisor.

Therefore, squaring 50 and multiplying by 3, we have 7500. Dividing 7500 into 32,464, we find the quotient to be 4; completing the divisor by adding  $3 \times 4 \times 50$  or 600, and  $4^2$  or 16, we find the divisor to be 7500 + 600 + 16 or 8116. Multiplying this by 4, we have 32,464. Hence we conclude that the cube root of 157,464 is 54. To prove the result correct, cube 54.

**280.** The work of extracting the cube root may be shortened thus:

		157'464   54
		125
$300 \times 5^2 = 7500$		32464
$30 \times 5 \times 4 = 600$		
$4^2 = 16$		
	8116	32464

Find the cube root of 926,859,375.

		926'859'375   975
		729
$300 \times 9^2 = 24300$		197859
$30 \times 9 \times 7 = 1890$		
$7^2 = 49$		
	26239	183673
$300 \times 97^2 = 2822700$		14186375
$30 \times 97 \times 5 = 14550$		
$5^2 = 25$		
	2837275	14186375



**281.** The cube of 3.19 is equal to 32.461759. From this it is evident that corresponding to the two figures in the decimal part of the number, viz. 19, we have two groups of three figures, viz. 461 and 759, in the decimal part of the cube. Hence in pointing off, begin at the decimal point and mark the number off into periods of three figures each to the right of the decimal and then again to the left.

(1) Find the cube root of 95.443993.

	95.443'993   4.57
	64
$300 \times 4^2 = 4800$	31443
$30 \times 4 \times 5 = 600$	
$5^2 = 25$	
5425	27125
$300 \times 45^2 = 607500$	4318993
$30 \times 45 \times 7 = 9450$	
$7^2 = 49$	
616999	4318993

(2) Extract the cube root of 16.

	16   2.519
	8
$300 \times 2^2 = 1200$	8000
$30 \times 2 \times 5 = 300$	
$5^2 = 25$	
1525	7625
$300 \times 25^2 = 187500$	375000
$30 \times 25 \times 1 = 750$	
$1^2 = 1$	
188251	188251
$300 \times 251^2 = 18900300$	186749000
$30 \times 251 \times 9 = 667700$	
$9^2 = 81$	
19568081	176112729

(3) To extract the cube root of such a number as 843.7295, add ciphers thus, 843. 729'500, and extract the cube root.

- (4) Extract the cube root of  $\frac{64}{343}$ .

$$\sqrt[3]{\frac{64}{343}} = \frac{\sqrt[3]{64}}{\sqrt[3]{343}} = \frac{4}{7}.$$

- (5) Extract the cube root of  $\frac{16}{343}$ .

$$\sqrt[3]{\frac{16}{343}} = \frac{\sqrt[3]{16}}{\sqrt[3]{343}} = \frac{2.519}{7} = .359.$$

- (6) Extract the cube root of  $\frac{16}{25}$ .

$$\begin{aligned}\frac{16}{25} &= .64 \text{ or } .640. \\ \sqrt[3]{.640} &= .861. \\ \therefore \sqrt[3]{\frac{16}{25}} &= .861.\end{aligned}$$

Which of the three denominators is not a perfect cube? When should a fraction be reduced to a decimal before extracting its cube root? Why?

### Exercise 178

Find the cube root of:

1. 29,791.

2. 54,872.

3. 110,592.

4. 804,357.

5. 941,192.

6. Compare the processes of long division, of extracting the square root, and the cube root of a number. Note in what respect they are similar and in what respect they are different.

7. 2,406,104.

13. .001906624.

8. 69,426,531.

14. 3, .3, .03, .003, .0003.

9. 8,365,427.

15.  $\frac{8}{27}$ ,  $\frac{125}{729}$ ,  $\frac{5832}{12167}$ .

10. 389.017.

16.  $\frac{250}{886}$ ,  $\frac{135}{320}$ .

11. 32.461759.

17.  $\frac{2}{3}$ ,  $\frac{49}{216}$ ,  $\frac{5}{7}$ .

12. .000912673.

18.  $3\frac{3}{8}$ ,  $405\frac{28}{125}$ ,  $7\frac{1}{5}$ .

19. A cubical block of stone contains 50,653 cu. ft. What is the area of its side?

20. A cube contains 56 cu. ft. 568 cu. in. Find its edge.

21. One gal. contains 231 cu. in. Find the edge of a cube equal to it.

22. Find the length of the inside edge of a cubical vessel which will just hold 10 gal.

23. Three cubes of lead, measuring respectively  $\frac{1}{2}$ ,  $\frac{2}{3}$ , and  $\frac{5}{8}$  of an in. on the edge, were melted together and cast into a single cube. Find the length of the edge of the cube thus formed, neglecting loss of lead in melting and casting.

24. Four cubes of lead, measuring respectively 6, 7, 8, and 9 in. on the edge, were melted together and cast into a single cube. Find the length of the edge of the cube thus formed, if 4% of the lead was lost in melting and casting.

25. Find the volume of a cube, the area of whose surface is 100.86 sq. in.

26. A cube measures 5 in. on the edge. A second cube is three times the volume of the first. By how much does the length of an edge of the second cube exceed that of an edge of the first cube?

27. By raising the temperature of a cube of iron, the length of each of its edges was increased by 5%. Find correct to four decimals the ratio of increase in the volume of the cube.

28. Each edge of a cube is diminished by  $\frac{1}{10}$  of its length. By what fraction of itself is the volume diminished? By what fraction of itself is the area of the surface diminished?

## CHAPTER XVII

### MENSURATION

**282.** The rectangle has been treated of in preceding paragraphs.

#### Exercise 179

1. The units of lengths being 1 in., 2 in., 3 in., 4 in., 5 in., 6 in., what are the units of area? Make mental pictures of the units of area.

2. If the length and breadth of a rectangle are respectively 7 and 5 times the unit of length, how many times the unit of area is the area of the rectangle? If the unit of length is 6 in., what is the area of the rectangle?

3. If the ratios of the length and breadth of a rectangle to the unit of length are respectively 5 and 3, what is the ratio of the area of the rectangle to the unit of area?

4. How do you find the number of units of area in a rectangle?

5. The measure of the area of a rectangle is 48, the area being 432 sq. in. What is the unit of area? Of length?

6. Draw a rectangle 2 in. by 3 in. in one corner of a rectangle 6 in. by 9 in. Are the figures similar in shape? Find the area of each rectangle. What is the ratio of the two breadths? Of the two lengths? Of the two areas? What is the relation between the ratio of the areas and the ratio of the lengths?

7. Draw a rectangle 3 in. by 4 in. in one corner of a rectangle 6 in. by 8 in. Are the figures similar? What is the ratio of the breadths? Of the lengths? What, then, is the ratio of the areas? Find the area of each rectangle and prove your last result correct.

8. The sides of a rectangle are 4 in. and 6 in. respectively. How often will it measure another rectangle whose sides are twice as long? 3 times as long? 4 times? 5 times?

9. The unit of area is a rectangle 3 in. by 5 in. How often will it measure a rectangle each of whose sides is 6 times as long? 7 times? 8 times? 9 times?

10. Two rectangles are similar in shape, and the second is 4 times as large as the first. Compare the lengths of their sides. Illustrate by a drawing and make a mental picture.

11. The areas of two similar rectangles are as 9:1. What is the ratio of their sides?

12. The ratio of the areas of two similar rectangles is 9:4. What is the ratio of their sides?

13. Make a rule showing how to find the ratio of the sides of two similar rectangles when you know the ratio of their areas.

14. What are the ratios of the areas of the following pairs of similar rectangles, the ratios of the sides being 4 to 1; 5 to 1; 3 to 4; 7 to 2? If the area of the smaller rectangle is 16 sq. ft., what is that of the larger in each case?

15. What are the ratios of the sides of the following pairs of similar rectangles, the ratios of the areas being 64 to 1; 121 to 1; 49 to 25; 144 to 100? If the sides of the smaller rectangle are 20 and 30 in., what are the sides of the larger rectangles?

**283.** (1) Find to the nearest inch the side of a square whose area is 1 A.

$$1 \text{ A.} = 6,272,640 \text{ sq. in.}$$

$$\text{The measure of the area} = 6,272,640.$$

$$\text{The measure of the side} = 2504.5 +.$$

$$\therefore \text{the side of the square, to the nearest in.} = 2505 \text{ in.} = 69 \text{ yd. } 1 \text{ ft. } 9 \text{ in.}$$

(2) The lengths of the sides of a rectangular piece of land are as 3 to 8 and its area is 60 A. Find the length of its sides.

Imagine the length of the field to be divided into 8 equal parts and the breadth into 3, and the field be divided into 24 equal squares by lines drawn through these points of division parallel to the sides of the field. Consider one of these squares the unit of area and one of its sides the unit of length.

Then  $24 \times$  the unit of area = 60 A. or 600 sq. ch.

The unit of area = 25 sq. ch.

The unit of length = 5 ch.

$\therefore$  the lengths of the sides = 15 ch. and 40 ch.

### Exercise 180

1. A square field contains exactly 8 A. Determine the length of a side of the field, correct to the nearest link.
2. The area of a chess-board marked in 8 rows of 8 squares each is 100 sq. in. Find the length of a side of a square.
3. On a certain map it is found that an area of 16,000 A. is represented by an area of 6.25 sq. in. Give the scale of the map in miles to the inch.
4. A rectangle measures 18' by 30'. Find the difference between its area and that of a square of equal perimeter.
5. Two rectangular fields are of equal area. One field measures 15 ch. by 20 ch.; the other is square. Find the length of a side of the latter field, correct to the nearest link.
6. How many stalks of wheat could grow on 1 A. of ground, allowing each stalk a rectangular space of 2" by 3"?
7. How many pieces of turf 3' 6" by 1' 3" will be required to sod a rectangular lawn 28' by 60'?
8. Sidewalks 12 ft. wide are laid on both sides of a street 440 yd. long. Find the cost of the sidewalks at \$1.35 per sq. yd. for the pavement and 75¢ per lineal yd. for curbing; deducting three crossings of 54 ft. each on both sides of the street.
9. The area of a rectangular field is 15 A.; the length of the field is double the width. Find the length of the field in chains.
10. What length must be cut off a board, which is  $7\frac{1}{2}$  in. broad, so that the area may contain 3 sq. ft.?

11. Find the area of each of the following rectangles, whose dimensions are: 1.  $L = 36$  ft.,  $B = 13$  ft. 2.  $L = 20$  ft. 3 in.,  $B = 20$  in. 3.  $L = 8$  ft. 9 in.,  $B = 3$  ft. 8 in.

12. Find the solid contents of the following: 1.  $L = 13$  ft. 4 in.,  $B = 7$  ft. 6 in.,  $H = 3$  ft. 10 in. 2.  $L = 20$  ft.,  $B = 1$  ft. 6 in.,  $H = 1$  ft. 2 in.

13. Find the length of the following rectangles: 1. Area = 40 sq. yd.,  $B = 20$  ft. 2. Area = 6 sq. ft.,  $B = 9$  in.

14. Find the area of the four walls of the following rooms: 1.  $L = 32$  ft.,  $B = 18$  ft.,  $H = 11$  ft. 2.  $L = 29$  ft.,  $B = 23\frac{1}{2}$  ft.,  $H = 11\frac{1}{4}$  ft.

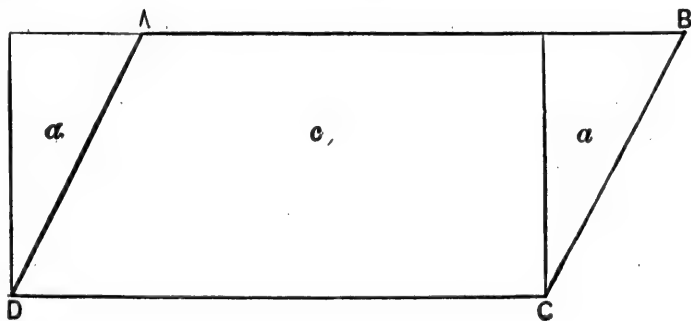
15. Find the cost of painting a surface: 1. 19 ft. 6 in. by 83 ft. 4 in., at 40¢ a sq. ft. 2. 25 ft. 8 in. long, and 16 ft. 9 in. wide, at 65¢ a sq. ft.

16. A gentleman wishes to set out a rectangular orchard of 1260 trees, so placed that the number of rows shall be to the number of trees in a row as 5 to 7. Find the number of rows and also the number of trees in a row.

17. The sides of a rectangular field containing 27 A. 48 sq. rd. are as 21 to 13. Find the perimeter of the field

**284. A Quadrilateral** is a plane figure having four sides.

A **Parallelogram** is a quadrilateral whose opposite sides are parallel.



**285.** To find the area of a parallelogram :

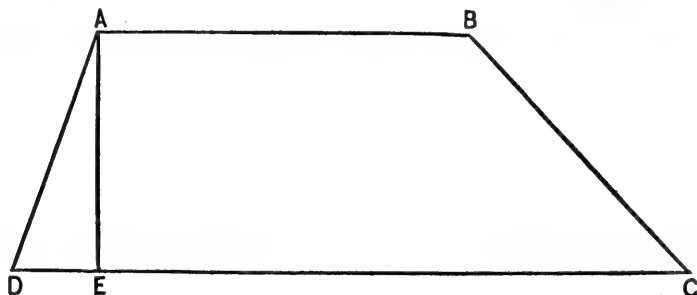
Let perpendiculars be drawn from  $C$  and  $D$  perpendicular to  $AB$ . Then it is evident that the triangles marked  $a$  are equal. Adding to each the quadrilateral marked  $c$ , it is evident that the parallelogram  $ABCD$  is equal to the rectangle upon the base  $CD$ .

*Hence, to find the measure of the area of a parallelogram, multiply the measure of its base by the measure of its altitude.*

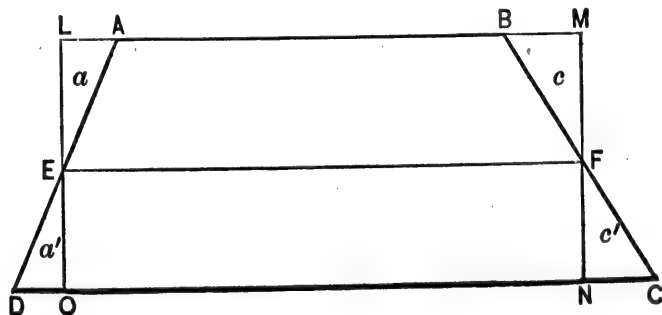
Draw any parallelogram and draw the perpendiculars as in the figure. Cut out the triangles and place them on each other, showing that they are equal.

**286.** A **Trapezoid** is a quadrilateral two of whose sides are parallel.

The parallel sides are called **Bases** and the perpendicular distance between the two bases is called the **Altitude**.



Thus, in the trapezoid,  $AB$  and  $CD$  are the bases, and  $AE$  the altitude.

**287.** To find the area of a trapezoid :



Let  $ABCD$  be a trapezoid, and let perpendiculars be drawn through  $E$  and  $F$ , the middle points of  $AD$  and  $BC$ , to  $AB$  and  $CD$ . Then it is evident that the triangles  $a$  and  $a'$  are equal and also  $c$  and  $c'$ .

To  $a'$  and  $c'$  and also to  $a$  and  $c$  add the figure  $ABFNOE$ , and we have the trapezoid equal to the rectangle  $LMNO$ .

Again,

$$EF = AB + AL + BM,$$

$$EF = CD - DO - NC.$$

Adding,

$$2 EF = AB + CD, \text{ since}$$

$$AL = DO \text{ and } BM = NC,$$

$$\text{i.e. } EF = \frac{1}{2}(AB + CD).$$

$$\therefore ON = \frac{1}{2}(AB + CD).$$

Therefore, to measure the area of the trapezoid, we multiply the measure of  $ON$ , i.e. of  $\frac{1}{2}(AB + CD)$ , by that of the altitude.

*Hence the area of a trapezoid is found by multiplying the measure of one-half the sum of its parallel sides by the measure of its altitude.*

**288.** Find the area of a trapezoid whose parallel sides are  $12' 7''$  and  $19' 3''$  respectively, the perpendicular distance between them being  $8' 5''$ .

$$\text{The sum of the bases} = 12' 7'' + 19' 3'' = 31' 10''.$$

$$\frac{1}{2} \text{ the sum of the bases} = 15' 11'' = 191''.$$

$$\text{The altitude} = 8' 5'' = 101''.$$

$$\therefore \text{the area} = 101 \times 191 \text{ sq. in.} = 19,291 \text{ sq. in.} = 133 \text{ sq. ft. } 139 \text{ sq. in.}$$

### Exercise 181

1. Fold a sheet of paper in the form of a trapezoid. Draw the lines as given in the figure in § 287, cut out the triangles  $a$  and  $c$ , and, by placing them on  $a'$  and  $c'$  respectively, show that  $a = a'$  and  $c = c'$ .

2. In the figure in question 1, produce  $DC$  to  $G$ , making  $CG = AB$ . Measure along  $DG$  twice with a line equal to  $EF$ , thus showing that twice  $EF = CD + AB$ .

3. The length of the base of a parallelogram is 45 ft.; the length of the perpendicular on the base from the opposite side is 28 ft. Find the area.

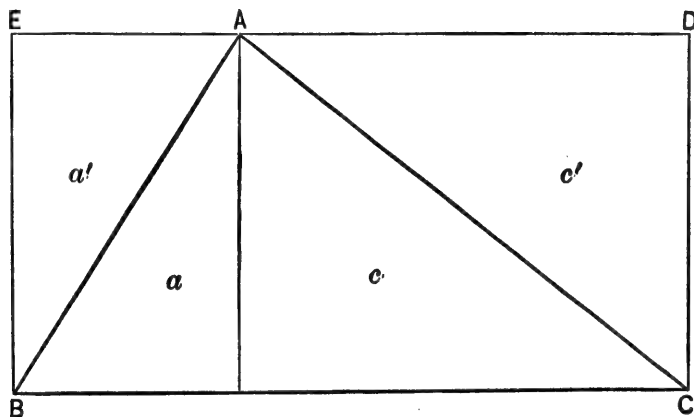
4. The adjacent sides of a parallelogram measure 132 ft. and 84 ft. respectively, and the area of the parallelogram is  $\frac{2}{3}$  of that of a square of equal perimeter. Find the perpendicular distance between each pair of parallel sides.

5. The lengths of the parallel sides of a trapezoid are 12 ft. and 17 ft., and the perpendicular distance between these sides is 8 ft. Find its area.

6. If the parallel sides of a garden are 84 and 92 ft. respectively, and their perpendicular distance  $82\frac{1}{2}$  ft., what did it cost at \$1200 an A.?

7. The area of a trapezoidal field is  $3\frac{1}{8}$  A., and the sum of the lengths of the parallel sides is 440 yd. Find the perpendicular distance between these sides. The lengths of the parallel sides being in the ratio of 5 to 6, find these lengths.

8. The area of the trapezoid is 9750 sq. yd., and the perpendicular distance between the parallel sides is 234 ft. If the length of one of the parallel sides be 410 ft., what will be the length of the other parallel side?



**289.** Let  $ABC$  be a triangle, and let the rectangle  $BCDE$  be drawn. Then it is evident that the triangles  $a$  and  $a'$  and  $c$  and  $c'$  are equal. Hence the triangle  $ABC$  is one-half of the rectangle  $BCDE$ . Hence, to find the area of a triangle, multiply one-half the measure of the base by that of the altitude.

**290.** To find the area of a triangle when the lengths of the sides are given :

*Find one-half of the sum of the measures of the sides; subtract from this the measure of each side separately. The square root of the product of these four results will give the measure of the area of the triangle.*

#### Exercise 182

1. Find the area of a triangle whose base is 45 ft., and altitude 17 ft.

2. A triangular piece of ground containing  $4\frac{1}{2}$  A. has a base of 135 yd. Find its altitude.

Find the areas of the triangles the lengths of whose sides are respectively :

3. 13 yd., 10 yd., and 13 yd.

4. 13 yd., 24 yd., and 13 yd.

5. 13 ft., 4 ft., and 15 ft.

6. 13 ft., 14 ft., and 15 ft.

7. 13 in., 11 in., and 20 in.

8. 13 in., 21 in., and 20 in.

9. 1.23 ch., 5.95 ch., and 6.76 ch.

10. 73.2 ch., 45.5 ch., and 87.6 ch.

11. Find the number of A., etc., that there are in a triangular field of which the sides are 7 ch. 60 l., 9 ch. 50 l., and 5 ch. 70 l.

12. The three sides of a triangle are 33 ft., 56 ft., and 65 ft. Find the measure of the triangle cut off by joining the points of bisections of the two greater sides.

13. Multiply the length of each side of the triangle in question 3 by 3. Find the area of the resulting triangle. What is the ratio of its area to that of the  $\Delta$  in question 3?

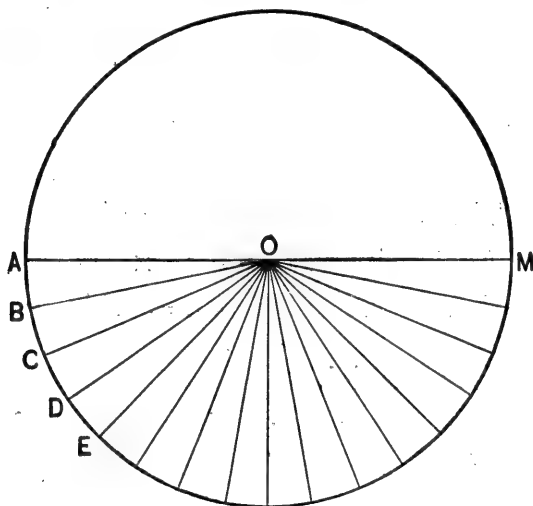
14. Similarly, multiply each side of the  $\Delta$  in question 4 by 5, and find its area. Find the area of the resulting triangle. What is the ratio of the areas?

15. If the sides of the triangle in question 5 be multiplied by the number 6, what will be the ratio of its area to that of the triangle in question 5? Hence find its area.

16. If each side of the triangle in question 7 be made 10 times as long, how much greater will be the area of the triangle?

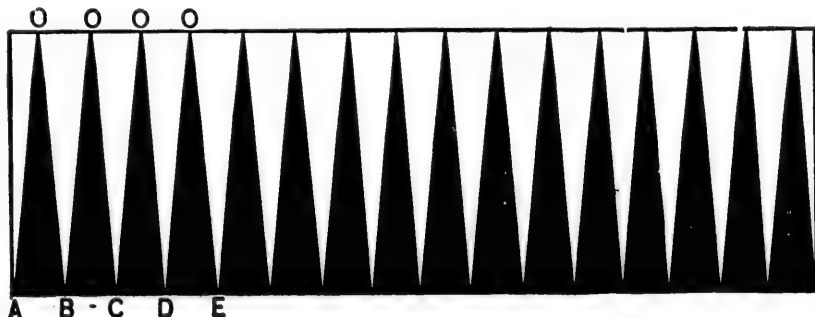
17. State in what ratio the area of a triangle is increased by increasing the length of each side.

291. To find the area of a circle :



Draw a circle on cardboard and cut it out — the larger the better. Divide each half of the circle as the semicircle in the figure is divided, the arcs  $A, B, C, D$ , etc., being as nearly equal as possible. Cut the circle into two equal parts along the line  $AOM$ .

Cut along  $OB, OC$ , etc., cutting nearly to the points  $B, C, D$ , but not separating the parts entirely at these points. Spread the resulting figure out as in the darker part of the figure below.



Then cut up the other semicircle in the same way ; spread open the parts and fit the two semicircles together as in the figure. The resulting figures will be nearly a rectangle. The smaller the arcs  $AB, BC$ , etc., the more nearly the area will be to a rectangle whose base is equal to one-half the circumference and whose altitude is equal to the radius of the circle.

*Hence the measure of the area of a circle is one-half the product of the measures of the circumference and the radius. It may also be expressed thus :*

The measure of the area  $= \frac{1}{2} cr$ .

Again, since  $c = 3.1416 \times 2r$ ,

*the measure of the area  $= 3.1416 r^2$ .*

Both formulas are useful.

The last rule may be read: *The measure of the area of a circle is found by multiplying the square of the measure of the radius by 3.1416.*

292. (1) Find the area of a circle whose diameter is  $7\frac{1}{2}$  in.

The measure of the area =  $3.1416 r^2$ .

The radius =  $7\frac{1}{2}$  in.  $\div 2 = 3.75$  in.

The measure of the area =  $3.1416 \times 3.75^2 = 44.18$ .

$\therefore$  the area = 44.18 sq. in.

- (2) If the arc of a circle is 2 ft. and the radius 6 ft., find how many degrees there are in the arc.

The length of the circumference =  $3.1416 \times 12$  ft. = 37.6992.

37.6992 ft. of circumference contain  $360^\circ$ .

1 ft. of circumference contains  $\frac{360^\circ}{37.6992}$ .

2 ft. of circumference contain  $\frac{720^\circ}{37.6992}$  or  $19^\circ 5' 54''$ .

$\therefore$  the arc contains  $19^\circ 5' 54''$ .

- (3) Find the length of a radius of a circle whose area is 4 A.

4 A. = 19,360 sq. yd.

The measure of the area =  $3.1416 r^2$ .

$\therefore r^2 = 19,360 \div 3.1416 = 6162.46$ .

$\therefore r = \sqrt{6162.46} = 78.5$  yd.

### Exercise 183

- Find the area of a circle 7 ft. in diameter.
- Find the area of a quadrant whose radius is 4 rd.
- Find the length of the radius of a circle whose area is 1 A.
- Find the length of the diameter of a circle whose area is 1 sq. mi.
- Find the length of the radius of a circle whose area is equal to the sum of the areas of four circles of 10 in., 15 in., 18 in., and 24 in. radius respectively.
- Find the total pressure on a plate 25 in. in diameter, the pressure per sq. in. being 65 lb.

7. A circular hole is cut in a circular metal plate of 7 in. radius, so that the weight of the plate is reduced by 40%. Find the length of the radius of the hole.

8. The area of a semicircle is 13.1 sq. in. Find the length of its perimeter.

9. The lengths of the sides of a triangle are 13 ft., 14 ft., and 15 ft. respectively. Find the difference between the area of the triangle and that of a circle of equal perimeter.

10. The perimeters of a circle, a square, and an equilateral triangle are each 17 ft. in length. Find by how much the area of the circle exceeds the area of the other figures.

11. Find the length of the diameter of a circle whose area is equal to that of a square whose sides are each 12 ft. long.

12. Out of a circle of radius 3 ft. is taken a circle of radius 2 ft. Find the area of the remainder.

13. Find the length of the arc which subtends an angle of  $60^\circ$  at the centre of a circle of 10 in. radius.

14. Find the length of the arc which subtends an angle of  $36^\circ$  at the centre of a circle of 25 in. radius.

15. How many degrees are there in the angle which an arc whose length is 1 ft. subtends at the centre of a circle of 2 ft. radius?

16. The length of the radius of a circle is 8 in. Find the length of the arc of which the angle is (a)  $90^\circ$ , (b)  $270^\circ$ .

17. There is a circular fish-pond of 90 ft. radius, surrounded by a walk 25 ft. in breadth. Find the area of the walk.

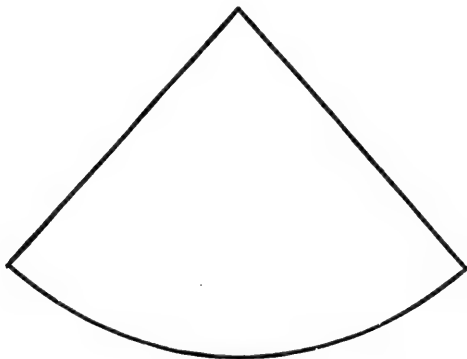
18. Within a circular garden 70 ch. in circumference is a circular pond 70 rd. in circumference. Find the width of the ring of land which surrounds the pond.

19. The radii of four circles are respectively 2 ft., 3 ft., 4 ft., 5 ft. Show that their areas are as the numbers 4, 9, 16, and 25.

**293.** If we wrap a rectangular sheet of paper about a cylinder, we find that the area of the curved surface of the cylinder is a rectangle whose base is the circumference of the cylinder and altitude the height of the cylinder. Hence we can find its area.

**294.** *To find the measure of the volume of a cylinder, take the product of the measures of the area of the base and the altitude.*

**295.** The curved surface of a cone can be unwrapped into a portion of a circle.



*Hence the measure of the curved surface is one-half the product of the measure of its base by its slant height.*

**296.** Make a cylinder out of paper and also a right circular cone having the same altitude and base. Fill the cone with some dry material and empty it into the cylinder. Do this three times and the cylinder will be just filled. Hence the volume of a right circular cone is one-third that of a cylinder of equal base and altitude. *Hence, to find the volume of a right circular cone, multiply one-third the measure of the area of the base by the measure of the altitude.*



**297.** (1) The length of the radius of a right circular cylinder is 5 in. and its altitude is 8 in. Find its volume and the area of its curved surface.

The measure of the area of the base =  $3.1416 \times 25 = 78.54$ .

The measure of the volume of the cylinder =  $8 \times 78.54 = 628.32$ .

$\therefore$  the volume = 628.32 cu. in.

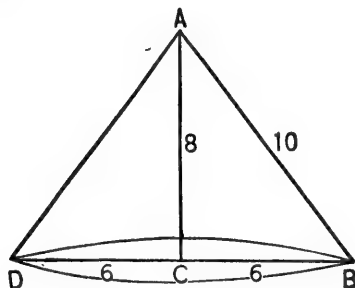
Again,

The measure of the circumference of the base =  $3.1416 \times 10 = 31.416$ .

The measure of the area of the curved surface =  $8 \times 31.416 = 251.328$ .

$\therefore$  the area = 251.328 sq. in.

(2) Find the area of the curved surface and also the volume of a cone whose altitude is 8 in. and whose base is 12 in. in diameter.



Since the altitude  $AC$  is perpendicular to the diameter  $BCD$ , the triangle  $ACB$  is a right triangle.

Hence  $AB$  is 10 in.

Also the circumference of the base =  $3.1416 \times 12$  in. = 37.6992 in.

The measure of the area =  $\frac{1}{2} \times 10 \times 37.6992 = 188.496$ .

$\therefore$  the area = 188.496 sq. in.

Again,

The altitude of the cone = 8 in.

The area of the base =  $3.1416 \times 6^2$  or 113.0976 sq. in.

The measure of the volume =  $\frac{1}{3} \times 8 \times 113.0976 = 301.5936$ .

$\therefore$  the volume = 301.59 cu. in.

**Exercise 184**

1. Make a cone and cylinder each of whose bases is 9 in. in circumference and altitudes 6 in.

Fill the cone and empty it three times, in succession, into the cylinder. What is the result?

2. Find the volume of a cylinder the radius of whose base is 10 in., the altitude being 18 in.

3. Find the volume of a cone the radius of whose base is 10 in., the altitude being 18 in.

4. How often can the cone in question 3 be filled and emptied into the cylinder in question 2?

5. The length of the radius of the base of a right circular cylinder is 9 in. and its altitude is 16 in. Find the volume.

6. Find the area of the curved surface of the cylinder in question 5. Find the area of its entire surface.

7. Find the volume of a cone whose altitude is 15 in., and whose base is a circle 10 in. in diameter.

8. Find the volume of a cone whose altitude is 12 in., and the diameter of whose base is 5 in.

9. Find the area of the curved surface of a cone whose altitude is 20 in., and the radius of whose base is 15 in. Find also its total area.

10. What must be the height of a cylindrical column of marble, the radius of whose base is 9 in., in order that it may contain  $5\frac{1}{2}$  cu. ft.?

11. If the diameter of a cylindrical well be 5 ft., and its depth 27 ft., how many cu. yd. of earth were removed in order to form it?

**298.** It can be shown that the area of the curved surface of a hemisphere is equal to twice the area of its flat surface;

hence the area of the surface of the sphere is equal to 4 times the area of this flat surface.

Thus the measure of the area of the surface of a sphere is 4 times the product of 3.1416 and the square of the measure of the length of the radius.

$$A = 4 \times 3.1416 r^2.$$

**299.** If we imagine a sphere to be divided into a large number of small cones, as in § 291 we divided the circle into triangles, the centre of the sphere being the vertex of each cone, and a small portion of the circumference being its base, we can think of the volume of the sphere as being equal to the sum of the volumes of the cones. The altitude of each cone is equal to the radius of the sphere, and the total area of their bases is equal to the area of its surface. Hence the volume of the sphere is given by the formula:

$$\begin{aligned} V &= \frac{1}{3} r (4 \times 3.1416 \times r^2) \\ &= \frac{4}{3} \times 3.1416 r^3. \end{aligned}$$

*Hence the measure of the volume of the sphere is  $\frac{4}{3}$  of 3.1416 times the cube of the measure of the radius.*

**300.** (1) Find the surface of a sphere whose radius is 6 in.

The measure of the area  $= 4 \times 3.1416 \times 6^2 = 452.3904$ .

$\therefore$  the area  $= 452.39$  sq. in.

(2) Find the volume of a sphere whose diameter is 8 in.

The measure of the volume  $= \frac{4}{3} \times 3.1416 \times 4^3 = 268.0832$ .

$\therefore$  the volume  $= 268.08$  cu. in.

### Exercise 185

1. Find the surface of a sphere whose radius is 3 in.
2. Find the surface of a sphere 12 in. in diameter.
3. Find the volumes of the spheres given in questions 1 and 2.

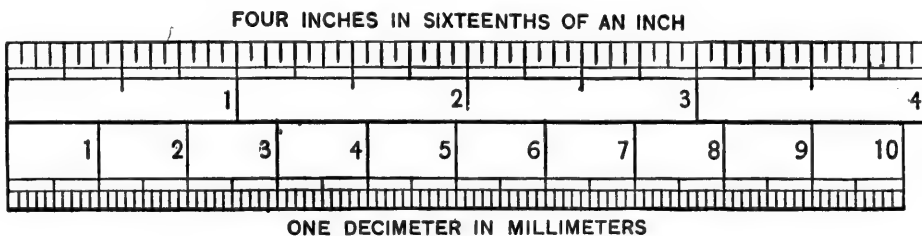
4. Find the surface of a sphere 5 ft. in diameter.
5. Find the volume of a sphere whose diameter is 16 ft.
6. Place a croquet or base ball between two chalk boxes. Place a foot measure in line with one edge of each box. What is the diameter of the ball? What is the area of its surface? What is its volume?
7. With a pair of compasses draw a circle with the diameter found in question 6. Cut out this circle and pass the ball through the hole.
8. If the pressure of the air is equal to 15 lb. a sq. in., what is the pressure on the surface of a sphere 6 in. in diameter?

## CHAPTER XVIII

### THE METRIC SYSTEM OF WEIGHTS AND MEASURES

**301.** The French or **Metric System** of Weights and Measures is based upon the decimal system. It is used in scientific treatises and has been adopted by most of the nations of Europe and South America. It is also in partial use in the United States and Canada.

**302.** The fundamental unit of the metric system is the **Meter**, which is 39.37 in. long. The original standard meter is a platinum rod, called the French Standard Meter, which is deposited in the Archives at Paris.



The length of the measure in the diagram is  $\frac{1}{10}$  of a meter, and is called a decimeter. It is divided into 10 equal parts, each of which is called a centimeter. Each of these is divided into 10 equal parts, each of which is called a millimeter.

**303.** In order that pupils may study this system of weights and measures to the best advantage, the school should be provided with a system of

metric weights and measures, and each pupil with a foot rule on which the decimeter, centimeter, and millimeter are marked.

An intelligent study of the system can, however, be made if the teacher has a metric stick and a liter for reference.

**304.** The names of the higher or lower units in the metric system are formed by attaching certain prefixes to the names of the standard units, thus:

*Deca* signifies 10 times the unit.

*Hecto* signifies 100 times the unit.

*Kilo* signifies 1000 times the unit.

*Deci* signifies the 10th part or .1 of the unit.

*Centi* signifies the 100th part or .01 of the unit.

*Milli* signifies the 1000th part or .001 of the unit.

UNITS OF LENGTH

1 millimeter (mm.)	= .001 meter
1 centimeter (cm.)	= .01 meter
1 decimeter (dm.)	= .1 meter
1 meter (m.)	= Standard unit
1 decameter (Dm.)	= 10 meters
1 hectometer (Hm.)	= 100 meters
1 kilometer (Km.)	= 1000 meters
1 myriameter (Mm.)	= 10,000 meters

The units in common use are the *millimeter*, *centimeter*, *meter*, and *kilometer*.

**305.** Write out the table of the units of length, thus:

(a) 10 millimeters = 1 centimeter, etc.

(b) 1 millimeter =  $\frac{1}{10}$  centimeter, etc.

Exercise 186

1. Measure with a yardstick in the school-yard a distance of 11 yd. Measure along the same distance 10 times with the meter. What is the difference in inches?

2. Reduce 11 yd. and also 10 m. to in., and verify the result obtained in the first question.

3. Measure the length of the schoolroom in m. and decimals of a m.

4. Find the hypotenuse of a right triangle whose sides are (1) 6 m. and 8 m., (2) 24 cm. and 45 cm.

5. Librarians frequently use the centimeter as the unit in registering the heights of books. Express in cm. the height of your (a) Arithmetic, (b) History, (c) Geography, (d) and also of several other books.

6. Measure and express in terms of the cm. the distances between the lines on ruled paper.

7. Measure and express in terms of the cm. the height of the schoolroom thermometer.

8. How many cm. are there in a full line of this book?

9. Press tightly together the leaves of this book. Make a layer just 1 cm. thick. Count the leaves and find the thickness of 1 leaf as a decimal of a mm.

10. Cut a slit 2 mm. wide, by 3 cm. long, in a sheet of paper.

11. How many mm. are there in the width of your pencil?

12. Find the number of in. in 1 Km., reduce the result to a decimal of a mi., and show that 1 Km. is nearly equal to  $\frac{5}{8}$  of a mi.

13. If a train travels at the rate of 20 m. a sec., what is the rate in Km. per hr.?

14. Show that 5 in. are very nearly equal to 127 mm.

15. For what do we use the in., the yd., and the mi.? For what are the mm., the cm., the m., and the Km. respectively used?

## UNITS OF AREA

**306.** The principal units for measuring land are the square meter, called the **Centare** (ca.), the square decameter called the **Are** (a.), and the square hektometer called the **Hectare** (ha.).

1 sq.  
cm.

1 sq. dm.

Each side of this square measures

1 dm., or

$3\frac{1}{8}$  in., very nearly.

A liter is a cube each face of which has the dimensions of this square.

A gram is the weight of a cubic centimeter (see small square above) of distilled water, weighed *in vacuo* at temperature of maximum density, 39.1 F. A liter or cubic decimeter of such water weighs 1 kg. or  $2\frac{1}{8}$  lb. nearly.

**307.** How many units of length are there in the side of a square meter, the decimeter being the unit?

How many units of area are there in a square meter, the square decimeter being the unit?



How many units of area are there in a square decimeter, the square centimeter being the unit of area?

What part of a square meter is a square decimeter?

Write a square decimeter as a decimal of a square meter.

Write a square centimeter as a decimal of a square decimeter. As a decimal of a square meter. What is the ratio of each unit of area in the metric system to the next smaller, and also to the next higher?

#### UNITS OF AREA

1 square millimeter (q. mm.)	= .000001 of a square meter
1 square centimeter (q. cm.)	= .0001 of a square meter
1 square decimeter	= .01 of a square meter
1 square meter (q. m.)	= <i>standard unit</i>
1 square decameter	= 100 square meters
1 square hektometer	= 10,000 square meters
1 square kilometer (q. Km.)	= 1,000,000 square meters
1 centare (ca.)	= .01 are (a.)
1 hectare (ha.)	= 100 ares

Note that the square meter is  $1\frac{1}{2}$  times as large as the square yard.

#### Exercise 187

1. Cut out of paper a q. dm. and also a q. cm. What is the ratio of the two areas?
2. Draw a sq. ft. and measure it with the q. dm. of paper as the unit of area.
3. A q. dm. contains .10764 sq. ft. Find the number of q. dm. contained in a sq. ft. correct to two decimal places, and compare the result with that obtained in question 2.
4. Make a drawing of a q. m., and draw a sq. yd. within it.
5. Mark out an *are* on the school-ground.
6. State the table of area, expressing each unit of area as equal to 100 times the next lower.
7. Find the area of a page of this book in q. cm.
8. Find the area of the room in ca.

## UNITS OF VOLUME

**308.** The principal units of volume are the cubic meter, also called the **Stere**, and the cubic decimeter, called the **Liter**.

**309.** How many units of length are there in a side of a meter, the decimeter being the unit?

How many units of volume are there in the cubic meter or stere, the cubic decimeter or liter being the unit?

What is the ratio of each unit of volume in the metric system to the next smaller unit? To the next larger?

## UNITS OF VOLUME

1 cubic millimeter (c. mm.) = .000000001 of a cubic meter

1 *cubic centimeter* (c. cm.) = .000001 of a cubic meter

1 cubic decimeter = .001 of a cubic meter

1 *cubic meter* (c. m.) = standard unit

**Exercise 188**

1. Make a liter out of paper.
2. Fill a qt., liquid measure, with sand and empty it into your liter. Which of the two measures is larger?
3. Fill a liter with sand and empty it into a qt., dry measure. Which of the two is larger?
4. Fill a gal. measure, using the liter as a dipper, and note how many liters are equivalent to the gal.
5. If 1 liter is equal to .264 gal., find correct to two decimal places the number of liters in a gallon. Compare this result with that obtained in question 4.
6. State some purposes for which the liter is used.
7. Make a c. cm. out of paper.
8. Make the necessary measurements and compute the volume of the room in c. m.

9. Make the necessary measurements and compute the volume of a box in liters.

10. Express as a decimal part of a c. m. the volume of a beam 3 m. long, 10 cm. wide, and 5 cm. thick.

11. A cylindrical vessel having a base of a q. m. is filled with water to the depth of 2 m. How many liters of water does it contain?

12. How many liters of water may be held by a vessel measuring  $25 \times 35 \times 75$  cm.?

13. What will it cost to build a wall 1 Hm. long,  $\frac{1}{2}$  dm. thick, and 1 m. high, at \$5 a c. m.?

#### UNITS OF WEIGHT

**310.** The principal units of weight are the **Gram** and the **Kilogram**.

The **Gram** is the weight of a cubic centimeter of distilled water at  $40^\circ$ , at which temperature water is at its maximum density.

A nickel weighs 5 g.

A liter of distilled water at  $40^\circ$  weighs 1 kg. The kilogram is nearly equal to  $2\frac{1}{5}$  lb. Avoir.

A cubic meter of water at  $40^\circ$  weighs a metric ton (1000 kg.).

#### Exercise 189

1. One gram is equal to 15.432 gr. Show that 1 kg. is approximately equal to  $2\frac{1}{5}$  lb. Avoir.

2. A cubic meter of distilled water at  $40^\circ$  weighs how many kg.? If 1 kg. is equal to  $2\frac{1}{5}$  lb. Avoir., how many pounds does 1 c. m. of water weigh?

3. Show that a metric ton weighs about 10% more than our short ton.

4. If sulphuric acid is 1.8 times as heavy as water, what weight of the acid will a two-liter bottle contain?

5. What part of a liter is 750 g. of water?

6. What is the weight of 1 deciliter of water?

7. If alcohol is 80% as heavy as water, what will 375 c. cm. of alcohol weigh?

8. If 20 c. cm. of lead weighs 227 g., what is the ratio of the weight of lead to that of an equal volume of water?

9. If a quantity of iron weighs 7.8 times as much as an equal quantity of water, what is the weight of an iron bar  $75 \times 4 \times 3$  cm.?

10. A body weighing 512 g. in air weighs 428 g. in water. What per cent of its weight is lost?

11. A liter flask was two-fifths filled with water; the remaining space being filled with sand, the weight was found to be 2050 g. Required the weight of a liter of sand.

12. If the pressure of the air on the surface of a body is 1 kg. to the q. cm., what is the pressure of the air on the surface of a sphere whose radius is 10 cm.?

13. A cubical block of ice measures 3 dm. along its edge. What will be its weight if ice weighs 94% as much as an equal volume of water?

14. What is the weight of air in a room 5 m. long, 3 m. wide, 4 m. high, if 1 c. dm. of air weighs .0018 kg.?

## CHAPTER XIX

### Miscellaneous Exercise 190

1. Write the following in figures :

- (a) Fifty thousand nine hundred and nine.
- (b) Nine hundred thousand and ninety.
- (c) Six hundred and fifty thousand seven hundred.
- (d) Eight hundred and seven thousand and eight.
- (e) Seven hundred and seventy thousand and sixty-seven.
- (f) Nine million ninety thousand and ninety-nine.
- (g) Eighty million nine hundred thousand and thirty.
- (h) Nine hundred and seventy million eight hundred and eighty-seven thousand.
- (i) Six hundred and seventeen million and ninety-three.
- (j) Nine hundred and nineteen thousand four hundred and eleven.

2. Write in figures :

Twenty-five thousand four hundred and ninety; ninety-nine thousand nine hundred and seventeen; nine hundred and seven thousand six hundred and six; one million; MDCCCXCV. And in words: 9009; 16,060; 7018; 207,509; 75,115.

3. (a) Define and give examples of quantity, unit, and number.  
(b) Explain the basis of our system of numeration.

4. Write in figures (placed for addition): Nine hundred and nineteen; three hundred and eleven; seven hundred and seventy; eight hundred and ninety-seven; six hundred and eight; XCVII.; LXVII.; CXIX.; CDL.; and DCXL.

5. Add:

4567890123  
 5678901234  
 6789012345  
 7890123456  
 8901234567  
 0912345678  
 6598695326  
 8396876549  
 7788995566  
3453453456

6. Write down neatly the following statement of six weeks' cash receipts; add the amounts vertically and horizontally, and prove the correctness of the work by adding your results:

	MON.	TUES.	WED.	THUR.	FRI.	SAT.	TOTAL
1st	\$ 28.79	\$ 34.71	\$ 35.33	\$ 30.10	\$ 27.97	\$ 47.81	
2d	23.87	30.03	29.38	33.84	26.77	48.77	
3d	16.99	27.09	28.77	30.16	24.95	43.07	
4th	29.13	33.72	30.81	39.17	28.47	50.05	
5th	18.47	32.29	26.73	34.45	28.88	54.39	
6th	19.02	27.06	29.04	29.89	29.51	61.93	
Total							

7. Solve, as in question 6:

	MON.	TUES.	WED.	THUR.	FRI.	SAT.	TOTAL
1st	\$ 65.95	\$ 24.89	\$ 79.79	\$ 40.78	\$ 37.59	\$ 89.61	
2d	58.71	41.65	24.67	94.26	70.26	42.51	
3d	47.58	99.57	50.60	80.71	91.82	89.76	
4th	29.69	70.80	87.91	74.93	36.63	21.90	
5th	81.45	56.93	54.82	96.57	12.72	96.67	
6th	42.63	68.77	81.79	60.86	31.87	75.82	
Total							

8. Mr. Jones bought one house for \$865 and another for \$984, and sold them both for \$1900. How much did he gain?

9. John has \$149, Will has \$87 more than John, and Sam has \$115 more than both. How many dollars has Sam?

10. Ned sold two bags of potatoes. With the money he got for them he bought a knife for 25¢, a saw for 65¢, a hatchet for 75¢, and had 45¢ left. How much did he get for his potatoes?

11. Two men together receive \$97.75, but one receives \$18.25 more than the other. How much does each receive?

12. A and B start together and walk in the same direction, A at the rate of 4 mi. an hr., and B at the rate of 3 mi. an hr. At the end of 7 hr. A turns and goes back. How many mi. will B have gone when he meets A?

13. In a factory, 12 men, 16 women, and 30 boys are employed. At the end of a week they receive \$330. A man is paid as much as 2 women, and a woman as much as 3 boys. What is the share of each?

14. A man bought a number of cows for \$1080; he sold half of them for \$810, thereby gaining \$15 on each one sold. What did each cow cost?

15. A clerk received a salary of \$650 a yr. He spent 50¢ a da. the first yr., \$4 a wk. the second yr., and \$22 a mo. the third yr. How much did he save in three years?

16. The subtrahend is 9564, the remainder is 1965. What is the minuend? The multiplier is 96 and the product is 82,848. What is the multiplicand?

17. The dividend is 1800, the quotient is 17, and the remainder 66. What is the divisor?

18. How many times can 506 be subtracted from the product of 6072 and 13,986?

19. The quotient of a division is 834. What quotient would have been obtained if both dividend and divisor had been first multiplied by 13? Why?

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20. Subtract  $847\frac{1}{2}$  from  $1003\frac{5}{12}$ , explaining fully each step. ✓
21. Simplify  $\frac{1}{2} - \frac{2}{3}$  of  $\frac{5}{8} + \frac{7}{8}$ , and find how many times the result is contained in  $\frac{3}{8} \div (\frac{3}{9} \text{ of } \frac{3}{14} - \frac{1}{8})$ .
22. Divide the sum of  $\frac{2}{3}$  of  $8\frac{1}{3}$  and  $2\frac{1}{7}$  of  $5\frac{5}{6}$  by the difference between  $\frac{3}{4}$  of  $3\frac{1}{2}$  and  $\frac{1}{2}$  of  $\frac{1}{3}$  of  $2\frac{2}{3}$ . ✓
23. Prove: (1)  $\frac{2}{3}$  of  $\frac{2}{5} = \frac{4}{15}$ ; (2)  $\frac{2}{5}$  of  $\frac{2}{3} = \frac{2}{3}$  of  $\frac{2}{5}$ .
24. Simplify  $3\frac{1}{2} + \frac{2\frac{1}{2} - \frac{1}{8} - \frac{7}{10}}{2\frac{1}{2} + \frac{1}{8}}$  of  $3\frac{1}{3} - \frac{5}{6}$ .
25. A boy's age now is  $\frac{1}{5}$  of his father's. In 6 yr. it will be  $\frac{1}{3}$  his father's present age. How old is he? ✓
26. A house and lot are together worth \$2100;  $\frac{1}{4}$  of the value of the house is equal to  $\frac{1}{3}$  of the value of the lot. Find the value of each. ✓
27. The circumference of a wheel is  $2\frac{2}{7}$  of its diameter. Find the diameter of a wagon wheel which makes 360 revolutions in going a mile.
28. A man owned a  $\frac{3}{5}$ -interest in a mill, and sold  $\frac{2}{7}$  of his interest to one man, and  $\frac{1}{5}$  of his interest to another. What part of the mill did each of the three men then own?
29. If to a certain number its  $\frac{1}{2}$ ,  $\frac{1}{3}$ , and  $\frac{1}{5}$  be added, the sum will be 122; required the number.
30. Find the number which is 207 more than the sum of  $\frac{1}{5}$  and  $\frac{1}{7}$  of itself.
31. A man spent  $\frac{5}{9}$  of his money for a house,  $\frac{2}{7}$  of the remainder for cattle, and the rest for a farm. If the farm cost him \$357 less than the house and cattle together, what did he pay for all?
32. A legacy of \$9500 is to be divided among A, B, and C, so that A will get  $\frac{5}{9}$  of the whole, and B will get  $\frac{2}{3}$  as much as C. Find the shares of each.
33. A man spent  $\frac{2}{7}$  of his money for provisions,  $\frac{5}{8}$  of the remainder for clothing,  $\frac{2}{15}$  of the remainder for charity, and had \$9.10 left. How much did he have at first? ✓



34. Nathan Curd sells a merchant 752 lb. of cheese at  $11\frac{3}{4}\text{¢}$  per lb., and receives the following goods in exchange:

11 yd. silk @ \$2.25;	96 lb. nails @ $3\frac{5}{8}\text{¢}$ ;
400 lb. sugar @ $4\frac{7}{8}\text{¢}$ ;	56 yd. gray cotton @ $9\frac{3}{8}\text{¢}$ ;
12 lb. raisins @ $11\text{¢}$ ;	11 yd. white cotton @ $10\text{¢}$ ;
3 pr. gloves @ $75\text{¢}$ .	

Find the balance due Nathan Curd.

35. A man owns a horse and saddle;  $\frac{1}{4}$  of the value of the horse is equal to 4 times the value of the saddle; the horse and saddle together are worth \$170. Find the value of each.

36. A man bought a horse and carriage for \$280, and  $\frac{2}{3}$  of the cost of the carriage was equal to  $\frac{2}{3}$  of the cost of the horse. What was the cost of each?

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37. Divide the product of .037 and .0025 by the sum of .9, .02, and .005.

38. Divide 6 by .000725, correct to four decimal places.

39. Add together 1.302, 3.2589, and 40.93. Multiply the sum by .00297 and divide the product by 90.09.

✓ 40. Multiply 350.4 by .0105 and divide the product by .0000219.

41. What decimal must be taken from the sum of  $69\frac{1}{6}$ , 8.2, 5.445, .065, and  $20\frac{1}{12}$ , so that it will contain 6.05 an exact number of times?

✓ 42. A drover lost .065 of his flock by wolves, .105 by disease, and .27 by theft. He then sold .75 of what remained, and has 280 sheep left. Find the number in his original flock.

43. Find the amount of the following bill:

1328 ft. siding, at \$1.62 $\frac{1}{2}$ per C.;
48,480 cu. ft. timber, at \$59.37 $\frac{1}{2}$ per M.;
7400 fence rails, at \$7.75 per C.;
8400 fence pickets, at \$15.00 per M.;
5680 lb. hay, at \$12.50 per T.

44. A cooper paid \$78.32 for 16,488 bbl. staves. Required the price per M.

45. A rectangular field is 7 ch. 75 l. long and 4 ch.  $87\frac{1}{2}$  l. wide.  $\times$   
How many rd. of fencing are required to enclose it?

46. How many mi. of road, 3 rd. wide will contain 8 A. of land?  $\checkmark$

47. Make a drawing that will show the number of sq. yd. in a sq. rd.

48. Find the value of a piece of land 20 ft.  $\times$  40 rd., at \$1000 per A.

49. A certain map is drawn on a scale of 8 mi. to an inch. On this map the township of Scott measures  $1\frac{5}{16}$  in. in length and  $1\frac{1}{8}$  in. in width. How many A. does it contain?

50. Find the expense of sodding a plot of ground which is 40 yd. long and 100 ft. wide, with sods each 1 yd. in length and 1 ft. in breadth, the sods, when laid, costing 75¢ per hundred.

51. A floor 16 ft. 8 in. by 14 ft. 2 in. is to be laid with square tiles. Find the dimensions of the largest tiles that can be used without cutting or fitting.

52. Find the cost of papering a room <sup>shingles</sup> 24 ft. long, 21 ft. wide, 12 ft. high, at 25¢ a roll, 12 yd. long and 21 in. wide.  $\checkmark$

53. How much will it cost to plaster the walls and ceiling of a room 15 ft. long, 12 ft. wide, and 11 ft. high, at  $32\frac{1}{2}$ ¢ per sq. yd.  $\checkmark$

54. A room 18 ft. by 16 ft. is carpeted with carpet  $\frac{3}{4}$  yd. wide, and the smallest possible number of yd. of the carpet is used. Find (a) the number of breadths, (b) the number of yd.

55. How many thousand shingles, 18 in. long and 4 in. wide, lying  $\frac{1}{4}$  to the weather, are required to shingle the roof of a build-

ing 54 ft. long, with rafters 22 ft. long, the first row of shingles being double ?

56. A schoolroom is 30 ft. long, 24 ft. wide, and 10 ft. high above the wainscoting. The trustees pay \$20 per M. for a new floor, \$15 per M. for a new board ceiling, 10¢ per sq. yd. for painting the ceiling, 4¢ per sq. yd. for tinting the walls, and \$2 per da. for 6 da. labor. Find the total cost.

57. A cubical cistern is 5 ft. deep. How many gal. of water will it hold if 231 cu. in. make a gal. ?

58. How many cubical blocks, each edge of which is  $\frac{1}{3}$  ft., are equivalent to a block of wood 8 ft. long, 4 ft. wide, and 2 ft. thick ?

59. If the ceiling of a square room is 15 ft. high, how many sq. ft. of floor must it have in order that 50 pupils and the teacher may each have 300 cu. ft. of air ?

60. Four-foot wood piled  $5\frac{1}{2}$  ft. high requires how many ft. in length of the pile for  $2\frac{1}{2}$  cd. ?

61. What is the value of a pile of wood 360 ft. long, 12 ft. wide, 6 ft. high, at \$3.20 per cd. ?

62. A square plot of ground that contains  $\frac{9}{40}$  A. is covered with cordwood (4 ft. long) to an average height of 12 ft. What is the wood worth at \$4.12 per cd. ?

63. Required the cost of 35 pieces of scantling, 18 ft. long, 4 in. wide, and 2 in. thick, at \$14 per M., board measure.

64. How many board feet are there in 12 scantlings 16 ft. by 4 in. by 2 in. ?

65. It is required to build a sidewalk  $\frac{1}{4}$  mi. in length, 8 ft. wide, and 2 in. thick, supported by three continuous lines of scantling 4 in. square. What will the lumber cost at \$17 per M. ?

66. Find the value of the following lumber at \$ 15 per M. :

20 pieces  $2 \times 4$ , 18 ft. long;

20 pieces  $4 \times 4$ , 12 ft. long;

20 pieces  $3 \times 10$ , 16 ft. long.

67. A farmer sold a lot of barley, weighing 2712 lb., when barley was 40¢ per bu. In weighing the grain, the dealer made a mistake and took it as rye, and paid for it at 49¢ per bu. How much did the farmer gain or lose by the mistake ?

68. The weight of a cu. ft. of water is  $62\frac{1}{2}$  lb., and 1 gal. contains 231 cu. in. Find the weight in oz. of 1 pt. of water.

69. A lake whose area is 45 A. is covered with ice 3 in. thick. Find the weight of the ice in T., if 1 cu. ft. weighs 920 oz. Avoir.

70. In what time would a field, 80 by 60 rd., pay for under-draining lengthwise, at 2¢ per ft., if the field yield 2 bu., at 66¢, per A. more than before draining ? The drains are 4 rd. apart, and the first drain runs down the centre of the field.

71. Find the amount of the following bill :

June 1, 1896, G. Murray & Co. sold to John Scott, 4886 bu. 36 lb. wheat @ 58¢ per bu., 4532 lb. peas @ 52¢ per bu., 38 bu. 3 pk. barley @ 54¢ per bu., 465 lb. flour @ \$ 1.50 per cwt., 4685 lb. bran @ \$ 15 per T. Write out a receipt in full for payment of account, June 26.

72. Find the length of the shortest line that can be exactly measured by a yard measure, a ten-foot pole, or a two-rod chain.

73. Required the cost of 1 doz. silver spoons, each weighing 18 pwt. 18 gr., at \$ 1.15 per oz.

74. Reduce 7 gal. 3 qt. 1 pt. to the fraction of a bbl.

75. I sow 11 bu. 2 pk. 4 qt. of wheat, and raise therefrom 215 bu. 2 qt. How much is the average yield per bu. of seed ?

76. The running time of the Empire State Express from New York to Buffalo is 8 hr. 30 min., and the distance is 440 mi. If stops of 5 min. each are made at Albany, Utica, Syracuse, and Rochester, what is its average speed per hr.?

77. Find (a) the exact number of da. from Jan. 17, 1896, to April 5, 1896; (b) the difference in time by subtraction of dates.

78. A railroad train moves 1 mi. in 65 sec. What is its speed per hr.?

79. A note given Aug. 15, 1896, for 90 da., will mature when?

80. A can walk  $3\frac{1}{3}$  mi. in 50 min., and B can walk  $2\frac{1}{4}$  mi. in 36 min. How many yd. will A be ahead of B when A has gone 6 mi., if they start together?

81. A farmer delivered at a warehouse four loads of wheat weighing respectively 2113 lb., 2310 lb., 2270 lb., and 2091 lb. How much should he have received at 72¢ per bu.?

82. The difference in longitude between two places being  $9^{\circ} 34' 25''$ , what is the difference in time?

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83. A man has a salary of \$400 a yr., and has \$500 in the bank. If he spends \$500 a yr., in what time will his money be all gone?

84. What is the shortest stick that can be cut into pieces, 9 in., 12 in., or 15 in. in length, with nothing remaining?

85. (a) What is meant by a Common Multiple of two or more fractions?

(b) Find the L. C. M. of  $2\frac{1}{4}$ ,  $3\frac{3}{8}$ ,  $3\frac{9}{32}$ ,  $14\frac{9}{14}$ .

86. (a) What is meant by the prime factors of a number?

(b) Find the prime factors of 13,230, 22,050, and 23,625; and

(c) By means of the prime factors find their G. C. M. and L. C. M.

87. Resolve 16,335 and 18,018 into their prime factors, and from inspection of these write the prime factors of their (a) L. C. M. and (b) G. C. M.

88. A farmer bought a number of horses and cows for \$2000. There were 3 times as many cows as horses, and a horse costs twice as much as a cow. If each horse cost \$80, how many cows did he buy?

89. The difference in weight of two chests of tea is 25 lb.; the value of both at 65¢ per lb. is \$113.75. How many lb. of tea are in each chest?

90. What is the smallest sum of money with which you can buy chickens at 25¢, or geese at 50¢, or turkeys at 75¢, or lambs at \$3, or sheep at \$5, or pigs at \$7, or cows at \$35, or horses at \$140, and have exactly \$15 left for expenses?

91. Ten cents will buy 3 oranges, 4 lemons, or 5 apples. How many apples are worth as much as 5 doz. oranges and 7 doz. lemons?

92. One workman charges \$3 for a day's work of 8 hr., and another \$3.50 for a day's work of 9 hr. Which had I better employ, and how much shall I have to pay him for work that he can do in a fortnight, working 6 hr. a day?

93. A can do a piece of work in  $\frac{2}{3}$  of a da., and B in  $\frac{1}{2}$  of a da. In what time can both together do it? If \$1.40 be paid for the work, how much should A receive?

94. How many oranges must a boy buy and sell to make a profit of \$9.30, if he buys at the rate of 5 for 3¢, and sells at the rate of 4 for 3¢?

95. A and B dig a ditch in 50 hr. With C's help they could have done it in  $18\frac{3}{4}$  hr. In what time could C do  $\frac{2}{3}$  of the work alone?

96. Three men can dig a certain drain in 8 da. They work at it for 5 da., when one of them falls ill, and the other two finish

the work in 5 da. more. How much of the work did the first man do before he fell ill?

97. A boy can run 6 times around a circular plot of ground in 52 sec.; another boy can run 9 times around the same plot in 80 sec. If they start from the same place at the same time, and run in the same direction, how many rounds will each make before the faster boy overtakes the slower?

98. Express in the form of a vulgar fraction the average of  $\frac{3}{8}$ ,  $\frac{3}{14}$ , .7,  $.4\frac{1}{5}$ , and  $.486\frac{1}{5}$ .

99. In a granary there are 4 bins, each 10 ft. long and 5 ft. wide. How high must they be boarded in front to be capable of holding 860 bu.?

100. The outfit of a livery stable is worth \$3000;  $\frac{1}{7}$  the value of the horses is equal to  $\frac{1}{5}$  the value of vehicles, harness, etc. Find the value of the horses.

101. A farmer agreed to pay his hired man 10 sheep and \$160 for 1 yr. labor. The man quit work at the end of 7 mo., receiving the sheep and \$60 as a fair settlement. Find the value of each sheep.

102. Divide \$1200 among A, B, and C, so that A may have \$70 more than B, and twice as much as C.

103. A train going 25 mi. an hr. starts at 1 o'clock P.M. on a trip of 280 mi.; another going 37 mi. an hr. starts for the same place at 12 min. past 4 o'clock P.M. When and where will the former be overtaken?

104. In the number, 28,672, the value expressed by the first two digits from the left is how many times the value expressed by the fourth digit from the left?

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V 105. A town whose population was 10,000 increased 10% every year for 3 yr. What was its population at the end of that period?

106. A house and lot was sold for \$7030, at a loss of  $16\frac{1}{3}\%$  of its cost. Find the cost.

107. Five men in a factory accomplish as much as 8 boys. What per cent of a man's work does a boy do? What per cent of a boy's work does a man do?

108. Forty-five per cent of a carload of melons were sold to one dealer, and  $33\frac{1}{3}\%$  of those left to another. How many were there in the car before any were sold, if after the second sale there remained 110?

109. In a certain school 48% of the pupils are boys, and there are 39 girls. Find the number of boys.

110. How many lb. of flour will be required to make 1000 lb. of bread, if the bread weigh 30% more than the flour used?

111. 93 lb. 6 oz. is what per cent of 43 lb. 12 oz.?

112. Water, in freezing, expands 10%. If 1 cu. ft. of water weighs 1000 oz., find the weight of 1 cu. ft. of ice.

113. Give answers to the following:

(a)  $15\frac{3}{4}\%$  of 660 =

(d) .2% of 40 =

(b) 660 is  $15\frac{3}{4}\%$  of what number? (e) 40 is .2% of what number?

(c)  $\frac{3}{7}$  is what per cent of  $\frac{3}{8}$ ?

(f) What per cent of itself must be added to a number so that the sum diminished by 10% of itself may be 17% more than the original number?

114. Brooms are bought wholesale at \$20 a gross. What per cent profit will be made by selling them at 20¢ each?

115. A merchant purchases sugar at \$4.50 per cwt. At what price per lb. must he sell it in order to gain  $5\frac{5}{8}\%$ ?

116. I bought a house for \$4000 and spent 40% of the cost in repairs. What must I rent it for a month in order to make a clear gain of 5% of the total cost, taxes and repairs amounting to \$72 yearly?



117. By selling a piano for \$260, a dealer loses 20%. How much should he have sold it for to gain 5%?

118. A man having lost 20% of his capital is worth exactly as much as another who has just gained 15% on his capital. The second man's capital was originally \$9000. What was the first man's capital?

119. A dealer sold an article for \$8.10 and lost 10%. At what selling price would he have gained 10%?

120. A bookseller deducts 10% from the market price of his books, and after this has a gain of 25%. He sells a book for \$7.20. Find the cost price of the book, and what per cent the marked price is in advance of the cost price.

121. A merchant bought 1000 yd. of carpet at 60¢ a yard, and sold  $\frac{2}{3}$  of it at a profit of 30%,  $\frac{1}{3}$  at a profit of 20%, and the rest at a loss of 20%. How much did he receive for the carpet?

122. A sells goods to B at a gain of 12%, and B sells the same goods to C at a gain of  $7\frac{1}{2}\%$ . C paid \$3762.50 for the goods. How much did A pay for them?

123. A machinist sold two seed-drills for equal sums of money. He gained 25% on the one and lost 25% on the other. His total loss was \$9.60. Find the cost of each drill.

124. R purchased a house and lot for \$3300, paid \$975 for repairs, and now rents the premises for \$30 a month. If he expends annually for taxes \$48.70, and for incidental repairs \$35, what is his per cent of annual income on his investment?

125. A merchant closed out a stock of cloaks for \$311.04, at a loss of 28%. Required the loss by the transaction.

126. By selling my cloth at \$1.26 per yd. I gain 11¢ more than I lose by selling it at \$1.05 per yd. What would I gain by selling 800 yd. at \$1.40 per yd?

127. A merchant marks his goods at 40% in advance of cost, and in selling uses a lb. weight  $\frac{1}{8}$  oz. too light. If he throws off 10% of his marked price, find his gain per cent.

128. State the relation between 1 lb. Troy and 1 lb. Avoir. What is the gain per cent when the selling price per oz. Avoir. is the same as the cost per oz. Troy?

129. A man bought a bankrupt stock at 60¢ on the dollar of the invoice price, which was \$4840. He sold half of it at 10% advance on invoice price, half the remainder at 20% below invoice price, and the balance at 50% of invoice price. His expenses were 10% of his investment. Find his loss or gain (a) in money, and (b) in rate per cent.

130. The list price of an article is \$150. If trade discounts of 25% and 16 $\frac{2}{3}$ % are allowed, what is the net price?

131. If a dealer buys stoves at a discount of 22% from list price, and sells them at list price, what is his per cent of gross profit on the investment?

132. Required the net price of an article listed at \$400, 30%, 10%, and 5% off.

133. From the list price of a line of goods a purchaser is allowed a trade discount of 20%; a further discount of 10% off the trade price for taking a quantity, and a still further discount of 5% off his bill for cash. Find his gain per cent by selling at 10% less than the list price.

134. The net price of a reaper is \$158.40, and the trade discounts allowed are 20% and 10%. Find the list price.

135. A commission merchant sold coffee for me and remitted \$1960, after deducting his commission of 2%. What is the value of the coffee?

136. If an agent receives \$1092 to buy pork, how many lb., at 6 $\frac{1}{2}$ ¢ per lb., can he buy and retain his commission of 5% for buying?

137. A commission merchant sold 1014 bu. of oats, at 41¢ per bu., paid \$33.74 freight charges, and retained 3 $\frac{1}{2}$ % commission. How much should he remit to the consignor?

138. A lad earned \$21.16 collecting accounts for a physician. He was allowed  $5\frac{3}{4}\%$ . What amount did he collect?

139. Find the premium paid to insure a house worth \$7500 for  $\frac{2}{3}$  of its value for 3 yr., the rate being  $\frac{3}{8}\%$  of the policy for each year.

140. What premium must be paid to insure a cargo of 4880 bu. of wheat, valued at 78¢ per bu., at  $1\frac{1}{2}\%$ , the policy being for only  $\frac{5}{8}$  of its value?

141. A building is insured for \$400 more than  $\frac{5}{8}$  of its cost at 4%. If destroyed, the loss will be \$216. Find the cost of the building.

142. A dealer shipped 200 bbl. of apples to Liverpool; the average cost of the apples was \$3.75 per bbl. For what sum must he have the apples insured at  $\frac{3}{4}\%$  premium to guard against all loss, in case of shipwreck, his other expenses being \$75?

143. If in a certain town \$3093.75 was raised from a  $\frac{3}{4}\%$  tax, what was the assessed valuation of the property in the town?

144. A tax of \$24,750 is levied on a town, the assessed valuation being 1.5 mills on a dollar. What tax does a man pay on an income of \$1100, of which \$400 is exempted?

145. A farmer whose property is assessed at \$9600 pays on the dollar,  $1\frac{3}{4}$  mills for township rates,  $1\frac{1}{4}$  for county rates,  $1\frac{1}{2}$  for railway bonus, and  $2\frac{1}{2}$  for school rate. How much does he pay in all?

146. B's tax was \$86.2755 when the rate was 7.635 mills on a dollar. What was the assessed valuation of his property?

147. A certain school section is assessed for \$150,000. The trustees have built a schoolhouse costing \$1800.

(a) What will the schoolhouse cost a ratepayer whose property is assessed for \$4500?

(b) What would be the rate of taxation per annum on the whole section if the house were paid for in six equal annual payments, without interest?

148. A clerk pays \$7.50 taxes on his salary. What is his total salary if \$400 of it is exempt from taxation and a  $2\frac{1}{2}\%$  rate is levied on the remainder?

149. What per cent must be assessed on \$1,500,000 to produce \$29,400 after paying 2% for collecting?

150. An importer receives an invoice of kid gloves billed at \$680, pays a duty of 50% ad valorem, and sells them at an advance of  $33\frac{1}{3}\%$  on their gross cost to him. How does the price paid by the purchaser compare with the exporter's price?

151. A merchant imports 75 cases of indigo, gross weight 196 lb. each, allowing 15% for tare. What was the duty at 5¢ per lb.?

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152. What will \$1 amount to in 3 yr. 216 da., at  $7\frac{1}{2}\%$  per annum, simple interest?

153. Find the simple interest on \$597.50 for 2 yr. 5 mo. 12 da., at 8% per annum.

154. How long will it take \$450, at 8%, to yield \$21.30 interest?

155. What amount will be due July 1, 1896, on a note of \$80, drawn Feb. 6, 1896, and bearing interest at  $5\frac{1}{4}\%$  per annum, exact interest?

156. Find the sum due Sept. 2, 1893, on a note for \$147.33, given Jan. 13, 1893, and bearing interest at 4% per annum.

157. Find the exact interest on \$225 from July 13, 1893, to Sept. 3, 1893, at 6%.

158. Find the interest on \$1, at  $7\frac{1}{2}\%$  per annum, from Jan. 1, 1895, to June 3, 1895. (Complete answer required.)

159. What sum will amount to \$354.09 in 7 mo., at 3% per annum?

160. In what time will \$1350 earn \$31.88 at 5% per annum?

161. Find the face of a draft that cost \$434.70, at  $\frac{5}{8}\%$  premium.

162. If the interest is \$12.57, the time 8 mo. 2 da., and the rate per annum  $5\frac{1}{2}\%$ , what is the principal?

163. Find the exact interest on \$150 from July 16 to Dec. 9, at  $5\%$  per annum.

164. A person borrows money for 6 yr. at  $3\frac{1}{2}\%$ , simple interest, and repays at the end of the time, as principal and interest, \$847. How much did he borrow?

165. Find the simple interest on \$912.50, at  $8\%$ , from Feb. 13, 1895, to Dec. 19, 1896.

166. A note of \$360, drawn April 20, 1895, is paid July 2, 1896, with interest at  $7\frac{1}{2}\%$  per annum. Find the amount paid, simple interest.

167. Oct. 15, 1895, a young man deposited in the savings bank the sum of \$860.75. May 20, 1896, he withdrew the principal and simple interest at  $4\%$  per annum. What amount did he withdraw?

168. Bought a horse for \$160, and gave in payment my note dated Aug. 15, 1896, with interest at  $7\frac{1}{2}\%$  per annum until paid. Jan. 9, 1897, I sold the horse for \$200 cash, and paid my note. What was my net gain?

169. If for \$7 I can have the use of \$35 for 3 yr. 4 mo., how much a month shall I have to pay for the use of \$8750?

170. Jan. 1, 1894, a person borrowed \$2417.50 at  $6\frac{3}{4}\%$ , simple interest, promising to return it as soon as it amounted to \$2582.50. On what day did the loan expire? (365 da. = 1 yr.)

171. March 1, 1896, a storekeeper bought goods amounting, at catalogue prices, to \$840, on which he was allowed successive discounts of  $33\frac{1}{3}\%$  and  $5\%$ . The account is payable in 60 da., after which time interest is to be charged at  $7\%$  per annum. June 1, 1896, he paid \$100. How much is due July 1, 1896?

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172. Find the proceeds of a note for \$200 given at Albany, N.Y., for 3 mo., and discounted at bank the day it was made at 6%.

173. Find the proceeds of a note for \$168 due Oct. 20, 1896, and discounted Sept. 25, 1896, at a Brooklyn, N.Y., bank, at 6% per annum.

174. \$1234  $\frac{50}{100}$ .

St. Louis, Jan. 15, 1894.

Ninety days after date, I promise to pay A. Bee, or order, the sum of one thousand two hundred and thirty-four  $\frac{50}{100}$  dollars, at the Bank of Commerce here. Value received.

C. DEE.

This note was discounted Feb. 10, 1894, at 6% per annum. Find the proceeds.

175. A note for \$230, drawn Jan. 2, 1896, at 3 mo., and bearing interest at 8% per annum, is discounted Feb. 1 at 7%. Find the proceeds.

176. Find the proceeds of the following note :

\$2400.

HAMILTON, OHIO, Feb. 3, 1896.

Five months after date, value received, I promise to pay Thomas Cowan, or order, the sum of two thousand four hundred dollars, at the Bank of Hamilton, with interest at 6% per annum.

VANCE ALLEN.

Discounted May 22, 1896, at 7%.

177. The discount on a note for \$3600, which matured April 21, 1896, and was discounted Feb. 24, 1896, was \$45.60. Find the rate of discount.

178. A buys 600 yd. of silk at 95¢ per yd., and sells it at once, receiving in payment a 90-day note for \$700, which he at once discounts at a bank at 6% per annum. Find the gain.

179. For what sum must a note be drawn June 1, 1896, payable in 90 da., so that when discounted June 14, at 8%, the proceeds will be \$717.20?

180. What rate of interest is made by a bank which discounts a 90-day note at 6% per annum?

181. Jan. 1, A owes a bank \$15,000. He offers for discount certain notes: \$2500 due Feb. 15, \$3700 due March 13, and \$7500 due April 1. If these are discounted at 8% per annum, how much cash must he pay?

182. What must be the face of a note so that when discounted at a bank for 90 da. at 6%, the proceeds will be \$1969?

183. What is the present worth of a note for \$540, due in 90 da., drawing interest at 6%, discounted at 8%, true discount?

184. Find the proceeds of a note for \$292.73, discounted at bank, for 35 da., at 6% per annum, exact interest method.

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185. Find the value of  $(1.03)^4$ .

186. A man has the choice of loaning his money at  $7\frac{1}{2}\%$ , compound interest, or at 8%, simple interest, money and interest to be paid at end of 3 yr. Show which is the better investment.

187. An annual deposit of \$250 is made with a loan company which pays 4% per annum on deposits, compounded half-yearly. Find the amount of all these deposits when the fourth has been made.

188. June 30, 1890, I borrow \$16.50, to be returned April 30, 1892. With compound interest at  $6\frac{1}{2}\%$ , what amount must I then pay?

189. A man puts \$350 in a savings bank each year, making his first deposit Dec. 31, 1893. How much will there be to his credit Jan. 1, 1897, the bank adding 4% per annum?

190. A owes B \$400 due in 1 yr., \$300 due in 2 yr., \$200 due in 3 yr. What sum paid now would cancel the debt, money being worth 5% per annum, compound interest?

191. A lent a sum of money for 2 yr., at 10% per annum, interest compounded yearly. B lent an equal sum for the same time at 10% per annum, interest compounded half-yearly. B gained \$220.25 more than A. Find the sum each lent.

192. A man rents a farm for two years at \$441 per annum, the rent for any year being supposed to be paid at end of that year. Money being worth 5% per annum, compound interest, find what sum would, in advance, pay the two years' rent.

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193. I invest \$39,900 in 6 per cents at 95. What is my income?

(a) What sum invested in 8% bonds at  $33\frac{1}{3}\%$  premium will yield an income of \$1200?

(b) What if the bonds were at  $33\frac{1}{3}\%$  discount?

194. Find the cost of 64 shares of railroad stock at  $107\frac{3}{4}$ , brokerage  $\frac{1}{8}\%$ .

195. Required the gain on 28 shares of stock bought at  $97\frac{1}{2}$  and sold at  $103\frac{1}{4}$ .

196. April 4, 1896, 20 shares of Chicago City Railway stock, quoted at 216, were sold, brokerage  $\frac{1}{8}\%$ . Find what was received by the owner of the stock.

197. A railroad company declared a dividend of  $1\frac{1}{4}\%$  for the quarter ending Sept. 30, 1894. If the stock was quoted at 105, what was the rate of income per annum on an investment in the stock of that company?

198. How much must be invested in U. S. 5's, at  $113\frac{1}{2}$ , to secure an annual income of \$175?

199. A man invests \$12,000 in 3% stock at 75. He sells out at 80 and invests  $\frac{1}{3}$  of the proceeds in  $3\frac{1}{2}\%$  stock at 96, and the remainder at 5% par. Find the change in his income.

200. A man owned \$8940 bank stock, which paid a yearly dividend of  $4\frac{1}{2}\%$ . He sold out  $102\frac{3}{8}$ , and invested the proceeds



in Michigan Central stock at  $74\frac{3}{8}$ , paying a yearly dividend of 3%. By how much was his yearly income changed by the transfer?

201. What must be the market value of 6% stock, so that after paying an income tax of 16 mills on the dollar, it may yield 5% on the investment?

202. A person bought stock at  $95\frac{1}{8}$ , and after receiving a half-yearly dividend of 7% per annum, sold out at  $92\frac{1}{2}$ , brokerage each way being  $\frac{1}{8}\%$ . If his net gain was \$25, how much stock did he buy?

203. If a 5% stock sells at 105, how much must be invested in it to yield a yearly income of \$794, after paying an income tax of 15 mills on the dollar, \$400 of income being exempted from taxation?

204. A man invests \$6000 in 5% stock at 120. At the end of 1 yr., having just received the yearly dividend, he sells at  $121\frac{1}{2}$ . How much better off is he than if he had loaned his money at 5% per annum?

205. I own \$6000 of bank stock, paying an annual dividend of 5%. How much will my annual revenue from the bank stock be reduced by selling enough of it at 72 to pay a note of \$3735 9 mo. before it is due, reckoning true discount at 5% per annum?

206. A man received \$495 as dividend on his bank stock. He sold 40 shares (\$100) at  $143\frac{1}{4}$ , and the remainder at  $144\frac{1}{2}$ , paying  $\frac{1}{8}\%$  brokerage on each transaction. What were the net proceeds of the sales?

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207. \$1200 is to be divided between two persons, A and B, so that A's share is to B's share as 2 to 7.

208. What is the ratio of  $3\frac{7}{8}$  to  $\frac{5}{8}$ ? Answer in per cent.

209. Divide 1026 into four parts that shall be in the ratio of 3, 11, 17, and 23.

210. An upright pole 16 ft. long casts a shadow 5 ft. 4 in. long, and at the same hour the shadow of a tree is found to be 26 ft. 9 in. Required the height of the tree.

211. The sum of three numbers is 940. The first number equals  $\frac{5}{9}$  of the second, and the second equals  $\frac{7}{10}$  of the third. Find the numbers.

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212. If 18 men do  $\frac{2}{3}$  of a piece of work in 30 da. of 10 hr., in what time should 15 men do the whole work, working 9 hr. a da.?

213. If 10 yd. of muslin,  $1\frac{1}{4}$  yd. wide, cost \$1.30, what is the cost of 12 yd.,  $1\frac{1}{8}$  yd. wide?

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214. One-sixth of the square of a certain number is 384. Find the number.

215. Find the square root of .6 correct to three decimal places.

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216. Find, within one inch, the side of a square whose area is 5 A.

217. A rectangular field whose length is  $\frac{4}{3}$  of its width contains 2 A. 112 sq. rd. Find the length of a diagonal.

218. Required the base of a right-angled triangle whose hypotenuse is  $16\frac{1}{4}$  ft., and perpendicular  $9\frac{3}{4}$  ft.

219. A ladder 78 ft. long stands perpendicularly against a building. How far must it be pulled out at the foot that the top may be lowered 6 ft.?

220. A road runs round a circular pond; the outer circumference is 440 yd., and the width of the road is 20 yd. Find the area of the pond.

221. In order to drain a swamp a ditch was dug 1 mi. long, 3 ft. deep, 6 ft. wide, at the surface, and 4 ft. wide at the bottom. Find the total cost at 9¢ per cu. yd.

222. How many gal. in a circular cistern 6 ft. in diameter and 7 ft. deep?

223. The surface of a cube is 432 sq. ft. What is its volume?

224. (a) A circular cistern, 8 ft. in diameter and 9 ft. in depth, is filled with water to the height of 6 ft. How many gal. of water in the cistern? (1 cu. ft. = 7.48 gal.)

(b) If a sphere whose diameter is 4 ft. is submerged in the water in the cistern, how high will it cause the water to rise?

225. How many cd. are there in a cylindrical log 20 ft. long and 3 ft. 6 in. in diameter?

226. Find the diameter of a circle whose area is equal to the sum of the areas of two circles whose diameters are 12 in. and 16 in. respectively.

227. Find the area of the curved surface of a right circular cone the radius of whose base is 3.5 in., and whose altitude is 7 in.

228. A chord of a circle, whose radius is 12 in., subtends a right angle at the centre of the circle. Find the area of the smaller segment cut off by this chord.

229. A spherical shell, internal diameter 14 in., is filled with water. Its contents are poured into a cylindrical vessel whose internal radius is 14 in. Find the depth of the water in the cylinder.

230. The sides of a triangle are 40, 45, and 50 ft. respectively. Find the length of the perpendicular from the vertex to the side 45 ft.

231. The diameter of a circular plate of lead is 13 in. From this is cut out a circular plate of radius 6 in., and the remainder of the lead is moulded into the form of a circular plate, with  $\frac{1}{4}$  of the former thickness. Find the diameter of this plate.

232. The sides of a triangle are 13, 14, and 15 ft. Find its area and the length of the three perpendiculars from the angles on the opposite sides.

233. The external dimensions of a rectangular covered box, made of inch stuff, are 7, 8, and 9 ft. Find the capacity of the box and the quantity of lumber in it.

234. A ball of yarn 3 in. in diameter makes one mitten. How many similar mittens will a ball 6 in. in diameter make?

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235. A farmer employs a number of men and 8 boys; he pays the boys \$.65 and the men \$1.10 per day. The amount that he paid to all was as much as if each had received \$.92 per day. How many men were employed?

236. Two men start from the same point at the same time to walk in the same direction around a block of land  $1\frac{1}{4}$  mi. on each side. A goes at the rate of 4 mi. and B 3 mi. an hr. How far will A walk before he overtakes B?

237. A commission merchant sells a consignment of wheat for \$27,500, on a commission of  $2\frac{1}{2}\%$ . He pays \$250 for freight and storage, and with the net proceeds buys pork at \$6.25 per cwt., charging  $2\frac{1}{2}\%$  for buying. How many cwt. of pork does he buy, and what is the amount of his two commissions?

238. Find the cost of the material required to fence  $2\frac{1}{2}$  mi. of railway (both sides), posts placed 8 ft. apart, an 8-in. base 1 in. thick, a  $2 \times 4$  in. rail at top, and 6 strands of wire. The posts cost  $12\frac{1}{2}\phi$  each, the lumber \$14 per M., and the wire 4¢ per lb. (A lb. of wire stretches 1 rd.)

239. A number of two digits is multiplied by 3, and the product placed to the left of the original number. Show that the number so formed is always exactly divisible by 7.

240. A merchant reduces the marked price of an article by a certain per cent. He gives the same per cent off this reduced price for cash. The cash price is now  $\frac{25}{36}$  of the original marked price. Find the rate per cent.

241. Divide \$916 among A, B, and C so that 5% of A's share may equal  $7\frac{1}{2}\%$  of B's, and  $12\frac{1}{2}\%$  of B's may equal 20% of C's.

242. A starts to walk from P to Q at the rate of 4 mi. an hr., and 1 hr. afterwards B starts from P and overtakes A in 4 hr. Walking on, B arrives at Q 2 hr. before A. Find the distance from P to Q.

243. A number is divisible by 9 if the sum of its digits is divisible by 9. Why?

244. At what two times between 3 and 4 o'clock are the hands of a watch equally distant from the figure III?

A's share  
of C's.

mi. an hr.,  
in 4 hr.  
distance

digits is

e hands